The Equilibrium Effect of Fundamentals on House Prices and Rents

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Abstract

This paper studies the joint dynamics of real house prices and rents. We build a dynamic equilibrium stochastic life cycle model of housing tenure choice with fully specified markets for homeownership and rental properties, and endogenous house prices and rents. Houses are modeled as discrete-size durable goods which provide shelter services, confer access to collateralized borrowing, provide sizable tax advantages, and generate rental income for homeowners who choose to become landlords. Mortgages are available, but home-buyers must satisfy a minimum down payment requirement, and home sales and purchases are subject to lumpy transaction costs. Lower interest rates, relaxed lending standards, and higher incomes are shown to account for about one-half of the increase in the U.S. house price-rent ratio between 1995 and 2006, and to generate the pattern of rapidly growing house prices, sluggish rents, increasing homeownership, and rising household indebtedness observed in the data. The model highlights the importance of accounting for equilibrium interactions between the markets for owned and rented property when analyzing the effect of changes in fundamentals on housing market.

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1. Introduction

The sharp increase and subsequent collapse in U.S. house prices over the past decade has been well documented. While real house prices rose by only 3.7 percent between 1985 and 1995, they increased by 50 percent between 1995 and 2006.[1] In sharp contrast, real rents remained virtually unchanged during the recent increase in house prices, so that in 2006 the house price-rent ratio peaked at approximately forty percent above its level in the year 1995 (Figure 1). Over the same time period, real interest rates reached historically low levels and availability of mortgage credit expanded greatly. Especially because of the significant drop in house prices since 2007, and the role played by the housing market in the Great Recession, there is considerable interest in understanding the driving forces behind changes in house prices and rents.

This paper quantitatively studies the effects of changes in fundamentals, such as the interest rate and required down payment, on endogenously determined house prices and rents using a dynamic equilibrium model of the housing market that features fully specified markets for both homeownership and rental properties. The primary contributions of this paper relative to the existing literature are incorporating household decisions about rental property investment into a model of the housing market, and endogenizing both house prices and rents. Our model is the first model of the housing market that allows rental supply to be determined endogenously by the optimizing investment decisions of heterogenous households, and where two distinct relative prices – rents and house prices – are determined in equilibrium via market clearing.

To study the impact of changes in interest rates and down payment requirements on house prices and rents, we build a stochastic life cycle Aiyagari-Bewley-Huggett economy with incomplete markets, uninsurable idiosyncratic earnings risk, exogenous down payment requirements and interest rates, and endogenous house prices and rents. We depart from a

[1] The house price and rent statistics are based on the FHFA house price index and the Bureau of Labor Statistics (BLS) Rent of Primary Residence.

representative-agent framework to build an economy where – as in the data – renters and homeowners differ in terms of income and wealth. Building on the idea of houses as durable, lumpy consumption goods that provide shelter services, confer access to collateralized borrowing, offer tax advantages to their owners, and can also be used as rental investments, we endogenize the buy vs. rent decision and also allow homeowners to lease out their properties in the rental market. The supply of rental housing is thus determined endogenously within the model, as homeowners weigh their utility from shelter space against rental income, taking into account the tax implications of their decisions. Mortgages are available to finance purchases of housing, but home-buyers must satisfy a minimum down payment requirement. The recent housing literature argues that housing market frictions such as non-convex transaction costs and higher depreciation of rental properties relative to owner-occupied properties are likely to be important determinants of housing demand and rental supply. We therefore incorporate these frictions into our model. Both house prices and rents are determined in equilibrium through clearing of housing and rental markets.

The calibrated model is well suited to study the impact of macroeconomic factors on equilibrium house prices and rents in a steady state, and along a deterministic transitional path between steady states. Our rational expectations model of the housing market demonstrates that rising incomes, historically low interest rates, and easing of down payment requirements can plausibly explain about one-half of the increase in U.S. house prices between 1995 and 2006.[2] In addition, the model predicts that changes in these factors will have only a small positive effect on equilibrium rents, a result that is consistent with the U.S. data. In our view, the fact that a rational expectations equilibrium model leaves a substantial fraction of the run-up in house prices during the boom unexplained provides indirect evidence that overly optimistic expectations about house prices contributed to a bubble in the housing market.

[2] A large body of empirical literature has investigated the relationship between house prices and macroeconomics aggregates. For example, regression analysis by by Englund and Ioannides (1997), Malpezzi (1999), and Muellbauer and Murphy (1997, 2008) show that real interest rates, income, income growth, and financial liberalization have a statistically significant effect on the dynamics of real house prices.

The key mechanism in the model generating the run-up in the equilibrium price-rent ratio in response to changed macroeconomic conditions is that the supply of rental property available on the rental market and the demand for rental units by tenants are endogenously determined jointly with the demand for housing. When the mortgage interest rate and required down payment fall, the demand for rental units by tenants falls because households switch from renting to owning as homeownership becomes more affordable. Simultaneously, the supply of rental property from landlords increases because investment in rental property becomes more attractive relative to the alternative of holding bank deposits as the interest rate falls.[3] As a result, the equilibrium rent falls. At the same time, the demand for housing increases because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the stock of housing is fixed, the equilibrium house price rises. Turning to the effects of income on the housing market, we find that an increase in income that is symmetric across all wage groups leads to a proportional increase in house prices and rents, but has little impact on the price-rent ratio.

The model provides a number of additional insights about the mechanisms that jointly determine house prices and rents. Both the house price and rent are relatively inelastic with respect to the down payment requirement, so a lessening of credit constraints cannot by itself account for the run-up in house prices observed in recent years. The key to understanding the small effect of decreases in the required down payment on equilibrium house prices is to realize that changes in equilibrium house prices from this source are primarily driven by renters entering the housing market when down payment requirements are relaxed. Since renters are marginal in terms of income and wealth, the increase in housing demand relative to the entire market demand for housing is small, so the resulting house price increase is small. The corresponding increase in household borrowing as credit constraints are relaxed is skewed toward low-income households, as poorer households gain access to mortgage markets

[3] In the United States, the buy-to-let markets have grown substantially since the mid-1990s (OECD, 2006). The portion of sales attributable to such investors has risen sharply since the late 1990s, reaching around 15 percent of all home purchases in 2004, much higher than the pre-1995 average of 5 percent (Morgan Stanley, 2005).

and borrow large amounts relative to their labor income to finance their home purchases.

Conversely, the model predicts that falling interest rates create large increases in house prices but reduced homeownership. Cheap credit reduces the cost of mortgage financing, boosting household willingness and ability to purchase big properties and to finance them using large mortgages. At the same time, a lower interest rate lowers the return on household savings, making it more difficult to save up for a down payment – itself now higher, thanks to higher house prices – and prompting investors to seek higher returns by becoming landlords. The equilibrium effects are higher house prices, higher rental supply, and a lower homeownership rate. Our finding that interest rates can be linked to a higher price-rent ratio is supported by the user cost literature (see, for example, seminal work by Poterba (1984) and follow-up work by Himmelberg et al. (2005), Hubbard and Mayer (2009), and Glaeser et al. (2010)). Moreover, our estimated interest rate elasticity of house prices closely matches the empirical estimate of Glaeser et al. (2010).

This paper builds on the growing body of literature which studies housing using quantitative macroeconomic models where household heterogeneity is generated via labor income shocks.[4] Díaz and Luengo-Prado (2008) build a partial equilibrium economy with a number of realistic features where housing and rental markets exist only insofar as both house prices and rents follow exogenous processes.[5] Chambers, Garriga, and Schlagenhauf (2009a,b,c) document that the vast majority of U.S. rental property is owned by households instead of firms, and develop a model where rental property is supplied by households who choose to become landlords as a result of optimal investment strategies. This paper adopts the structure of rental markets from Chambers, Garriga, and Schlagenhauf, but also allows both house prices and rents to be determined in an equilibrium. Gervais (2002) provides an alternative framework for modeling the rental market where a representative financial intermediary supplies rental properties.[6] In this framework, the price of housing is always equal to the

[4] Since these studies are numerous, we review recent studies that are most closely related to this paper.

[5] The features considered include, for example, collateralized borrowing, non-convex adjustment costs, taxes, and idiosyncratic earnings risk.

[6] This framework is also adopted in Nakajima (2008).

price of consumption, which is normalized to unity, so this framework is not appropriate for examining the response of house prices to changes in fundamentals.

Kiyotaki, Michaelides, and Nikolov (2011) explore the relationship between the price of housing equity and rent in a model where households costlessly trade housing shares (the only asset in the economy), and where the relative holding of the shares with respect to the size of the shelter services consumed determines the homeownership status of the household. Production capital can be costlessly transformed to provide shelter services, so rent is determined as a factor price of this production capital. Favilukis, Ludvigson, and Van Nieuwerburgh (2011) study the housing boom in a two sector RBC model where fluctuations in the price-rent ratio are driven by changing risk premia in response to aggregate shocks. However, their model does not include a rental market. Instead, they impute rent from a distribution of the marginal rate of substitution (MRS) between homeowners' consumption of nondurable goods and housing. Related to Ortalo-Magné and Rady (2006), Ríos-Rull and Sánchez-Marcos (2008) study house price dynamics in a general equilibrium model with two different size properties, called houses and flats, where both house prices are endogenous, and where, as in our model, the supply of housing is inelastic. Lastly, Chatterjee and Eyigungor (2011) study the effects of changes in housing supply on house price dynamics and mortgage default using a calibrated model with a representative stand-in rental firm. They study the housing market bust, while we focus on the behavior of U.S. house prices and rents during the housing market boom of 1995-2006.

2. The Model Economy

We consider an Aiyagari-Bewley-Huggett style economy with heterogeneous households. Households derive utility from nondurable consumption and from shelter services which are obtained either via renting or through ownership. Households supply labor inelastically, receive an idiosyncratic uninsurable stream of earnings in the form of endowments, and make joint decisions about their consumption of nondurable goods and shelter services, house

size, mortgage size, and holdings of deposits. Idiosyncratic earnings shocks can be partially insured through precautionary savings (deposits), or through collateralized borrowing in the form of liquid home equity lines of credit (HELOCs). Households prefer homeownership to renting, in part because of the tax advantages to homeownership embedded in the U.S. tax code, but may be forced to rent due to the down payment requirement and high transaction costs. An important feature of our model is that houses can be used as a rental investment: they provide a source of income when leased out, and tax deductions available to landlords can be used to offset non-rental income and rental property related depreciation expenses. House prices and rents are determined in equilibrium through clearing of housing and rental markets.

2.1. Demography and Labor Income

The model economy is inhabited by a continuum of overlapping generations households with identical preferences. The model period is one year. Following Castaneda, Díaz-Gimenez, and Ríos-Rull (2003) and Heathcote (2005), we model the life cycle as a stochastic transition between various labor productivity states that also allows household's expected income to rise over time.[7]

In our stochastic life cycle model, households transit from state w via two mechanisms: (i) aging and (ii) productivity shocks, where the events of aging and receiving productivity shocks are assumed to be mutually exclusive. The probability of transiting from a state w_j via aging is equal to $\phi_j = 1/(p_j L)$, where p_j is the fraction of population with productivity w_j in the ergodic distribution over the discrete support $\mathcal{W}$, and L is a constant equal to the expected lifetime. Similarly, the conditional probability of transiting from a working-age state w_j to a working-age state w_i due to a productivity shock is defined as $P(w_i|w_j)$. The overall probability of moving from state j to state i, denoted by π_{ji}, is therefore defined as:

[7] The stochastic-aging economy is designed to capture the idea that liquidity constraints may be most important for younger individuals who are at the bottom of an upward-sloping lifetime labor income profile without requiring that household age be incorporated into our already large state space.

$$\Pi = \begin{bmatrix} 0 & \phi_1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \phi_{J-1} \\ \phi_J & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} (1-\phi_1) & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & (1-\phi_{J-1}) & 0 \\ 0 & 0 & 0 & (1-\phi_J) \end{bmatrix} P. \tag{1}$$

The fractions p_j are the solutions to the system of equations $p = p\Pi$.[8]

Young households are born as renters. In this model, we do not allow for intergenerational transfers of wealth (financial or non-financial) or human capital. Instead, we assume that, upon death, estates are taxed at a 100 percent rate by the government and immediately resold. All proceeds of these sales are not re-distributed, but are instead used to finance government expenditures that do not affect individuals.

2.2. Preferences

We assume that each household has a per-period utility function of the form $U(c, s, h')$, where c stands for nondurable consumption, s represents the consumption of shelter services, and h' is the household's current period holdings of the housing stock after the within-period labor income shock has been realized. Shelter services can be obtained either via the rental market at price ρ per unit or though homeownership at price q per unit of housing.[9] A linear technology is available that transforms one unit of housing stock, h', into one unit of shelter services, s.

The household's choices about the amount of housing services consumed relative to the housing stock owned, $(h' - s)$, determine whether a household is renter $(h' = 0)$, owner-occupier $(h' = s)$, or landlord $(h' > s)$. Landlords lease $(h' - s) =: l$ to renters at rental

[8] A detailed description of this process is available in the Appendix of Heathcote's paper.

[9] Our specification of a per-unit price of housing follows recent work in quantitative macroeconomics that studies the housing market; see for example Chatterjee and Eyigungor (2011). Allowing for multiple house prices would be very difficult because a model with N house prices and M rents requires searching in $N \times M$-dimensional space for the equilibrium house prices and rents while repeatedly re-solving the household optimization problem. See Ríos-Rull and Sánchez-Marcos (2008) for a model with multiple house prices, but no rental market.

rate ρ. Many recent studies assume that renters receive lower utility from a unit of housing services than homeowners (see, for example, Kiyotaki, Michaelides, and Nikolov (2011)). In this model, we assume that renters receive the same utility from housing services as homeowners, but landlords face a utility loss caused by the burden of maintaining and managing a rental property.

2.3. Assets and market arrangements

There are three assets in the economy: houses ($h \geq 0$), deposits ($d \geq 0$) with an interest rate r, and collateral debt ($m \geq 0$) with a mortgage rate r^m. Households may alter their individual holdings of the assets h, d, and m to the new levels h', d', and m' at the beginning of the period after observing their within-period income shock w.

Houses are big items that are available in $K = 11$ discrete sizes, $h \in \{0, h(1), ..., h(K)\}$. Households may choose not to own a house ($h' = 0$), in which case they obtain shelter through the rental market. Agents also make a discrete choice about shelter consumption. Households can rent a small unit of shelter, $\underline{s}$, which is smaller that than the minimum house size available for purchase, $0 < \underline{s} < h(1)$. Renters are also free to rent a larger amount of shelter. To maintain symmetry between shelter sizes available to homeowners and renters, we assume that all levels of shelter consumption must match a point on the housing grid, so $s \in \{\underline{s}, h(1), ..., h(K)\}$. The total housing stock, H, is fully owned by households and its size does not change over time.[10]

Houses are costly to buy and sell. Households pay a non-convex transactions costs of τ^b percent of the house value when buying a house, and pay τ^s percent of the value of the house when selling a house. Thus, the total transactions costs incurred when buying or selling a house are $\tau^b q h'$ and $\tau^s q h$. The presence of transactions costs reduces the transaction volume

[10] Although the stock of housing (as well as population size) is fixed in our model, there is evidence that the stock of housing increased over the boom period. For example, according to the National Income and Product Accounts (NIPA) tables, residential investment as a fraction of fixed investment hovered at about 15 percent between 1949 and 2000, while it rose from 18.2% to 25.2% between 2000 and 2005. We discuss the effect of changes in the housing supply on the housing market equilibrium in Section 4.4..

in the economy, and generates inaction regions with regard to the household decision to buy or sell. Therefore, only a part of the total housing stock is traded every period. The total housing supply and demand are thus determined endogenously, and are respectively upward and downward sloping functions of the house price. Similarly, the demand and supply of property in the rental market are endogenously determined, with rental supply determined by the individual demands for housing and shelter, $h' - s$.

Homeowners incur maintenance expenses, which offset physical depreciation of housing properties, so that housing does not deteriorate over time. Under this assumption, the total stock of housing, H, in the economy is fixed. The actual expense depends both upon the value of housing and the quantity of owned property that is rented to other households, $h' - s$. We assume that housing occupied by a renter depreciates more rapidly than owner occupied housing.[11] This problem arises because renters decide how intensely to utilize a house but may not actually pay the resulting cost, which creates an incentive to overutilize the property. Housing which is consumed by the owner depreciates at rate δ_o while the depreciation rate for rented property is δ_r, with $\delta_r > \delta_o$. Thus, current total maintenance costs facing an agent who has just chosen housing capital equal to h' are given by

$$M(h', s) = (\delta_0 s + \delta_r \max\{h' - s, 0\}). \tag{2}$$

Modeling depreciation as proportional to housing value follows the existing literature in quantitative macro and in user cost theory,[12] and is also consistent with the tradition of National Income and Product Accounts (NIPA).[13]

Homeownership confers access to collateralized borrowing at a constant markup over the risk-free deposit rate, r, so that $r^m = r + \kappa$. Borrowers must, however, satisfy a minimum

[11] See Chambers, Garriga, and Schlagenhauf (2009b) for a detailed discussion of this assumption.

[12] For quantitative studies, see, for example, Díaz and Luengo-Prado (2008) and Chambers et al. (2009b). For user cost studies, see Poterba (1984), Himmelberg, Mayer, and Sinai (2005), or Glaeser, Gottlieb, and Gyourko (2010).

[13] NIPA defines depreciation as "the change in value associated with the aging of an asset" (Fraumeni (1997)).

equity requirement. In a steady state where the house price does not change across time, the minimum equity requirement is given by the constraint

$$m' \leq (1-\theta)qh', \tag{3}$$

with $\theta > 0$. The equity requirement limits entry to the housing market, since households interested in buying a house with a market value qh' must put down at least a fraction θ of the value of the house. By the same token, households who wish to sell their house and move to a different size house or become renters must repay all the outstanding debt, since the option of mortgage default is not available. The accumulated housing equity above the down payment can, however, be used as collateral for home equity loans. Along the transitional path where house prices fluctuate, the operational constraint becomes

$$m' I^{\{(m'>m)\cup(h'\neq h)\}} \leq (1-\theta)\, qh'. \tag{4}$$

This modified version of the constraint shown in equation 3 implies that homeowners need not decrease their collateral debt balance during house price declines, as long as they do not sell their house. On the other hand, when house prices rise, households can access the additional housing equity through a pre-approved home equity loan. In a steady-state environment where prices are constant, equation 4 reduces to equation 3.

We follow the existing literature in modeling mortgages in a HELOC fashion,[14] where loans are essentially payment-option mortgages with a required interest rate payment and a pre-approved home equity line of credit. Effectively, this means that a borrower must cover mortgage interest payments every period, but is not obligated to pay down principle. We discuss how these features of mortgages in the model affect the calibration in Section 3.3..

[14] See, for example, Díaz and Luengo-Prado (2008), Gervais (2002), Ríos-Rull and Sánchez-Marcos (2008) or Nakajima (2008). Chambers, Garriga, and Schlagenhauf (2009c) and Campbell and Cocco (2003) offer a more complete analysis of mortgage choice.

2.4. The Government

We follow Díaz and Luengo-Prado (2008) in modeling a tax system with a preferential tax treatment of owner-occupied housing that mimics the U.S. system in a stylized way. In addition to the taxation of household labor and asset income, the government imposes a proportional property tax on housing which is fully deductible from income taxes, and allows deductions for interest payments on collateral debt (mortgages and home equity). As in the U.S. tax code, the imputed rental value of owner-occupied housing is excluded from taxable income. As discussed below, we expand on the tax treatment of rental property in existing models of the housing market by allowing landlords to deduct depreciation of the rental property from their taxable income. For simplicity, we assume proportional income taxation at the rate τ^y.

The total taxable income is thus defined as

$$\widetilde{y} = w + rd + I^{h'\neq 0}\left[-\tau^m r^m m - \tau^h q h'\right] + I^{h'>s}[\rho\left(h'-s\right) - \tau^{LL} q\left(h'-s\right) - \delta_r q\left(h'-s\right)], \quad (5)$$

where $w + rd$ represents household labor income plus earned interest. The first term in brackets represents the tax deduction received by homeowners, where $\tau^m r^m m$ is the mortgage interest deduction, and $\tau^h q h'$ is the fully deductible property tax payment made by the household. The next term in brackets represents the taxable rental income of landlords, which equals total rents received, $\rho\left(h'-s\right)$, minus the tax deductions available to landlords, which are discussed next.

In the current U.S. tax treatment, landlords must pay income taxes on rental income, but are permitted to deduct many different expenses associated with operating a rental property from their gross rental income. In addition to the mortgage interest and property tax deductions available to owner-occupiers, landlords can also deduct depreciation of the rental structure and maintenance expenditures.

Accordingly, the term $\tau^{LL} q\left(h'-s\right)$ in equation 5 represents the tax deduction for de-

preciation of rental property, where τ^{LL} represents the fraction of the total value of the rental property that is tax deductible in each year, and $\delta_r q\,(h' - s)$ represents tax deductible maintenance expenses. In our model, as in the U.S. tax code, if the tax deductions from the rental property exceed rental income, then rental losses will reduce the households' tax liability by offsetting income from wages and interest.[15] Finally, we follow Díaz and Luengo-Prado (2008) and assume that the entire proceeds from taxation are used to finance government expenditures that do not affect individuals.[16]

2.5. The Dynamic Programming Problem and Definition of Stationary Equilibrium

A household starts any given period t with a stock of residential capital, $h \geq 0$, deposits, $d \geq 0$, and collateral debt (mortgage and equity loans), $m \geq 0$. Households observe the idiosyncratic earnings shocks, w, and – given the current prices (q, ρ) – solve the following problem:

$$v(w, d, m, h) = \max_{c,s,h',d',m'} U(c, s, h') + \beta \sum_{w' \in W} \pi(w'|w) v(w', d', m', h') \tag{6}$$

subject to

$$\begin{aligned} & c + \rho\,(s - h') + d' - m' + q(h' - h) + I^s \tau^s q h + I^b \tau^b q h' \\ & \leq w \;\; + (1 + r)d - (1 + r^m)\, m - \tau^y \widetilde{y} - \tau^h q h' - qM\,(h', s) \end{aligned} \tag{7}$$

$$m' I^{\{(m' > m) \cup (h' \neq h)\}} \leq (1 - \theta)\, q h' \tag{8}$$

[15] A maximum of \$25,000 in rental property losses can be used to offset income from other sources, and this deduction is phased out between \$100,000 and \$150,000 of income. In our stylized model we abstract away from these upper limits.

[16] The treatment of proceeds from taxation is consistent with our treatment of proceeds from sales of estates of deceased agents, previously discussed in Section 2.1.

$$m' \geq 0 \text{ and } d' \geq 0 \tag{9}$$

$$h' \geq s \ > 0 \ if \ \ h' > 0 \tag{10}$$

$$s > 0 \ if \ \ h' = 0, \tag{11}$$

by choosing non-durable consumption, c, shelter services consumption, s, as well as current levels of housing, h', deposits, d', and collateral debt, m'. The term $\rho\,(s - h')$ represents either a rental payment by renters (i.e., households with $h' = 0$), or the rental income received by landlords (i.e., households with $h' > s$). The term $q(h' - h)$ captures the difference between the value of the housing purchased at the start of the time period (h') and the stock of housing that the household entered the period with (h). Transactions costs enter into the budget constraint when housing is sold ($\tau^s qh$) or bought ($\tau^b qh'$), with the binary indicators I^s and I^b indicating the events of selling and buying, respectively. Household labor income is represented by w, and it follows the process $\pi_w(w_t|w_{t-1})$ described in Section 2.1.. Households earn interest income rd on their holdings of deposits in the previous period, and pay mortgage interest $r^m m$ on their outstanding collateral debt in the last period. The income and property tax payments are represented by $\tau^y \widetilde{y}$ and $\tau^h qh'$, with τ^y denoting the marginal income tax rate, $\widetilde{y}$ representing the total taxable income from equation 5, and τ^h being the property tax rate. $qM\,(h', s)$ represents the maintenance expenses for homeowners which is described in equation 2. Finally, equation 8 indicates that a household that either increases the size of its mortgage ($m' > m$) or moves to a different-sized home ($h' \neq h$) must satisfy the down payment requirement $m' \leq (1 - \theta)qh'$.

Stationary Equilibrium: A stationary equilibrium is a collection of optimal household policy functions $\{c(x), s(x), d'(x), m'(x), h'(x)\}$ which are functions of the state vector $x = (w, d, m, h)$, a probability measure, λ, and a stationary price vector (q, ρ) such that decision rules are optimal, markets clear, and λ is a stationary probability measure. A formal definition of the equilibrium is presented in an Appendix which is available as supplementary material.

3. Calibration

The model is calibrated in two stages; all parameters are shown in Table 1. In the first stage, values are assigned to parameters that can be determined from the data without the need to solve the model. In the second stage, the remaining parameters are estimated by the simulated method of moments (SMM).

3.1. Demography and Labor Income

To calibrate the stochastic aging economy, we assume that households live, on average, 50 periods (e.g., $L = 50$). In terms of the process for household productivity, many papers in the quantitative macroeconomics literature adopt simple AR(1) specification to capture the earnings dynamics for working-age households that is characterized by the serial correlation coefficient, ρ_w, and the standard deviation of the innovation term, σ_w. For the purposes of this paper, we set ρ_w and σ_w to 0.90 and 0.20, respectively, and follow Tauchen (1986) to approximate an otherwise continuous process with a discrete number (7) of states.[17]

3.2. Preferences

Following the literature on housing choice (see, for example, Díaz and Luengo-Prado (2008), Chatterjee and Eyigungor (2011), and Kiyotaki, Michaelides, and Nikolov (2011)), the preferences over the consumption of non-durable goods (c) and housing services (s) are modeled as non-separable of the form

$$U(c, s, h') = (1 - \chi I^{h'>s}) \frac{\left(c^{\alpha} s^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}. \tag{12}$$

The binary variable $I^{h'>s}$ indicates that a homeowner is also a landlord. The risk aversion parameter, σ, is set to 2. The remaining parameters that characterize preferences are the

[17] Using data from the Panel Study of Income Dynamics (PSID), work by Card (1994), Hubbard, Skinner, and Zeldes (1995) and Heathcote, Storesletten, and Violante (2010) indicates a ρ_w in the range 0.88 to 0.96, and a σ_w in the range 0.12 to 0.25.

weight on non-durable consumption of the Cobb-Douglas aggregator, α, the discount factor, β, and the landlord utility loss parameter, χ.[18] These three parameters are estimated in the second stage. Section 3.5. discusses our strategy for identifying these parameters and explains the role of the landlord utility loss in the model.

3.3. Market Arrangements

Using data from the Consumer Expenditure Survey (CE), Gruber and Martin (2003) document that selling costs for housing are on average 7 percent, while buying costs are around 2.5 percent. We use these authors' estimates and set $\tau^b = 0.025$ and $\tau^s = 0.07$. In terms of the maintenance cost function $M(h', s)$ in Equation (2), Harding, Rosenthal, and Sirmans (2007) estimate that the depreciation rate for housing units used as shelter is between 2.5 and 3 percent. We thus set $\delta_0 = 0.025$. The depreciation rate of rental property, δ_r, is estimated in the second stage (see Section 3.5.). To calibrate the cost of mortgage credit, we set the interest rate on deposits r to 4 percent and let $\kappa = 0.015$ so that the mortgage rate $r^m = r + \kappa$ is equal to 5.5 percent in the baseline model.[19]

Consistent with the existing literature, we set the baseline down payment requirement θ at 20 percent.[20] In this class of model, homeownership is preferred to renting, mortgages are modeled in the HELOC fashion with no fixed payments, and there is no loan approval, so real world features of mortgages such as credit score or income requirements are absent. As such, the minimum down payment requirement in the model serves as a proxy for the overall

[18] Cobb-Douglas preferences imply that households tend to spend constant fractions of their income on housing and non-housing consumption. This is consistent with empirical findings in Davis and Ortalo-Magné (2010). At the same time, Cobb-Douglas preferences have implications for the degree of substitutability between c and s. Various studies have attempted to estimate the degree of complementarity between housing and non-housing consumption. Piazzesi, Schneider, and Tuzel (2007) estimate a high degree of substitutability in aggregate data. In contrast, combining PSID and AHS data, Flavin and Nakagawa (2008) estimate that the intra-temporal substitutability between housing and non-durable consumption is low.

[19] Using data from the Federal Reserve Statistical Release – H15 – Selected Interest Rates, the interest rate r is based on the average 30-year constant maturity Treasury deflated by year-to-year headline CPI inflation for the period between 1977 and 2008. The markup κ is based on the average spread between the nominal interest rate on a 30-year fixed-rate mortgage and 30-year Treasury.

[20] See, for example, Díaz and Luengo-Prado (2008), Gervais (2002), Ríos-Rull and Sánchez-Marcos (2008) or Nakajima (2008).

tightness of credit conditions and is, therefore, set conservatively in the baseline model. Empirically, Foote, Gerardi, and Willen (2012) shows that low or no down payment loans were commonly available to qualified borrowers in the pre-boom period.

3.4. Taxes

We set property tax rate $\tau^h = 0.01$, and allow mortgages to be fully deductible so that $\tau^m = 1$. The U.S. tax code assumes that a rental structure depreciates over a 27.5 year horizon, which implies an annual depreciation rate of 3.63 percent. However, only structures are depreciable for tax purposes, and the value of a house in our model includes both the value of the structure and the land that the house is situated on. Davis and Heathcote (2007) find that on average, land accounts for 36 percent of the value of a house in the U.S. between 1975 and 2006. Based on their findings, we set the depreciation rate of rental property for tax purposes to $\tau^{LL} = (1 - .36) \times .0363 = .023$. Lastly, we follow Díaz and Luengo-Prado (2008) and Prescott (2004) and set the income tax rate, τ^y, to 0.20.

3.5. Estimation

Based on the previous discussion, four structural parameters must be estimated: the Cobb-Douglas consumption share, α, the discount factor, β, the landlord utility loss, χ, and the depreciation rate of rental property, δ_r. Let $\Phi = \{\alpha, \beta, \chi, \delta_r\}$ represent the vector of parameters to be estimated. Let m_k represent the $k-$th moment in the data, and let $m_k(\Phi)$ represent the corresponding simulated moment generated by the model. The SMM estimate of the parameter vector is chosen to minimize the squared difference between the simulated and empirical moments:[21]

$$\widehat{\Phi} = \arg\min_{\Phi} \sum_{k=1}^{4} (m_k - m_k(\Phi))^2. \tag{13}$$

[21] Minimizing this function is computationally expensive because it requires numerically solving the agents' optimization problem and finding the equilibrium house price and rent for each trial value of the parameter vector.

The four moments targeted during estimation are the homeownership rate, the landlord rate, the imputed rent-to-wage ratio, and the fraction of homeowners who hold collateral debt. The remainder of this section details the data sources for the targeted moments and discusses how the parameters (Φ) impact the simulated moments. The share parameter α affects the allocation of income between non-durable consumption and shelter by agents in the model. This motivates our use of the imputed rent-to-wage ratio as a targeted moment. As outlined in Section 3.2., Davis and Ortalo-Magné (2010) estimate the share of expenditures on housing services by renters to be roughly 0.25 over the last several decades. The discount factor, β, directly impacts the willingness of agents to borrow, so we attempt to match the fraction of owner-occupiers with collateral debt. According to data from the 1994-1998 AHS, approximately 65 percent of homeowners report collateral debt balances.

The final two targeted moments are the homeownership rate and landlord rate. The homeownership rate averaged 0.66 in the United States between 1995 and 2005. Chambers, Garriga, and Schlagenhauf (2009b) use the AHS data to compute the fraction of homeowners who claim to receive rental income. The authors find that approximately 10 percent of the sampled homeowners receive rental income. Targeting the homeownership and landlord moments implies that we are also implicitly targeting the fraction of households who are renters (0.34) and owner-occupiers (0.56) because the landlord, renter, and owner-occupier categories are mutually exclusive and collectively exhaustive. The homeownership and landlord moments provide information about the magnitude of the landlord utility loss parameter (χ) and the depreciation rate of rental property (δ_r).[22]

Estimated Parameters (Φ): Table 1 shows the estimated parameters and demonstrates that the model closely matches the targeted empirical moments. The estimate of the

[22] The parameters χ and δ_r both impact the decision to become a landlord, but they have different implications for household behavior. When χ is greater than zero, a household will only become a landlord if rental income increases utility by enough to offset the fact that χ reduces the utility received by a landlord from all consumption of housing and shelter. Owners of large houses are able to rent out more space, and consequently are able to obtain more rental income than owners of small houses, so they are more likely to find it optimal to pay the landlord utility cost χ. In this sense, χ operates much as a fixed cost of being a landlord. In contrast, an increase in δ_r, holding ρ fixed, reduces the profitability of renting out a unit of housing for all households, and is effectively an increase in the marginal cost of being a landlord.

discount factor, 0.959, appears reasonable. To put the estimate of δ_r in context, recall that we assume that owner occupied housing depreciates at rate $\delta_0 = 0.025$, so our estimate of δ_r indicates that the depreciation rate for rented property is 1.2 percentage points greater than the depreciation rate of owner occupied property. The estimate of the landlord utility loss parameter, χ, indicates that landlords incur only a 2.4 percent utility loss due to the burden of managing a rental property.

3.6. Moments not Targeted in the Estimation

As an external test of our model, Table 1 reports several other key statistics generated by the model that were not targeted in the estimation and compares them to statistics that are either drawn from other studies or computed from the 1998 Survey of Consumer Finances (SCF). An Appendix which is available as supplementary material describes how we compute the SCF statistics. Encouragingly, the model predictions fall well within the range of estimates based on U.S. data.

3.7. Cross-sectional Implications of the Model

In our economy, renters are typically hand-to-mouth agents at the bottom of the wealth distribution who consume little housing. Nearly 68 percent of renters live in the smallest shelter space that is not available for sale, which we call a "room".[23] All other renters inhabit the smallest-sized house. In the model, homeownership is preferred to renting – mostly due to the favorable tax-treatment of homeownership – so households who can afford a down payment on a house typically purchase one.[24]

Interestingly, the option to become a landlord exerts an important influence on agents' decisions in our model economy, and represents an additional reason why ownership may

[23] The smallest-size shelter unit captures the idea that agents can also rent a very small living space that is not, however, available for sale. For example, a person can share a room with a roommate, or can rent a room while sharing a kitchen.

[24] An Appendix which is available as supplementary material shows how the exclusion of imputed rental income from taxable income impacts the cost of housing for homeowners.

be preferred to renting. First, rental units provide investment income. Second, the option to become a landlord is an important risk-management mechanism for homeowners facing adverse income shocks; exercising this option allows some to maintain higher consumption than would be otherwise possible, and to avoid sizable transactions costs associated with downsizing. It follows that transactions costs have an effect on household tenure in a home.[25]

While only ten percent of agents are landlords in the baseline steady state, the varied motivations to become a landlord lead to a diverse landlord pool. Indeed, Table 2 indicates that landlords can be found among all income groups. While the majority of landlords are either middle or high income households owning large properties, nearly 20% of landlords are in the poorest two categories. These predictions are qualitatively consistent with the statistics reported by Chambers, Garriga, and Schlagenhauf (2009b) who, using the 1996 Property Owners and Managers Survey, find that 25 percent of households receiving rental income are low-income households with annual earnings below $30,000, compared to 30 percent of high-income households with annual earnings over $100,000 (see their Table 2).

4. House Prices and Rents: Impact of Changed Fundamentals

The estimated model is employed to analyze the observed changes in house prices, rents, and the price-to-rent ratio between 1995 and 2006. The analysis is conducted in two steps. First, we study the model's predictions about the responsiveness of house prices and rents, and the price-rent ratio, to changes in interest rates, borrowing constraints, household incomes, and the combination of these macroeconomic factors in the steady-state housing market equilibrium. As such, all of the analysis in Section 4. is based on comparisons of different steady state economies. In the second step, presented in Section 5., we extend the model to include deterministic dynamics of house prices and rents.

[25] Using the 1985-2001 AHS data, Harding, Rosenthal, and Sirmans (2007) estimate that time between sales for owner-occupiers averages roughly 6 years. Rosenthal (1988) estimate average duration in the home for homeowners at 7 years in the PSID. Ferreira, Gyourko, and Tracy (2010) report average duration in house at 8.2 years using AHS data between 1985 and 2007. In comparison, our model predicts the average time in home at time of sale at 10.9 years, with the median homeowner living in the residence for 8 years.

4.1. Relaxation of Down Payment Requirements in a Steady-State Economy

Figure 2 illustrates the impact of changes in the minimum down payment requirement, θ, on equilibrium housing market outcomes. A reduction in θ from the baseline value of 0.20 to 0.05 leads to a 3 percent increase in the house price, and a 3.4 percent decrease in rent.[26] Neither house prices or rents respond strongly to θ, so lessening of credit constraints cannot by itself explain the large run-up in house prices observed in recent years.

That said, the homeownership rate does respond strongly to changes in down payment requirements. When θ is lowered from 0.30 to 0.20, the homeownership rate rises from 65 to 66 percent; when θ falls further to 0.05, the homeownership rate increases to 83 percent. The loan-to-wage ratio and the fraction of homeowners in debt also rise as θ falls.

The key to understanding the large effect of θ on homeownership, but the small effect of θ on house prices, is the fact that housing market responses are primarily driven by households for whom the minimum down payment is a binding constraint: low-income, low-savings households. These households are numerous, but since these entrants are marginal in terms of income and wealth, their impact on house prices is modest. As low income households enter the housing market by purchasing small-size houses, there is a corresponding decrease in the amount of rented property. Thus, the equilibrium response of landlords is effectively to sell housing to their tenants, and they are willing to do this because the house price has risen while the rent has remained virtually constant.[27]

Table 3 highlights the important role of general equilibrium forces in generating housing market outcomes. Column (2) displays a partial equilibrium experiment, which counterfactually holds both house prices and rents fixed at the baseline market-clearing level while

[26] The fraction of households with high loan-to-value (LTV) mortgages increased sharply between 2000 and 2005 (Gerardi, Lehnert, Sherlund, and Willen (2008)). Chambers et al. (2008) and Duca, Muellbauer, and Murphy (2011) document a decrease in the average down payments for all borrowers and first-time homebuyers, respectively, since 1995 and use these shifts to proxy the relaxation of lending standards during the housing boom. Chomsisengphet and Pennington-Cross (2006) document similar trends in the subprime lending markets.

[27] This statement is not literally true, since the decrease in the downpayment discussed in this section is based on a comparison of two different steady state economies.

reducing θ from 20 to 5 percent. In contrast, Column (3) reports the general equilibrium impact of this decrease in θ; here both house prices and rents adjust to clear the housing and rental markets. As noted above, rent is fairly unresponsive to θ in general equilibrium; the initial rent level of approximately 0.22 holds in both the partial and general equilibrium experiments. When prices are held fixed, homeownership sharply increases when the minimum down payment falls because previously credit constrained households shift from renting to owning. However, in the general equilibrium experiment, the increase in the house price dampens the responsiveness of homeownership to decreases in θ. As noted above, despite the large increase in homeownership, only a 3 percent increase in the house price is needed to clear the market and accommodate these new homeowners.

Our results are consistent with several recent studies which document a strong relationship between the size of the down payment requirement and homeownership (e.g., Chambers, Garriga, and Schlagenhauf (2009a), Díaz and Luengo-Prado (2008), and Ortalo-Magné and Rady (2006)). These studies suggest that, while financial sector innovations have minimal impact for existing homeowners, lower down payment requirements strongly affect households for whom the high down payment is a binding constraint; the initially excluded households enter the housing market and the homeownership rate rises.[28]

In summary, the model clearly indicates that in the absence of changes in other factors, a relaxation of borrowing constraints is important for understanding homeownership rates but cannot by itself account for the magnitude of the recent increase in house prices. With this result in mind, the next sections of the paper examine the impact of changes in the interest rate and income on the equilibrium house prices and rents.

[28] Further support is found in the empirical findings of Ortalo-Magné and Rady (1999) and Muellbauer and Murphy (1990, 1997), who document that decreases in the down payment requirements in England and Wales after the financial liberalization of the early 1980s were one of the two most important factors associated with unprecedented increases in young-household homeownership (the second factor being optimistic appreciation expectations).

4.2. Changes in the Interest Rate in a Steady-State Economy

Figure 3 illustrates the impact of changes in the real risk-free rate, r, on equilibrium housing market outcomes. Since the mortgage interest rate r^m is determined by a constant markup, κ, over r, changes in r directly translate into changes in r^m; hence changes in r affect both the cost of borrowing and the rate of return on saving. As can be seen in the figure, interest rate changes have a large effect on house prices and rents. When r is lowered from 6 percent to 1 percent, the equilibrium house price increases by 33 percent, while the equilibrium rent decreases by 14 percent, leading to a 54 percent increase in the price-rent ratio (from 9.9 to 15.2). When r is lowered from 4 percent to 2 percent, a decrease broadly consistent with the actual decline between 1995 and 2006, the house price level rises by 16.4 percent, the rent falls by 2.6 percent, and the price-rent ratio rises by 20 percent from its initial level of 11.3.[29]

Our simulated interest rate elasticity of house prices is consistent with recent empirical estimates in Glaeser et al. (2010). They find that (i) the interest rate elasticity of house prices is larger when real interest rates are already low (i.e., below 3.5 percent), and that (ii) a 200 basis point decrease in real rates is associated with approximately a 16 percent increase in real house prices.[30] These empirical findings are both qualitatively and quantitatively consistent with our simulated results in Figure 3. In our model, the relationship between interest rates and house prices is also non-linear with a break at about the 3-percent interest rate mark. Moreover, as noted above, a 200 basis point decrease in interest rate from 4 to 2 percent is associated with 16.4 percent increase in house prices.[31]

[29] Based on Federal Housing Financing Board data deflated by headline CPI inflation, the real effective mortgage rate for single-family residential property oscillated around the 5 percent mark between 1990 and 1997, but then started tending downwards, reaching 2.5 percent in 2005.

[30] Using the deflated FHFA price index and real 10-year Treasury rates between 1980 and 2008, Glaeser et al. (2010) estimate a piecewise linear house price function, with a break at an interest rate of 3.45 percent (i.e, the median real rate between 1980 and 2008).

[31] The estimated effect of interest rates on house prices in Glaeser et al. (2010) is positive but indistinguishable from zero when real rates are high (i.e., above 3.5 percent). Our model predicts that a 200 basis point decrease in the long-term interest rate in the high interest rate regime (from 6 to 4 percent) would lead to only a 3 percent increase in house prices.

In the model, lower interest rates reduce the cost of household borrowing and reduce the rate of return on household savings. Both effects increase demand for owned housing. On the intensive margin, demand for housing services rises, due both to the lower opportunity cost and to the lower costs of financing a given mortgage; thus, house prices rise, which prices out some of the less wealthy. At the same time, there is a portfolio shift: rental property investment becomes relatively more attractive as borrowing costs fall and as the return to the alternative investment, deposits, falls. Despite the rise in house prices, which ceteris paribus raises the cost of becoming a landlord, the net effect is that the supply of rental properties rises, and rents fall. For example, when the interest rate decreases from 4 to 2 percent, the aggregate supply of rental property increases by 4 percent, while the rent falls from 0.22 to 0.21.

Perhaps surprisingly, the 50 percent reduction in the interest rate from 4 percent to 2 percent has almost no impact on the homeownership rate (Figure 3). This is caused by general equilibrium price effects, which are illustrated in Table 3. Column (4) displays the partial equilibrium counterfactual, while column (5) displays the general equilibrium outcome. If house prices and rents did not adjust, the homeownership rate would rise from 66 percent to 81 percent, reflecting the lower cost of consuming owned housing services and the reduced attractiveness of saving relative to housing investment. At the same time, the fraction of landlords in the economy would rise from 10 percent to nearly 50 percent, because when q and ρ are held constant, a decrease in r increases the rate of return to being a landlord and decreases the rate of return to the alternative of holding deposits. In equilibrium, however, higher house prices increase the minimum down payment, and the lower interest rate makes it difficult for prospective homeowners to save up for it. Furthermore, equilibrium rent decreases from 0.22 to 0.21, despite the fact that house prices are rising, making renting relatively more attractive, and reducing the return obtained by landlords.

To provide some intuition for the inverse relationship between the price-rent ratio and interest rates in our model, it is useful to examine a simplified model that abstracts from

credit constraints, discreteness, transactions costs, or utility loss for landlords.[32] In such an environment, an optimizing landlord holding a mortgage chooses h' in order to satisfy

$$\rho = \frac{q(1 + (1-\tau^y)\left(\tau^h + \delta_r\right) - \tau^y \tau^{LL} - \frac{1}{1+(1-\tau^y)r^m})}{(1-\tau^y)}. \tag{14}$$

The steady state price-rent ratio (q/ρ) is inversely related to the interest rate. Of course, equation 14 will not hold exactly in our full model with discreteness and frictions. Furthermore, it cannot on its own predict the individual responses of house prices and rents to a change in the interest rate; determining those responses requires solving for the the pair of market prices which jointly clear the markets for shelter and owned housing.

It is also useful to place our results in the context of the widely-used representative rental firm model of the housing market developed by Gervais (2002). In this model, the price-rent ratio is determined by the arbitrage condition

$$\frac{q}{\rho} = \frac{1}{r + \delta_r},$$

which equates the returns between housing and deposits. As in our model, there is an inverse relationship between q/ρ and r, and the sensitivity of the price-rent ratio to the interest rate depends upon the housing depreciation rate. However, in this class of model, there is no theory of the individual responses of house prices and rents to a change in the interest rate; the price of housing (q) is normalized to one due to the assumption of costless conversion between the consumption good and the housing good, and the above relationship only determines a price-rent ratio. Our model is distinctive in providing a quantitative theory of how both equilibrium house prices and rents respond to changes in various parameters, given the existence of important frictions and tax wedges. In our view, this is a significant distinction, particularly when viewed within the context of the recent housing boom and bust.

[32] An Appendix, which is available as supplementary material, describes this simplified model in detail.

4.3. Changes in Income in a Steady-State Economy

A large body of empirical literature identifies the level and growth rate of income as an important determinant of house price dynamics. In the United States, real hourly wages increased by 9.4 percent between 1995 and 2005.[33]

Figure 4 summarizes the impact of changes in income on the housing market equilibrium. In our experiment, we assume that household wages rise at the same rate across all wage groups.[34] The model suggests that both house prices and rents increase at about the same rate as wages. Since the relative price of obtaining housing services via homeownership versus renting remains unchanged, symmetric changes in income of the sort examined here have no effect on the homeownership and landlord rates.

Once again, equilibrium price effects play a central role, as illustrated in Columns (6) and (7) in Table 3. When house prices and rents are not allowed to adjust, rising income has a substantial impact on the housing market, with the homeownership rate increasing from 66 to 92 percent, reflecting the fact that more households are able to afford the down payment and mortgage payments required to purchase a house. In addition, many households stop renting out their units as they can more easily cover their mortgage payments: the share of owner-occupied housing increases from 0.56 to 0.71. But in equilibrium, house prices and rents rise proportionally, leaving the proportions of renters, homeowners, and landlords unchanged.

4.4. Combined Changes in Market Fundamentals

As discussed in the preceding sections, neither declines in the real interest rate, nor relaxation of borrowing constraints, nor rising incomes can on their own account for the

[33] This calculation is based on the BLS Current Employment Statistics (CES) real wage data.

[34] The actual changes in the income levels were not, however, symmetric. Heathcote, Perri, and Violante (2010) document the changes in the U.S. earnings inequality between 1967 and 2006. Using the Current Population Survey data, the authors find that the real earnings of the bottom decile of the earnings distribution did not, on average, grow between 1985 and 2000, although the earnings of the top earnings distribution grew steadily over the sample period. Given our stylized income process, we leave the exploration of asymmetric income changes to further work.

increases in the price-rent ratio, homeownership rate, and household debt between 1995 and 2006. This section examines the combined effects of changes in these fundamentals on equilibrium housing market outcomes. Figure 5 depicts the percentage deviation of the steady state price-rent ratio from the baseline economy for a range of interest rates and required down payments. Point A represents the calibrated baseline economy with an interest rate on deposits, r, of 4 percent and a required down payment, θ, of 20 percent. As illustrated in the figure, the price-rent ratio rises with a falling interest rate and lower down payment requirement. However, even when the minimum down payment is reduced all the way to 5 percent, the model generates an increase in the price-rent ratio that is significantly less than the 40 percent increase observed in the U.S. between 1995 and 2006.

In addition, the maximum effect of fundamentals shown in Figure 5 likely represents an upper bound on the ability of a rational expectations model to explain the housing boom for several reasons. First, the supply of housing is fixed in the model, so there is no supply response to attenuate price increases driven by demand shifts.[35] Second, the relaxation of down payment requirements from 20 percent to 5 percent likely overstates the extent of relaxation of credit constraints in the economy, as available evidence (i.e., Foote, Gerardi, and Willen (2012)) suggests that low-down payment loans had been commonly available to qualified borrowers in the period preceding the housing boom. Moreover, any household satisfying the down payment requirement qualifies for a high loan-to-value (LTV) mortgage in our model. In reality, additional underwriting criteria are put in place in order to qualify for high LTV loans, even though these were admittedly vastly relaxed during the boom. Finally, we treat the relaxation of credit constraints as an exogenous event. However, relaxed underwriting standards during the boom may have in part been an endogenous response by lenders to expectations of high future house price growth. To the extent that this is true, the

[35] Although the assumption of a fixed stock of housing implies that our model produces an upper bound on house price movements, the price-rent *ratio* is quite unresponsive to changes in supply. A 5-percent increase in the supply of housing combined with a lowered interest rate (from 4 to 2 percent) and down payment requirement (from 20 to 15 percent) leads to a proportional decrease in both the house price and rent, and does not affect the sensitivity of the price-rent ratio to changes in θ and r.

causality may run from house prices to credit conditions, rather than in the reverse direction. As a result, the reduction of the down payment from 20 percent to 5 percent is designed with the intent of determining an upper bound of the effect of relaxed credit constraints in this type of model. With this in mind, the remainder of this section studies a range of different decreases in θ, because there is no particular decrease that unambiguously matches the relaxation observed during the housing boom.

Table 4 provides a more comprehensive analysis of the simulated effects of parameter changes by showing the percentage deviations in house prices, rents, and the price-rent ratio from their baseline values (column (1)). To facilitate a comparison of the model's predictions to the data, column (6) shows the actual changes over the U.S. housing boom. Column (2) shows that when income is held constant, lowering θ and r raises house prices, lowers rents, and consequently increases the price-rent ratio. Column (3) of Table 4 shows that increasing wages by 10 percent while decreasing θ and r does not change the price-rent ratio compared to the scenarios where income is held constant.[36] However, the model also predicts that higher income will cause a small increase in rents that is quite close to the growth in rents observed in the data. Columns (4) and (5) show that further reductions in θ lead to relatively small additional increases in house prices and the price-rent ratio. We conclude that a plausible calibration of the model can account for approximately one-half of the changes in house prices and rents observed during the boom. An upper bound estimate of the fraction explained by fundamentals is approximately 60 percent (column 5). These results suggest that the changes in the interest rate and required down payment observed in the United States had a considerable impact on the price-rent ratio, but also leave a significant portion of the run-up unexplained by a rational expectations model.

Turning to the mechanism, in our model, holding house prices and rents constant, when the mortgage interest rate and required down payment fall, the demand curve for rental property shifts inward because households switch from renting to owning as homeownership

[36] A 10 percent increase in real wages is approximately what was observed in the U.S. between 1995 and 2005.

becomes more affordable. At the same time, the supply curve for rental property shifts to the right because when θ and r decrease, more households are able to afford down payments and mortgage payments on rental properties. In addition, since both the mortgage rate and rate of return on deposits fall when interest rates decrease, investing in rental property becomes more attractive relative to the alternative of holding bank deposits. The net result of the declining demand and increasing supply in the rental market is a decrease in the equilibrium rent. At the same time, the demand for housing (or homeownership) increases when the interest rate and the required down payment decrease because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the supply of housing is fixed, the equilibrium house price rises. It follows that the price-rent ratio increases as the house price increases and rent falls in response to the change in fundamentals.

5. Transitional Dynamics

Up to this point, we have confined our analysis to comparisons of different steady state economies. This section studies the transitional dynamics of the housing market between two steady states. We assume that the economy is initially in a steady state that corresponds to the baseline calibration of the model, where the interest rate is 4 percent and the required down payment is 20 percent. Starting from this initial steady state, the interest rate and required down payment unexpectedly and permanently fall to $r = 0.02$ and $\theta = 0.15$. We solve for equilibrium movements of house prices and rents along the perfect foresight transition path that ends at the new steady state.[37]

Figure 6 shows the transition path for the house price, rent, and price-rent ratio. In the first period of the transition, both the house price and rent overshoot their long-run steady state values: the house price increases by 25 percent while the rent increases by 15 percent.

[37] Along the transition path, all agents correctly forecast the sequence of equilibrium house prices and rents which leads to the new steady state, and the housing market clears in each time period. An Appendix which is available as supplementary material describes the solution of the model along the transition path in more detail.

Although the price-rent ratio does not overshoot, it jumps by 9 percent on impact. After the initial spike in the house price and rent, the equilibrium prices decline gradually over time, and the price-rent ratio steadily increases to its new steady state level.

Why do house price and rent overshoot? Several mechanisms are jointly operative. The first mechanism is a portfolio reallocation between deposits and housing by households. The initial steady state features a relatively large amount of financial wealth (deposits), owing to the high (4 percent) rate of return and relatively high (20 percent) down payment requirement. An unexpected decline in the interest rate and the down payment requirement give households an incentive to shift their portfolios from deposits into housing, since the rate of return to deposits falls, the mortgage cost of financing falls, and housing consumption becomes more attractive relative to saving. In conjunction with this, a capital gains mechanism is operative. The initial increase in the house price allows existing homeowners to capitalize gains, trade-up, and move to larger homes. This mechanism operates in the same manner as the one discussed in Ortalo-Magné and Rady (2006), where a fixed supply of start-up homes bolsters the overshooting relative to our model. As a result, housing demand rises sharply, but there is a fixed housing supply, so the surge of funds into housing drives up the house price. Turning to the rental market, two forces jointly operate: on one hand, the lower interest rate reduces the cost of rental investment; on the other, the higher house price increases it. Initially, the house price effect dominates, and rent increases to compensate landlords for the increased cost of rental space. Initially, the homeownership rate stays roughly constant, in part due to higher shelter cost in both the homeownership and rental markets.

The initial spikes in the house price and rent are not sustainable as a long run equilibrium, because they are fueled by the large amount of financial wealth that households accumulated in the high interest rate steady state. Over time, the house price and rent decrease as households draw down their financial wealth, and live for more time periods with the low interest rate. Moreover, as the overshooting in prices fades away, more renters shift into homeownership, the homeownership rate rises to its new long run equilibrium level, and

rents fall because of the reduced demand for rental space.

6. Sensitivity Analysis: The Cost of High LTV Credit

Section 4.1. examines the effect of a relaxation in down payment requirements through a decrease in θ on the housing market. However, there are alternative ways of modeling a reduction in credit constraints. In this section, we quantify the impact of a relaxation of credit constraints that operates primarily through the cost of high LTV credit. In particular, we conduct an experiment where the minimum down payment is set loosely to start with, but high LTV credit is more costly. Then, credit constraints are reduced though a simultaneous reduction in θ and a decrease in the cost of high LTV credit. More specifically, the baseline minimum down payment is $\theta = 0.10$, and borrowers with LTV ratios greater that 0.80 must pay an additional 2 percentage point mortgage insurance premium. In the subsequent experiment, the down payment is reduced to $\theta = 0.05$, and the insurance premium for high LTV mortgages is reduced to 1 percentage point.

In this experiment, the house price increases by 1.2 percent, the rent falls by 1.4 percent, and the price-rent ratio increases by only 2.7 percent. The explanation for these relatively small price effects follows the one first presented in Section 4.1.. Relaxed credit conditions allow low income, low wealth households to enter the housing market. Since these agents are marginal in terms of income and wealth, their entry produces only a small increase in house prices. The experiment confirms that a lessening of credit constraints is unable to generate a large run-up in house prices in this rational expectations model.

7. Conclusion

This paper develops a dynamic equilibrium model of the housing market in which both house prices and rents, not merely their ratio, are determined endogenously. We use the model to study the effect of changes in fundamentals such as the interest rate, required down payment, and income on equilibrium in the housing market. This analysis is motivated by the

fact that although house prices, rents, and their ratio are widely used economic indicators, there is no existing quantitative model of how these objects are simultaneously determined by the clearing of markets for owned and rented housing. Without an understanding of this theoretical relationship, it is not possible to determine whether observed changes in the relationship between house prices and rents reflect changing fundamentals or an asset price bubble.

We document that interest rates and credit constraints reached very low levels by historical standards during the housing boom of 1995-2006, and use our model as a tool to quantitatively evaluate the effects of changes in these fundamentals on house prices and rents. The model predicts that the combination of low interest rates and reduced down payment requirements leads to a large increase in the rational expectations equilibrium house price, but rents remain approximately constant. As a result, the house price-rent ratio increases. However, although our model illustrates that large increases in house prices accompanied by comparatively constant rents are consistent with the equilibrium response of the housing market to low interest rates and relaxed down payment requirements, changes in these fundamentals are capable of plausibly explaining only about one-half of the nearly 40 percent increase in the price-rent ratio observed between 1995 and 2006. In our view, the fact that a detailed rational expectations equilibrium model can explain only a fraction of the observed increase in house prices provides indirect support for the hypothesis that overly optimistic expectations about house price growth contributed to a bubble in the housing market.

Although this paper studies the housing market boom, the housing market crash also illustrates the inherent limitations of using rational expectations, fundamentals-based models to explain recent events in the housing market. In this class of model, increased house prices and approximately constant rents are the fully sustainable, equilibrium response of the market to low interest rates and relaxed credit conditions. Within this framework, the housing market can only crash if there is an exogenous, unexpected increase in interest rates and tightening of credit conditions. We view this as an implausible explanation for the recent

housing market bust.[38]

Throughout our analysis, we have maintained the assumption of rational expectations about future house prices and rents. Piazzesi and Schneider (2009) examine household beliefs during the housing boom, and develop a tractable search model where optimistic traders can push up house prices. Consistent with our focus on credit conditions, these authors find that in the 2003 Michigan Survey of Consumers, the primary reason reported by households for believing that it was a good time to buy a house was favorable credit conditions. Later in the boom, they find that the proportion of households who believed that house prices would continue to increase reached a 25-year high. It is well recognized that incorporating this type of information about expectations into a model like ours raises many difficult conceptual and practical questions. However, given that our model leaves a large fraction of observed changes in house prices unexplained, we view this as an important avenue for future work.

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[38] See Foote, Gerardi, and Willen (2012) for a detailed discussion of rational expectations models such as Favilukis, Ludvigson, and Van Nieuwerburgh (2011), which attempt to account for both the boom and bust through a single set of fundamentals.

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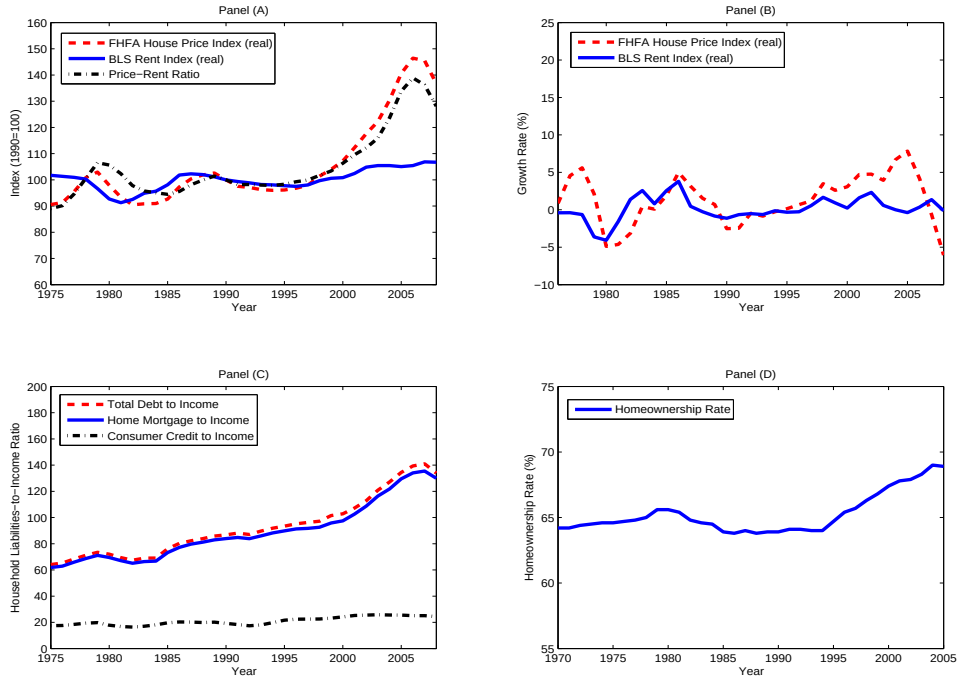

Notes: Variables in Panel (C) are from NIPA. Homeownership rate in Panel (D) is from the Census tables.

Figure 1: FHFA House Price Index and BLS Rent of Primary Residence Index

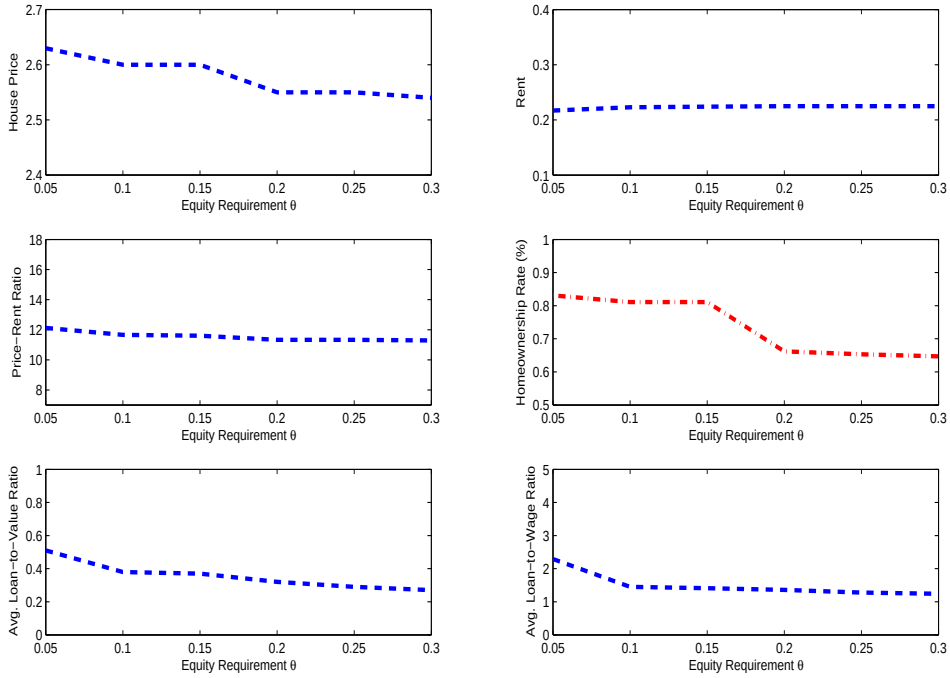

Figure 2: The Housing Market Equilibrium Under Different Equity Requirements

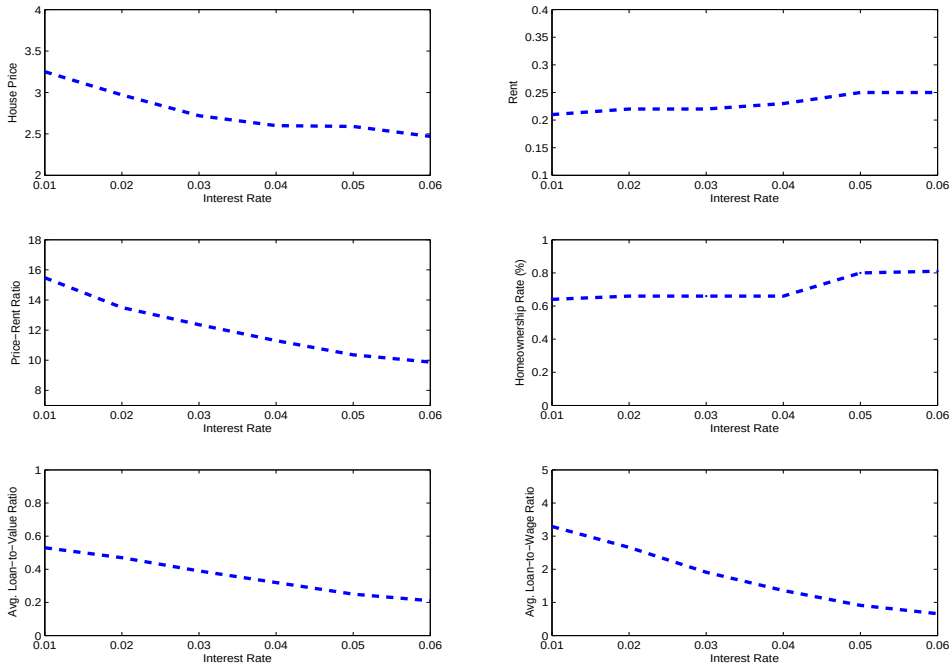

Figure 3: The Housing Market Equilibrium Under Different Interest Rates

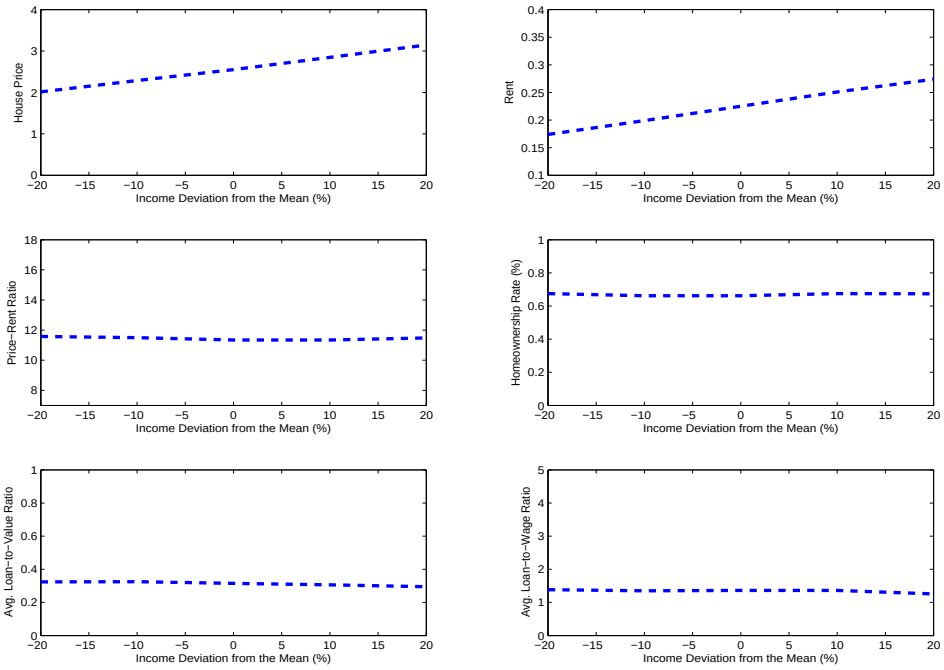

Figure 4: The Housing Market Equilibrium Under Different Income Levels

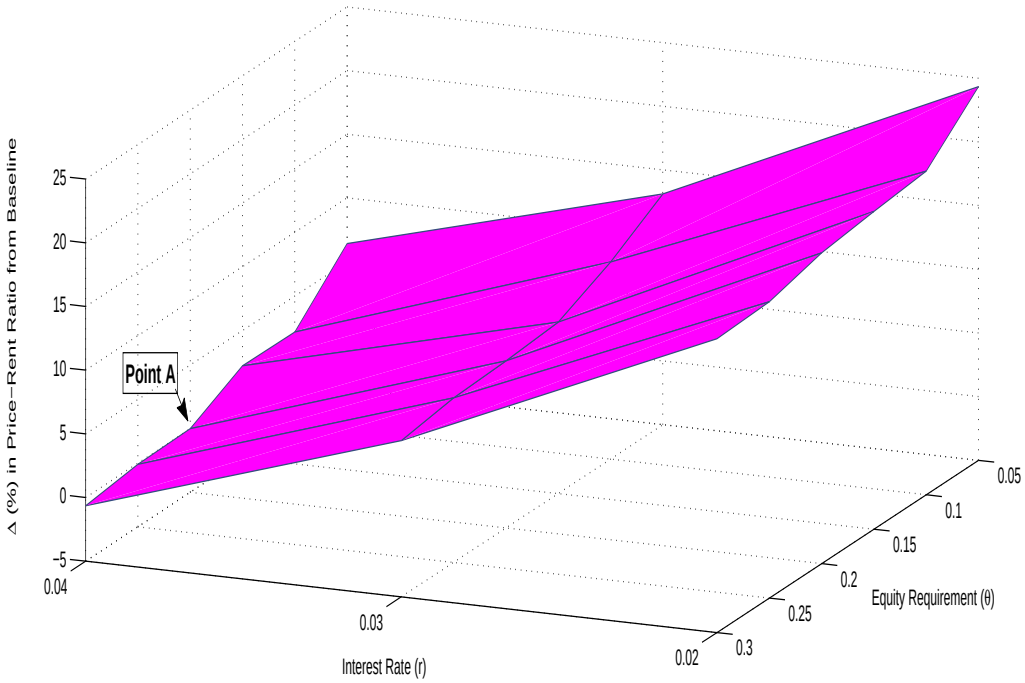

Figure 5: Percentage Deviations of the House Price-Rent Ratio from the Baseline (Point A) Under Different Interest Rates and Required Downpayment

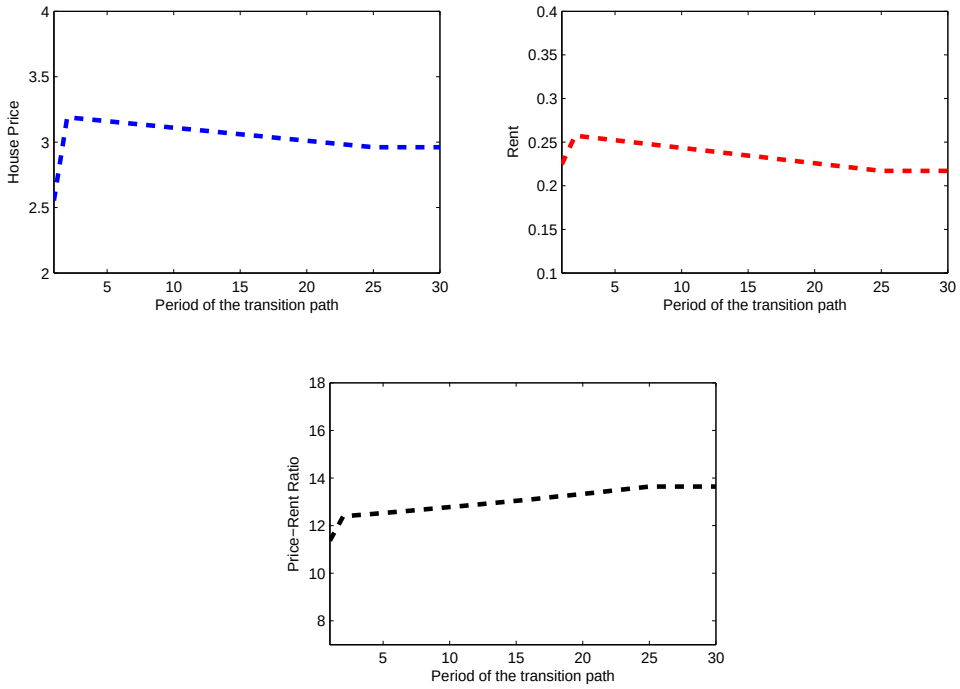

Figure 6: Transition Paths: Simultaneous Decrease in Interest Rate and Minimum Down-payment

Table 1: Parameters and Moments

Exogenous Parameters	Value	
Autocorrelation ρ_w	0.90	
Standard Deviation σ_w	0.20	
Risk Aversion σ	2.00	
Down Payment Requirement θ	0.20	
Selling Cost τ^s	0.07	
Buying Cost τ^b	0.025	
Risk-free Interest Rate r	0.04	
Spread κ	0.015	
Depreciation Rate for Homeowner-Occupiers δ_0	0.025	
Property Tax Rate τ^h	0.01	
Mortgage Deductibility Rate τ^m	1.00	
Deductibility Rate for Depreciation of Rental Property τ^{LL}	0.023	
Income Tax τ^y	0.20	
Estimated Parameters		
Discount Factor β	0.959	
Consumption Share α	0.720	
Depreciation of Rental Property δ_r	0.037	
Landlord Utility Loss χ	0.024	
Targeted Moments	Model	Data
Home-ownership rate	0.66	0.66
Landlord rate	0.10	0.10
Imputed rent-to-wage ratio	0.25	0.25
Fraction of homeowners with collateral debt	0.64	0.65
Other Moments (average ratios)	Model	Data
Loan to value ratio for homeowners	0.31	0.35[&]
Housing value to total income ratio for homeowners	4.02	4.43[&]
Loan to total income ratio for homeowners	1.34	1.28[&]
Net worth to total income ratio for homeowners	3.16	3.53[&]
House price-rent ratio	11.3	8 - 15.5*

Notes: (&): Calculated using the 1998 SCF.

(*): The U.S. Department of Housing and Urban Development and the U.S. Census Bureau report a price-rent ratio of 10 in the 2001 Residential Finance Survey (chapter 4, Table 4-2). Garner and Verbrugge (2009), using Consumer Expenditure Survey (CE) data drawn from five cities over the years 1982-2002, report that the house price to rent ratio ranges from 8 to 15.5 with a mean of approximately 12. The cities included in this analysis are Chicago, Houston, Los Angeles, New York, and Philadelphia.

Table 2: Distribution of Landlords by Labor Income

Labor Income Group	% Landlords	% Total Rental Property
Group 1	3.32	1.7
Group 2	15.02	10.2
Group 3	33.85	20.7
Group 4	15.44	20.8
Group 5	14.47	20.8
Group 6	12.32	17.7
Group 7	5.58	7.8

Note: Labor income group refers to the seven discrete wage levels that are used to approximate the continuous wage process.

Table 3: The Partial and Equilibrium Effects of Changes in Fundamentals

	Baseline	5% Equity Requirement		2% Interest Rate		10% Increase in Income	
		Fixed Prices	Equil. Prices	Fixed Prices	Equil. Prices	Fixed Prices	Equil. Prices
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
House Price	2.55	2.55	2.62	2.55	2.97	2.55	2.85
Rent	0.22	0.22	0.22	0.22	0.21	0.22	0.25
Share of Homeowners	0.66	0.91	0.83	0.81	0.66	0.92	0.67
Share of Renters	0.34	0.09	0.17	0.19	0.34	0.21	0.33
Share of Landlords	0.10	0.14	0.16	0.49	0.10	0.08	0.11
Share of Owner-Occupiers	0.56	0.77	0.67	0.32	0.55	0.71	0.56
Share of Homeowners in Debt	0.64	0.72	0.64	0.94	0.84	0.70	0.64

Notes:"Fixed Prices" columns hold prices at the baseline values from column (1). "Equil. Prices" columns allow prices to adjust to clear markets

Table 4: The Combined Effects of Interest Rate, Required Downpayment, and Income Changes

	Baseline	Changes in r and θ (%Δ from Baseline)				U.S. Data (%Δ)
	r=0.04	r=0.02	r=0.02	r=0.02	r=0.02	1995-2006
	θ=0.20	$\theta = 0.15$	$\theta = 0.15$	$\theta = 0.10$	$\theta = 0.05$	
		$\Delta w = 0\%$	$\Delta w = +10\%$	$\Delta w = +10\%$	$\Delta w = +10\%$	
	(1)	(2)	(3)	(4)	(5)	(6)
house price	2.55	16.1%	28.1%	30.2%	31.7%	50.4%
rental price	0.22	−3.5%	7.1%	6.6%	6.2%	7.9%
price-rent ratio	11.3	20.0%	20.0%	22.4%	24.4%	39.4%

Notes: Columns (2) - (5) show percent changes in the equilibrium value of each variable from the baseline model shown in column (1). Column (6) shows the actual percent changes observed in the U.S.

8. Appendix A: Definition of Stationary Equilibrium

In the benchmark economy, we restrict ourselves to stationary equilibria. The individual state variables are deposit holdings, d, mortgage balances, m, housing stock holdings, h, and the household wage, w; with $x = (w, d, m, h)$ denoting the individual state vector. Let $d \in \mathcal{D} = \mathbb{R}_+$, $m \in \mathcal{M} = \mathbb{R}_+$, $h \in \mathcal{H} = \{0, h_1, ..., h_{11}\}$, and $w \in \mathcal{W} = \{w_1, ..., w_7\}$, and let $\mathcal{S} = \mathcal{D} \times \mathcal{M} \times \mathcal{H} \times \mathcal{W}$ denote the individual state space. Next, let λ be a probability measure on $(\mathcal{S}, \mathcal{B}_s)$, where $\mathcal{B}_s$ is the Borel $\sigma-$algebra. For every Borel set $B \in \mathcal{B}_s$, let $\lambda(B)$ indicate the mass of agents whose individual state vectors lie in B. Finally, define a transition function $P : \mathcal{S} \times \mathcal{B}_s \rightarrow [0, 1]$ so that $P(x, B)$ defines the probability that a household with state x will have an individual state vector lying in B next period.

Definition (Stationary Equilibrium): A stationary equilibrium is a collection of value functions $v(x)$, a household policy $\{c(x), s(x), d'(x), m'(x), h'(x)\}$, probability measure, λ, and price vector (q, ρ) such that:

1. $c(x), s(x), d'(x), m'(x)$, and $h'(x)$ are optimal decision rules to the households' decision problem from Section 2.5., given prices q and ρ.

2. Markets clear:

   (a) Housing market clearing: $\int_{\mathcal{S}} h'(x) d\lambda = H$, where H is fixed;

   (b) Rental market clearing: $\int_{\mathcal{S}} (h'(x) - s(x)) d\lambda = 0$;

   where $\mathcal{S} = \mathcal{D} \times \mathcal{M} \times \mathcal{H} \times \mathcal{W}$.

3. λ is a stationary probability measure: $\lambda(B) = \int_{\mathcal{S}} P(x, B) d\lambda$ for any Borel set $B \in \mathcal{B}_s$.

9. Appendix B: Frictionless Analytical Results

Consider a problem of a homeowner who consumes all housing services yielded by the owned property (e.g., $s = h'$) but also chooses how much to invest into a rental property, h'_r.

For simplicity, we assume that mortgage interest payments are fully tax deductible ($\tau^m = 1$), and that there are no borrowing constraints, buying and selling costs, income uncertainty, or landlord utility penalty. The homeowner thus chooses (c, h', h'_r, m', d') to optimally solve:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c, h')$$

subject to initial conditions and

$$\begin{aligned} & c + d' - m' + qh' + qh'_r \\ & \leq w \ + (1+r)d - (1+r^m)\, m + \rho h'_r + qh + qh_r - \tau^y \widetilde{y} - \tau^h qh' - \tau^h qh'_r - \delta_0 qh' - \delta_r qh'_r, \end{aligned}$$

where

$$\widetilde{y} = w + rd + \rho h'_r - \left[r^m m + \tau^h qh' + \tau^h qh'_r + \delta_r qh'_r + \tau^{LL} qh'_r \right].$$

The corresponding first order conditions are:

$$\begin{aligned} c &: \beta u_c(c, h') - \lambda = 0, \\ h' &: \beta u_h(c, h') + \lambda(-\tau^y \frac{\partial \widetilde{y}}{\partial h'} - \tau^h q - \delta_0 q - q) + \lambda' q' = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}}{\partial h'} = -\tau^y \tau^h q, \\ h'_r &: \lambda(\rho - \tau^y \frac{\partial \widetilde{y}}{\partial h'_r} - \tau^h q - \delta_r q - q) + \lambda' q' = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}}{\partial h'_r} = \tau^y(\rho - \tau^h q - \delta_r q - \tau^{LL} q), \\ d' &: -\lambda + \lambda'(-\tau^y \frac{\partial \widetilde{y'}}{\partial d'} + (1+r)) = 0 \text{ where } \tau^y \frac{\partial \widetilde{y'}}{\partial d'} = \tau^y r, \\ m' &: \lambda + \lambda'(-\tau^y \frac{\partial \widetilde{y'}}{\partial m'} - (1+r^m)) = 0 \text{ where } \tau^y \frac{\partial \widetilde{y'}}{\partial m'} = -\tau^y r^m. \end{aligned}$$

Combining the first order conditions with respect to c and h', we obtain the expression representing the user cost of a homeowner,

$$\frac{u_h(c_t, h')}{u_c(c_t, h')} = q(1 + (1 - \tau^y)\tau^h + \delta_0) - \frac{\lambda'}{\lambda} q'. \tag{15}$$

Similarly, the first order condition with respect to h'_r gives the asset pricing equation for a landlord in this frictionless economy:

$$\rho = \frac{q(1 + (1-\tau^y)\tau^h + (1-\tau^y)\delta_r - \tau^y\tau^{LL}) - \frac{\lambda'}{\lambda}q'}{(1-\tau^y)}. \tag{16}$$

Equations 15 and 16 can be used to compare the cost of housing of a renter to that of a homeowner. Landlords can access deductions not available to homeowners, such as physical depreciation of the rental property and maintenance costs. However, rental property depreciates at a higher rate, and rental income (unlike user-occupied space) is taxable. Letting $C := 1 + (1-\tau^y)\tau^h - \frac{\lambda'}{\lambda}$, then in steady-state, equations 15 and 16 become

$$\frac{u_h(.)}{u_c(.)} = q(C + \delta_0)$$
$$(1-\tau^y)\rho = q(C + (1-\tau^y)\delta_r - \tau^y\tau^{LL})$$

Clearly, the fact that imputed rental income from owner-occupied shelter is excluded from taxable income is of central importance when examining the decision of a homeowner to supply rental property. At our calibrated parameter values, $\frac{u_h(.)}{u_c(.)} < \rho$, primarily due to the tax treatment of rental income – a result consistent with Díaz and Luengo-Prado (2008). Moreover, Díaz and Luengo-Prado (2008) show that when there is a spread between the return on deposits and the mortgage rate (as in here), then households do not simultaneously hold deposits and debt; see their Proposition 2. As a result, using the first order conditions with d' and m', the user cost and the landlord asset pricing equations above can be further simplified by substituting $\frac{\lambda'}{\lambda} = \frac{1}{1+(1-\tau^y)r}$ if the homeowner holds deposits, or $\frac{\lambda'}{\lambda} = \frac{1}{1+(1-\tau^y)r^m}$ if the homeowner holds a mortgage loan.

10. Appendix C: Solving the Model

Finding Equilibrium in the Housing and Rental Markets

Equilibrium in the housing and rental markets is formally defined by the conditions presented in Section 8.. In practice, the market clearing rent (ρ^*) and house price (q^*) are found by finding the (q^*, ρ^*) pair that simultaneously clear both the housing and shelter markets in a simulated economy. The market clearing conditions for a simulated cross section of N agents are

$$\sum_{i=1}^{N} h_i'(q^*, \rho^*|x) = H \tag{17}$$

$$\sum_{i=1}^{N} s_i'(q^*, \rho^*|x) = H. \tag{18}$$

The optimal housing and shelter demands for each agent are functions of the market clearing steady state prices and the agents other state variables (x). Solving for the equilibrium of the housing market is a time consuming process because it involves repeatedly re-solving the optimization problem at potential equilibrium prices and simulating data to check for market clearing until the equilibrium prices are found. The algorithm outlined in the following section exploits theoretical properties of the model such as downward sloping demand when searching for market clearing prices. Taking advantage of these properties dramatically decreases the amount of time required to find the equilibrium relative to a more naive search algorithm.

10.1. The Algorithm

Let q_k represent the kth guess of the market clearing house price, let ρ_k represent a guess of the equilibrium rent, and let $\rho_k(q_k)$ represent the rent that clears the market for housing conditional on house price q_k. The algorithm that searches for equilibrium is based on the

following excess demand functions

$$ED_k^h(q_k, \rho_k) = \sum_{i=1}^{N} h_i'(q_k, \rho_k|x) - H \tag{19}$$

$$ED_k^s(q_k, \rho_k) = \sum_{i=1}^{N} s_i'(q_k, \rho_k|x) - H. \tag{20}$$

The equilibrium prices q^* and ρ^* simultaneously clear the markets for housing and shelter, so

$$ED_k^h(q^*, \rho^*) = 0 \tag{21}$$

$$ED_k^s(q^*, \rho^*) = 0. \tag{22}$$

The following algorithm is used to find the market clearing house price and rent.

1. Make an initial guess of the market clearing house price q_k.

2. Search for the rent $\rho_k(q_k)$ which clears the market for owned housing conditional on the current guess of the equilibrium house price, q_k. The problem is to find the value of $\rho_k(q_k)$ such that $ED_k^h(q_k, \rho_k(q_k)) = 0$. This step of the algorithm requires re-solving the agents' optimization problem at each trial value of $\rho_k(q_k)$, simulating data using the policy functions, and checking for market clearing in the simulated data. One useful property of the excess demand function $ED_k^h(q_k, \rho_k(q_k))$ is that conditional on q_k, it is a strictly decreasing function of ρ_k. Based on this property, $\rho_k(q_k)$ can be found efficiently using bisection.

3. Given that the *housing* market clears at prices $(q_k, \rho_k(q_k))$, check if this pair of prices also clears the market for *shelter* by evaluating $ED_k^s(q_k, \rho_k(q_k))$.

   (a) If $ED_k^s(q_k, \rho_k(q_k)) < 0$ and $k = 1$, the initial guess q_1 is too high, so set $q_{k+1} = q_k - \varepsilon$ and go to step (2). This initial house price guess q_1 is too high if $ED_k^s(q_k, \rho_k(q_k)) < 0$ because $ED_k^s(q_k, \rho_k(q_k))$ is decreasing in q_k.

(b) If $ED_k^s(q_k, \rho_k(q_k)) > 0$ set $k = k + 1$ and $q_{k+1} = q_k + \varepsilon$ and go to step (2).

(c) If $ED_k^s(q_k, \rho_k(q_k)) = 0$, the equilibrium prices are $q^* = q_k$, $\rho^* = \rho_k(q_k)$, so stop.

10.2. Solving for the Transition Path

This appendix describes the solution of the model along the perfect foresight transition path between two steady states. In the first time period, the economy is in the initial, high interest rate, high down payment steady state. In time period $t = 2$, the interest rate and minimum down payment unexpectedly, and permanently, decline. Let T represent the number of time periods that it takes for the economy to converge to the final steady state.[39] Let (q^*, ρ^*) and (q^{**}, ρ^{**}) represent the initial and final steady state equilibrium house price and rent. The transition path is a sequence of prices, $\{q_t, \rho_t\}_{t=1}^T$, along which the optimal decisions of households clear both the markets for shelter and housing. Solving the household optimization problem along the transition path requires adding time to the state variables listed in the steady state problem described in Section 8. because both current-period prices and future prices affect households' optimal decisions. Given a sequence of prices $\{q_t, \rho_t\}_{t=1}^T$, the dynamic programming problem is solved recursively, moving backwards in time from time period T.

The algorithm begins by setting the market clearing prices in periods $t = 1$ and $t = T$ equal to their initial and final steady state values, so $q_1 = q^*$, $\rho_1 = \rho^*, q_T = q^{**}$, $\rho_T = \rho^{**}$. Next, a guess is made for the remaining prices along the transition path, $\{q_t, \rho_t\}_{t=2}^{T-1}$. The transition path is found using the following algorithm:

1. Solve the household problem recursively, moving backward from period T, taking the sequence of prices $\{q_t, \rho_t\}_{t=1}^T$ as given.

2. Use the optimal decision rules to simulate data from the model for each period along the transition path.

[39] In practice, we set $T = 30$, but find that the economy converges to the new steady state after 25 periods. The computed equilibrium is unchanged by extending the horizon to $T > 30$.

3. Check for market clearing in each time period using the conditions listed in section 8.. If markets clear in all time periods, stop because the transition path has been found. If markets do not clear, make a new guess of the transition path and go back to step 1.

11. Appendix D: SCF Data

The Survey of Consumer Finances (SCF) 1998 is used to construct the moments summarized in Table 1. The SCF is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of U.S. families. The total housing wealth is constructed as the total sum of all residential real estate owned by a household, and is taken to represent the housing wealth qh' in the model. Secured debt (i.e., debt secured by primary or other residence) is used as a model analog of the collateralized debt, m'. The model analogue of the total net worth (i.e., $d' + qh' - m'$) is constructed as the sum of household's deposits in the transaction accounts and the housing wealth (as defined above), net of the secured debt. The total household income reported in the SCF is taken to represent the total household income defined in the model as $y = w + rd' + I^{h'>s}[\rho(h' - s)]$. Both data and the SAS code are available at request, or can be found at the official Survey of Consumer Finances website.