Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2003

Problem Set 1
Due Friday, September 12

Note: Optional recitations will be held in 4246 Randall on Thursdays from 4:00-5:30.

1. Problem 1 from handout.

2. In a cubical resonant cavity, the lowest order TE mode is a 101 mode with $n_x = n_z = 1$ and $n_y = 0$, $k^2 = (n_x^2 + n_y^2 + n_z^2)\pi^2/L^2$ where L is the length of one side of the cube. Take $L = 1.0$ cm and calculate the lowest frequency mode the cavity can support. In experiments involving entangled states and quantum information, atoms in Rydberg (highly excited) states are sent through such cavities and have transitions that are resonant with this field frequency. How cold must the cavity be kept to avoid transitions resulting from thermal noise?

3. Calculate the probability distribution as a function of time for the propagation of a Gaussian wave packet with $\psi(\mathbf{r},0) = \frac{1}{(2\pi\sigma^2)^{3/4}} e^{i\mathbf{k}_0\cdot\mathbf{r}} e^{-r^2/2\sigma^2}$

4-5. The k-space distribution function for a one-dimensional problem is given by $\phi(k) = A$ for $-k_0 \le k \le k_0$ and $\phi(k) = 0$ for $|k| > k_0$.

(a) Plot $|\phi(k)|^2$. Find A such that $|\phi(k)|^2$ is normalized.

(b) Calculate $\psi(x,0)$ and show that $\Delta x \Delta k$ is of order unity. How does $|\psi(x,0)|^2$ differ qualitatively from the result for a Gaussian wave packet? Why?

(c) Write down an integral expression for $\psi(x,t)$ and estimate the times for which spreading of the wave packet becomes appreciable.

(d) Now explicitly evaluate $\psi(x,t)$ in terms of the dimensionless variables $a = (\hbar t/2m)k_0^2$ and $b = k_0 x$. (You can evaluate the integral and do the plotting using a program such as Mathematica.) The answer will be given in terms of the error function, for example

$$\psi(b,a) = \sqrt{k_0}\,\frac{(-1)^{3/4} e^{ib^2/4a}}{4\sqrt{a}} \left\{ \mathrm{Erf}\left(\frac{(-1)^{1/4}[b-2a]}{2\sqrt{a}} \right) - \mathrm{Erf}\left(\frac{(-1)^{1/4}[b+2a]}{2\sqrt{a}} \right) \right\}.$$

(e) Plot $|\psi(b,a)|^2$ as a function of b for values of a ranging from 0 to 20. Show that the width of the wavepacket is of order $4a$ when $a \gg 1$. Interpret this result. Optional: Create a movie of the wave packet propagation

6. Go to the web and find some sights where wave packet propagation is shown. Report on the URL locations, contents of the sights, and what you learned from them. With a very quick search, I found one sight, http://www.nhn.ou.edu/~mason/quantum/quantumapplets.html.