Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2003

Study Guide

These are review questions which can be used as a study guide. They are **not** *questions which will necessarily appear on the exam.*

Recitation on Thursday, December 11, in Room 4246 West Hall from 4-6.

Final Exam: Friday, December 12, 3:00-6:00, Room 340, West Hall

1. State three properties of eigenvalues or eigenvectors of Hermitian operators.

2. Of what use are commuting operators in quantum mechanics?

3. Calculate the average force that a particle exerts on the wall of an infinite one dimensional potential well.

4. Show how one can get the energy levels in a one-dimensional, finite potential well.

5. Write down equations that can be used to get the transmission coefficient for a one-dimensional barrier. Without solving the equations, indicate how you would go about finding the transmission coefficient.

6. The initial state of a particle moving in a one-dimensional simple harmonic potential is $\psi(\xi,0) = 1/\sqrt{6}$ for $|\xi| < 3$ and is zero otherwise. Estimate the maximum number of eigenstates that enter into the expansion of this wave function.

7. A particle moves in a potential $V(x) = \alpha x^4$. For large quantum number n, does the energy grow linearly with n? Explain.

8. Prove that, on average, a particle moving in a potential $V(\mathbf{r})$ obeys Newton's equations of motion.

9. Give some general arguments as to why a symmetry in nature is connected with energy degeneracy of the energy eigenfunctions.

10. Find an integral expression for the eigenfunctions of a particle moving in the potential $V(x) = \alpha x$ for $x > 0$ and $V(x) = \infty$ for $x < 0$.

11. Prove that the wave function for a particle moving in an infinite potential well is periodic, and find its period.

12. The initial state of an harmonic oscillator is $\psi(\xi,0) = (3\xi^2 + 2\xi)e^{-\xi^2/2}$. Find $|\psi(\xi,t)|^2$.

13. As the size of an infinite one dimensional well is decreased, does the energy level spacing increase or decrease? Expalin this result on physical grounds.

14. Calculate the scattering in first Born approximation for a spherically symmetric potential.

15. Consider scattering by a repulsive potential $V = V_0 > 0$ for $r < a$ and $V = 0$ for $r > a$. In the case of low energy scattering, ka<<1 and $E < V_0$, calculate the differential scattering cross section. Use the method of the effective potential to get a qualitative picture of the scattering for all energies.

16. Consider low energy scattering by an arbitrary attractive potential. Obtain an expression for the total cross section in terms of the scattering length.

17. An electron moves in a potential $V = \frac{1}{2}k\rho^2$ (2-D harmonic oscillator). The wave function is a product of the 1-D wave functions, but can also be written in polar coordinates. The energy eigenvalues are $E = (n+1)\hbar\omega$ where $\omega = \sqrt{k/\mu}$ and $n = n_x + n_y$, and $n_x, n_y = 0, 1, 2\ldots$ Find the degeneracy of a state of given n. Solve the problem in polar coordinates.

18. Write expressions for $\hat{x}$ and $\hat{p}$ in the $|x\rangle$ and $|p\rangle$ bases. Starting with the commutation relation $[x,p] = i\hbar$ obtain an expression for $\langle x|\hat{p}|x'\rangle$. How are the $|E\rangle$ and $|x\rangle$ bases related.

19. Compare the predictions of the Bohr theory with the results that follow from the Schrodinger equation.

20. Qualitatively outline the steps that are needed to solve a problem with spherical symmetry. Of what use are the operators L^2 and $\mathbf{L}$ in this procedure?

21. Find $\langle \ell m|L_x^2 + L_y^2|\ell' m'\rangle$. Use raising and lowering operators to evaluate $\langle \ell m|L_x|\ell' m'\rangle$.

22. Use the uncertainty principle to estimate the total scattering cross section for a potential that falls off as r^{-4}. What is the corresponding classical cross section?

23. Obtain eigenvalues and eigenkets of the operator $\begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}$.

24. Why must high energy electrons be used to probe within the proton?

25. Explain the general conditions needed to obtain a useful set of ladder operators for a given Hamiltonian H or operator B.

26. Explain why the dipole moment of a *free* 1-D harmonic oscillator must necessarily be time dependent.

27. Obtain an equation for $d^2\langle \mathbf{r}\rangle/dt^2$ for the electron in the hydrogen atom. Why is it difficult to obtain a solution to this equation?

28. What does it mean to say that the angular momentum operator is the generator of rotations?

29. Estimate the number of partial waves that enter the partial wave expansion for scattering of particles having energy E by a potential having range a.

30. Explain in general terms situations in which the partial wave expansion or the Born approximation are the most efficient ways of solving a scattering problem.