The Equilibrium Effect of Fundamentals on House Prices and Rents*

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Abstract

This paper studies the joint dynamics of real house prices and rents over the past decade. We build a dynamic equilibrium stochastic life cycle model of housing tenure choice with fully specified markets for homeownership and rental properties, and endogenous house prices and rents. Houses are modeled as discrete-size durable goods which provide shelter services, confer access to collateralized borrowing, provide sizeable tax advantages, and generate rental income for homeowners who choose to become landlords. Mortgages are available, but home-buyers must satisfy a minimum down payment requirement, and home sales and purchases are subject to lumpy adjustment costs. Lower interest rates, relaxed lending standards, and higher incomes are shown to account for over one-half of the increase in the U.S. house price-rent ratio between 1995 and 2005, and to generate the pattern of rapidly growing house prices, sluggish rents, increasing homeownership, and rising household indebtedness observed in the data. The model highlights the importance of accounting for equilibrium interactions between the markets for owned and rented property when analyzing the effect of changes in fundamentals on housing market.

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1 Introduction

The sharp increase and subsequent collapse in U.S. house prices over the past decade has been well documented. While real house prices rose by only 3.7 percent between 1985 and 1995, they increased by 46 percent between 1995 and 2005. In sharp contrast, real rents remained virtually unchanged during the recent increase in house prices, so that in 2006 the house price-rent ratio peaked at approximately forty percent above its level in the year 2000 (Figure 1). Over the same time period, the real interest rate and minimum down payment required to purchase a home reached historically low levels. Especially because of the significant drop in house prices since 2007, and the role played by the housing market in the Great Recession, there is considerable interest in understanding the driving forces behind changes in house prices and rents.

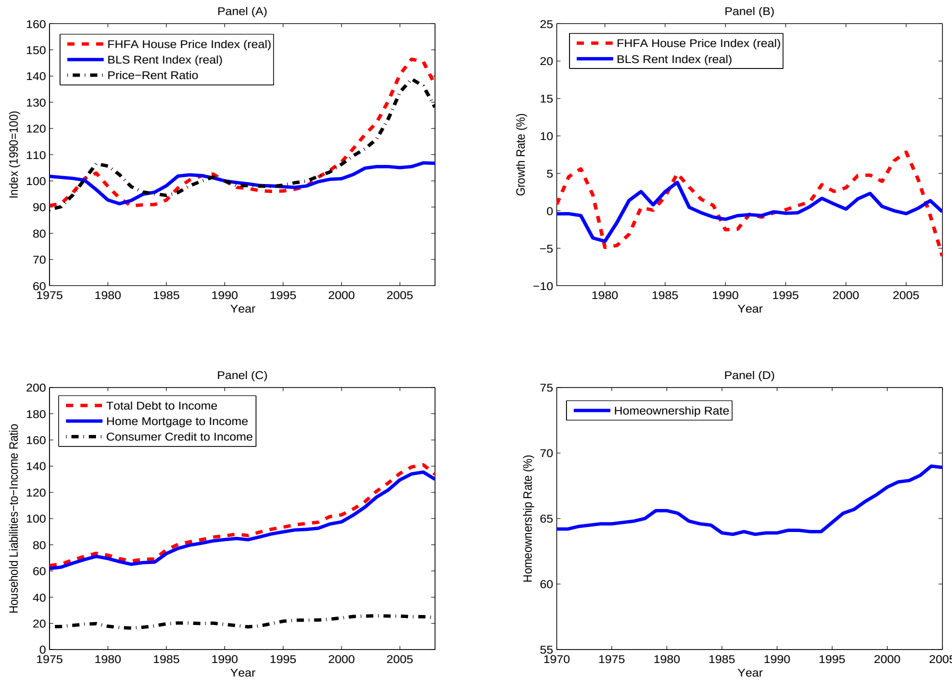
This paper quantitatively studies the effects of changes in fundamentals, such as the interest rate and required downpayment, on endogenously determined house prices and rents using a dynamic equilibrium model of the housing market that features fully specified markets for both homeownership and rental properties. The primary contributions of this paper relative to the existing literature are incorporating household decisions about rental property investment into a model of the housing market, and endogenizing both house prices and rents. Our model is the first model of the housing market that allows rental supply to be determined endogenously by the optimizing investment decisions of heterogenous households, and where two distinct relative prices – rents and house prices – are determined in equilibrium via market clearing.

To study the impact of changes in interest rates and down payment requirements on house prices and rents, we build a stochastic life cycle Aiyagari-Bewley-Huggett economy with incomplete markets, uninsurable idiosyncratic earnings risk, exogenous down payment requirements and interest rates, and endogenous house prices and rents. We depart from a representative-agent framework to build an economy where – as in the data – renters and homeowners differ in terms of income and wealth. Building on the idea of houses as durable, lumpy consumption goods that provide shelter services, confer access to collateralized borrowing, offer tax advantages to their owners, and can also be used as rental investments, we endogenize the buy vs. rent decision and also allow homeowners to lease out their properties in the rental market. The supply of rental housing is thus determined endogenously within the model, as homeowners weigh their utility from shelter space against rental income, taking into account the tax implications of their decisions. Mortgages are available to finance purchases of housing, but home-buyers must satisfy a minimum down payment requirement. The recent housing literature argues that housing market frictions such as non-convex transaction costs and higher depreciation of rental properties relative to owner-occupied properties are likely to be important determinants of housing demand and rental supply. We therefore incorporate these frictions into our model economy to quantitatively assess their importance in determining the price-rent ratio.[1] Both house prices and rents are determined in equilibrium through clearing of housing and rental markets.

The calibrated model is well suited to study the impact of macroeconomic factors on equilibrium house prices and rents in a steady state, and along a deterministic transitional path between steady states. Our rational expectations model of the housing market demonstrates that the rising incomes, historically low interest rates, and easing of down payment requirements observed in the data can explain about one-half of the increase in U.S. house prices between 1995 and 2005.[2] In addition, the model predicts that changes in these factors will have only a small positive effect on equilibrium rents, a result that is consistent with the U.S. data. Consequently, the model indicates that approximately one-half of the run-up in the U.S. house price-rent ratio between 1995 and 2005 can be explained as the equilibrium response of

[1] Appendix A highlights the important role that these features of the housing market play in determining demand for housing and rental properties, as well as rental supply.

[2] A large body of empirical literature has investigated the relationship between house prices and macroeconomics aggregates. For example, regression analysis by by Englund and Ioannides (1997), Malpezzi (1999), Muellbauer and Murphy (1997) and Muellbauer and Murphy (2008) show that real interest rates, income, income growth, and financial liberalization have a statistically significant effect on the dynamics of real house prices.



Notes: Variables in Panel (C) are from NIPA. Homeownership rate in Panel (D) is from the Census tables.

Figure 1: FHFA House Price Index and BLS Rent of Primary Residence Index

the housing market to changes in fundamentals. The price and rent dynamics generated by the model coincide with increases in the homeownership rate and household debt-to-income ratio that are also similar to the actual developments in the U.S. housing market between 1995 and 2005.[3]

The key mechanism in the model generating the run-up in the equilibrium price-rent ratio in response to changed macroeconomic conditions is that the supply of rental property available on the rental market and the demand for rental units by tenants are endogenously determined jointly with the demand for housing. When the mortgage interest rate and required down payment fall, the demand for rental units by tenants falls because households switch from renting to owning as homeownership becomes more affordable. Simultaneously, the supply of rental property from landlords increases because investment in rental property becomes more

[3]The total household debt to disposable income ratio has increased from 80 percent in 1985 to 93 percent in 1995 and to a whopping 141 percent in 2007. At the same time, the U.S. homeownership rate, initially flat at 64 percent between 1983 and 1995, rose to 69 percent by 2005.

attractive relative to the alternative of holding bank deposits as the interest rate falls.[4] As a result, the equilibrium rent falls. At the same time, the demand for housing increases because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the stock of housing is fixed, the equilibrium house price rises. Turning to the effects of income on the housing market, we find that an increase in income that is symmetric across all wage groups leads to a proportional increase in house prices and rents, but has little impact on the price-rent ratio.

The model provides a number of additional insights about the mechanisms that jointly determine house prices and rents. Both the house price and rent are relatively inelastic with respect to the down payment requirement, so a lessening of credit constraints cannot by itself account for the run-up in house prices observed in recent years. The key to understanding the small effect of decreases in the required down payment on equilibrium house prices is to realize that changes in equilibrium house prices from this source are primarily driven by renters entering the housing market when down payment requirements are relaxed. Since renters are marginal in terms of income and wealth, the increase in housing demand relative to the entire market demand for housing is small, so the resulting house price increase is small. The corresponding increase in household borrowing as credit constraints are relaxed is skewed toward low-income households, as poorer households gain access to mortgage markets and borrow large amounts relative to their labor income to finance their home purchases.

Conversely, the model predicts that falling interest rates create large increases in house prices but reduced homeownership. Cheap credit reduces the cost of mortgage financing, boosting household willingness and ability to purchase big properties and to finance them using large mortgages. At the same time, a lower interest rate lowers the return on household savings,

[4] In the United States, the buy-to-let markets have grown substantially since the mid-1990s (OECD, 2006). The portion of sales attributable to such investors has risen sharply since the late 1990s, reaching around 15 percent of all home purchases in 2004, much higher than the pre-1995 average of 5 percent (Morgan Stanley, 2005).

making it more difficult to save up for a down payment – itself now higher, thanks to higher house prices – and prompting investors to seek higher returns by becoming landlords. The equilibrium effects are higher house prices, higher rental supply, and a lower homeownership rate. Our finding that interest rates can be linked to a higher price-rent ratio is empirically supported by the user cost literature (see, for example, seminal work by Poterba (1984) and follow-up work by Himmelberg et al. (2005), Hubbard and Mayer (2009), and Glaeser et al. (2010)).

This paper builds on the growing body of literature which studies housing using quantitative macroeconomics models where household heterogeneity is generated via labor income shocks. We will briefly review several recent studies that are most closely related to this paper.

Díaz and Luengo-Prado (2008) build a partial equilibrium economy with a number of realistic features such as collateralized borrowing, non-convex adjustment costs, taxes, and idiosyncratic earnings risk. However, in their model, housing and rental markets exist only insofar as both house prices and rents follow exogenous processes. Chambers, Garriga and Schlagenhauf (2009a, 2009b, 2009c) document that the vast majority of U.S. rental property is owned by households instead of firms, and develop a model where rental property is supplied by households who choose to become landlords as a result of optimal investment strategies. This paper adopts the structure of rental markets from Chambers, Garriga, and Schlagenhauf (2009a), but also allows both house prices and rents to be determined in an equilibrium. Gervais (2002) provides an alternative framework for modeling the rental market where a representative financial intermediary supplies rental properties.[5] In this framework, the price of housing is always equal to the price of consumption, which is normalized to unity, so this framework is not appropriate for examining the response of house prices to changes in fundamentals.

Kiyotaki, Michaelides, and Nikolov (2011) explore the relationship between the price of housing equity and rent in a model where households costlessly trade housing shares (the only

[5] This framework is also adopted in Nakajima (2008).

asset in the economy), and where the relative holding of the shares with respect to the size of the shelter services consumed determines the homeownership status of the household. Production capital can be costlessly transformed to provide shelter services, so rent is determined as a factor price of this production capital. Favilukis, Ludvigson, and Van Nieuwerburgh (2011) study the housing boom in a two sector RBC model where fluctuations in the price-rent ratio are driven by changing risk premia in response to aggregate shocks. However, their model does not include a rental market. Instead, they impute rent from a distribution of the marginal rate of substitution (MRS) between homeowners' consumption of nondurable goods and housing. Related to Ortalo-Magné and Rady (2006), Ríos-Rull and Sánchez-Marcos (2008) study house price dynamics in a general equilibrium model with two different size properties, called houses and flats, where both house prices are endogenous, and where, as in our model, the supply of housing is inelastic. Lastly, Chatterjee and Eyigungor (2009) study the effects of changes in housing supply on house price dynamics and mortgage default using a calibrated model with a representative stand-in rental firm. They study the housing market bust, while we focus on the behavior of U.S. house prices and rents during the housing market boom of 1995-2005.

This paper is organized as follows. Section 2 presents a quantitative model of the housing and rental markets, and Section 3 defines the equilibrium. Section 4 describes the model's estimation and discusses the fit of the benchmark model. Section 5 examines the effect of changes in fundamentals on house prices and rents in the model, and compares these predictions to the U.S. housing market boom. Section 6 studies the transition path of the economy between steady states. Section 7 concludes.

2 The Model Economy

We consider an Aiyagari-Bewley-Huggett style economy with heterogeneous households. Households derive utility from nondurable consumption and from shelter services which are obtained either via renting or through ownership. Households supply labor inelastically, receive an idiosyncratic uninsurable stream of earnings in the form of endowments, and make joint decisions about their consumption of nondurable goods and shelter services, house size, mortgage size, and holdings of deposits. Idiosyncratic earnings shocks can be partially insured through precautionary savings (deposits), or through collateralized borrowing in the form of liquid home equity lines of credit (HELOCs). Households prefer homeownership to renting, in part because of the tax advantages to homeownership embedded in the U.S. tax code, but may be forced to rent due to the down payment requirement and high transaction costs. An important feature of our model is that houses can be used as a rental investment: they provide a source of income when leased out, and tax deductions available to landlords can be used to offset non-rental income and rental property related depreciation expenses. House prices and rents are determined in equilibrium through clearing of housing and rental markets.

2.1 Demography and Labor Income

The model economy is inhabited by a continuum of overlapping generations households with identical preferences. The model period is one year. Following Heathcote (2005) and Castaneda, Díaz-Gimenez, and Ríos-Rull (2003), we model the life cycle as a stochastic transition between various labor productivity states that also allows household's expected income to rise over time. The stochastic-aging economy is designed to capture the idea that liquidity constraints may be most important for younger individuals who are at the bottom of an upward-sloping lifetime labor income profile without requiring that household age be incorporated into our already large state space.

In our stochastic life cycle model, households transit from state w via two mechanisms: (i) aging and (ii) productivity shocks, where the events of aging and receiving productivity shocks are assumed to be mutually exclusive. The probability of transiting from a state w_j via aging is equal to $\phi_j = 1/(p_j L)$, where p_j is the fraction of population with productivity w_j in the ergodic distribution over the discrete support $\mathcal{W}$, and L is a constant equal to the expected lifetime. Similarly, the conditional probability of transiting from a working-age state w_j to a working-age state w_i due to a productivity shock is defined as $P(w_i|w_j)$. The overall probability of moving from state j to state i, denoted by π_{ji}, is therefore equal to the probability of transition from j to i via aging, plus the probability of transition from j to i via a productivity shock, conditional on not aging, so that

$$\Pi = \begin{bmatrix} 0 & \phi_1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \phi_{J-1} \\ \phi_J & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} (1-\phi_1) & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & (1-\phi_{J-1}) & 0 \\ 0 & 0 & 0 & (1-\phi_J) \end{bmatrix} P. \tag{1}$$

The fractions p_j are the solutions to the system of equations $p = p\Pi$. A detailed description of this process is available in the Appendix of Heathcote's paper.

Young households are born as renters. In this model, we do not allow for inter-generational transfers of wealth (financial or non-financial) or human capital. Instead, we assume that, upon death, estates are taxed at a 100 percent rate by the government and immediately resold.

2.2 Preferences

In the spirit of Ríos-Rull and Sánchez-Marcos (2008), Kiyotaki, Michaelides, and Nikolov (2011) and Chatterjee and Eyigungor (2009), we assume that each household has a per-period utility function of the form:

$$U(c, s, h')$$

where c stands for nondurable consumption, s represents the consumption of shelter services, and h' is the household's current period holdings of the housing stock after the within-period labor income shock has been realized. Shelter services can be obtained either via the rental market at price ρ per unit or though homeownership at price q per unit of housing.[6] A linear technology is available that transforms one unit of housing stock, h', into one unit of shelter services, s.

The household's choices about the amount of housing services consumed relative to the housing stock owned, $(h' - s)$, determine whether a household is renter $(h' = 0)$, owner-occupier $(h' = s)$, or landlord $(h' > s)$. Landlords lease $(h' - s) =: l$ to renters at rental rate ρ. Many recent studies assume that renters receive lower utility from a unit of housing services than homeowners (see, for example, the studies named above). In this model, we assume that renters receive the same utility from housing services as homeowners, but landlords face a utility loss caused by the burden of maintaining and managing a rental property.

2.3 Assets and market arrangements

There are three assets in the economy: houses $(h \geq 0)$, deposits $(d \geq 0)$ with an interest rate r, and collateral debt $(m \geq 0)$ with a mortgage rate r^m. Households may alter their individual holdings of the assets h, d, and m to the new levels h', d', and m' at the beginning of the period after observing their within-period income shock w.

Houses are big items that are available in $K = 11$ discrete sizes, $h \in \{0, h(1), ..., h(K)\}$. Households may choose not to own a house $(h' = 0)$, in which case they obtain shelter through the rental market. Agents also make a discrete choice about shelter consumption. Households can rent a small unit of shelter, $\underline{s}$, which is smaller that than the minimum house size available for purchase, $0 < \underline{s} < h(1)$. Renters are also free to rent a larger amount of shelter. To maintain symmetry between shelter sizes available to homeowners and renters, we assume that all levels

[6] Our specification of a per-unit price of housing follows recent work in quantitative macroeconomics that studies the housing market; see for example Chatterjee and Eyigungor (2009). Allowing for multiple house prices would be very difficult because a model with N house prices and M rents requires searching in $N \times M$-dimensional space for the equilibrium house prices and rents while repeatedly re-solving the household optimization problem. See Ríos-Rull and Sánchez-Marcos (2008) for a model with multiple house prices, but no rental market.

of shelter consumption must match a point on the housing grid, so $s \in \{\underline{s}, h(1), ..., h(K)\}$. The total housing stock, H, is fully owned by households and its size does not change over time.[7] Our set-up with endogenous house prices and inflexible housing supply thus represents an alternative to a production economy where land – the input factor into the housing production – is in fixed supply.

Houses are costly to buy and sell. Households pay a non-convex transactions costs of τ^b percent of the house value when buying a house, and pay τ^s percent of the value of the house when selling a house. Thus, the total transactions costs incurred when buying or selling a house are $\tau^b qh'$ and $\tau^s qh$. The presence of transactions costs reduces the transaction volume in the economy, and generates sizeable inaction regions with regard to the household decision to buy or sell. Therefore, only a part of the total housing stock is traded every period. The total housing supply and demand are thus determined endogenously, and are respectively upward and downward sloping functions of the house price. Similarly, the demand and supply of property in the rental market are endogenously determined, with rental supply determined by the individual demands for housing and shelter, $h' - s$.

Homeowners incur maintenance expenses, which offset physical depreciation of housing properties, so that housing does not deteriorate over time. Under this assumption, the total stock of housing, H, in the economy is fixed. The actual expense depends both upon the value of housing and the quantity of owned property that is rented to other households, $h' - s$. We assume that housing occupied by a renter depreciates more rapidly than owner occupied housing. This problem arises because renters decide how intensely to utilize a house but may not actually pay the resulting cost, which creates an incentive to overutilize the property. Housing which is consumed by the owner depreciates at rate δ_o while the depreciation rate for rented

[7] Although the stock of housing (as well as population size) is fixed in our model, there is evidence that the stock of housing increased over the boom period. For example, according to the National Income and Product Accounts (NIPA) tables, residential investment as a fraction of fixed investment hovered at about 15 percent between 1949 and 2000, while it rose from 18.2% to 25.2% between 2000 and 2005. However, Section 5.4 of the paper demonstrates that the generated increase in the price-rent ratio in our model is robust to allowing for increases in the stock of housing.

property is δ_r, with $\delta_r > \delta_o$. Thus, current total maintenance costs facing an agent who has just chosen housing capital equal to h' are given by

$$M(h', s) = (\delta_0 s + \delta_r \max\{h' - s, 0\}). \tag{2}$$

Homeownership confers access to collateralized borrowing at a constant markup over the risk-free deposit rate, r, so that $r^m = r + \kappa$. Borrowers must, however, satisfy a minimum equity requirement. In a steady state where the house price does not change across time, the minimum equity requirement is given by the constraint

$$m' \leq (1 - \theta) q h', \tag{3}$$

with $\theta > 0$. The equity requirement limits entry to the housing market, since households interested in buying a house with a market value qh' must put down at least a fraction θ of the value of the house. By the same token, households who wish to sell their house and move to a different size house or become renters must repay all the outstanding debt, since the option of mortgage default is not available. The accumulated housing equity above the down payment can, however, be used as collateral for home equity loans.[8] Along the transitional path where house prices fluctuate, the operational constraint becomes

$$m' I^{\{(m' > m) \cup (h' \neq h)\}} \leq (1 - \theta)\, q h'. \tag{4}$$

This modified version of the constraint shown in equation 3 implies that homeowners need not decrease their collateral debt balance during house price declines, as long as they do not sell their house. On the other hand, when house prices rise, households can access the additional housing

[8] Similarly to Díaz and Luengo-Prado (2008), we abstract from income requirements when purchasing houses. See their paper for further discussion. Chambers, Garriga and Schlagenhauf (2006) and Campbell and Cocco (2003) offer a more complete analysis of mortgage choice. See Li and Yao (2007) for an alternative model with refinancing costs.

equity through costless refinancing or a home equity loan. In a steady-state environment where prices are constant, equation 4 reduces to equation 3.

2.4 The Government

We follow Díaz and Luengo-Prado (2008) in modeling a tax system with a preferential tax treatment of owner-occupied housing that mimics the U.S. system in a stylized way. In addition to the taxation of household labor and asset income, the government imposes a proportional property tax on housing which is fully deductible from income taxes, and allows deductions for interest payments on collateral debt (mortgages and home equity). As in the U.S. tax code, the imputed rental value of owner-occupied housing is excluded from taxable income. As discussed below, we expand on the tax treatment of rental property in existing models of the housing market by allowing landlords to deduct depreciation of the rental property from their taxable income. For simplicity, we assume proportional income taxation at the rate τ^y.

The total taxable income is thus defined as

$$\widetilde{y} = w + rd + I^{h' \neq 0}\left[-\tau^m r^m m - \tau^h q h'\right] + I^{h' > s}[\rho\left(h' - s\right) - \tau^{LL} q\left(h' - s\right) - \delta_r q\left(h' - s\right)], \quad (5)$$

where $w + rd$ represents household labor income plus earned interest. The first term in brackets represents the tax deduction received by homeowners, where $\tau^m r^m m$ is the mortgage interest deduction, and $\tau^h q h'$ is the fully deductible property tax payment made by the household. The next term in brackets represents the taxable rental income of landlords, which equals total rents received, $\rho\left(h' - s\right)$, minus the tax deductions available to landlords, which are discussed next.

In the current U.S. tax treatment, landlords must pay income taxes on rental income, but are permitted to deduct many different expenses associated with operating a rental property from their gross rental income. In addition to the mortgage interest and property tax deductions available to owner-occupiers, landlords can also deduct depreciation of the rental structure and maintenance expenditures.[9] Accordingly, the term $\tau^{LL} q\left(h' - s\right)$ in equation 5 represents the tax deduction for depreciation of rental property, where τ^{LL} represents the fraction of the total value of the rental property that is tax deductible in each year, and $\delta_r q\left(h' - s\right)$ represents tax deductible maintenance expenses. In our model, as in the U.S. tax code, if the tax deductions from the rental property exceed rental income, then rental losses will reduce the households' tax liability by offsetting income from wages and interest.[10] Finally, we follow Díaz and Luengo-Prado (2008) and assume that the entire proceeds from taxation are used to finance government expenditures that do not affect individuals.

2.5 The Dynamic Programming Problem

A household starts any given period t with a stock of residential capital, $h \geq 0$, deposits, $d \geq 0$, and collateral debt (mortgage and equity loans), $m \geq 0$. Households observe the idiosyncratic earnings shocks, w, and – given the current prices (q, ρ) – solve the following problem:

$$v(w, d, m, h) = \max_{c,s,h',d',m'} U(c, s, h') + \beta \sum_{w' \in W} \pi(w'|w) v(w', d', m', h') \tag{6}$$

subject to

$$\begin{aligned} & c + \rho\left(s - h'\right) + d' - m' + q(h' - h) + I^s \tau^s q h + I^b \tau^b q h' \\ & \leq w \ + (1 + r)d - \left(1 + r^m\right) m - \tau^y \widetilde{y} - \tau^h q h' - q M\left(h', s\right) \end{aligned} \tag{7}$$

$$m' I^{\{(m' > m) \cup (h' \neq h)\}} \leq \left(1 - \theta\right) q h' \tag{8}$$

$$m' \geq 0 \ \text{ and } d' \geq 0 \tag{9}$$

[9] Other expenses that are tax deductable but not incorporated in out model are expenses related to advertising, travel to the rental property, comissions, insurance, legal and professional fees, management fees, supplies, and utilities. See IRS publication 527 for details on the tax treatment of residential rental property.

[10] A maximum of $25,000 in rental property losses can be used to offset income from other sources, and this deduction is phased out between $100,000 and $150,000 of income. In our stylized model we abstract away from these upper limits.

$$h' \geq s \ > 0 \ if \ \ h' > 0 \tag{10}$$

$$s > 0 \ if \ \ h' = 0, \tag{11}$$

by choosing non-durable consumption, c, shelter services consumption, s, as well as current levels of housing, h', deposits, d', and collateral debt, m'. The term $\rho(s - h')$ represents either a rental payment by renters (i.e., households with $h' = 0$), or the rental income received by landlords (i.e., households with $h' > s$). The term $q(h' - h)$ captures the difference between the value of the housing purchased at the start of the time period (h') and the stock of housing that the household entered the period with (h). Transactions costs enter into the budget constraint when housing is sold $(\tau^s qh)$ or bought $(\tau^b qh')$, with the binary indicators I^s and I^b indicating the events of selling and buying, respectively. Household labor income is represented by w, and it follows the process $\pi_w(w_t|w_{t-1})$ described in Section 2.1. Households earn interest income rd on their holdings of deposits in the previous period, and pay mortgage interest $r^m m$ on their outstanding collateral debt in the last period. The income and property tax payments are represented by $\tau^y \widetilde{y}$ and $\tau^h qh'$, with τ^y denoting the marginal income tax rate, $\widetilde{y}$ representing the total taxable income from equation 5, and τ^h being the property tax rate. $qM(h', s)$ represents the maintenance expenses for homeowners which is described in equation 2. Finally, equation 8 indicates that a household that either increases the size of its mortgage $(m' > m)$ or moves to a different-sized home $(h' \neq h)$ must satisfy the down payment requirement $m' \leq (1 - \theta)qh'$.

3 Definition of a Stationary Equilibrium

In the benchmark economy, we restrict ourselves to stationary equilibria. The individual state variables are deposit holdings, d, mortgage balances, m, housing stock holdings, h, and the household wage, w; with $x = (w, d, m, h)$ denoting the individual state vector. Let $d \in \mathcal{D} = \mathbb{R}_+$, $m \in \mathcal{M} = \mathbb{R}_+$, $h \in \mathcal{H} = \{0, h_1, ..., h_{11}\}$, and $w \in \mathcal{W} = \{w_1, ..., w_7\}$, and let $\mathcal{S} =$

$\mathcal{D}\times\mathcal{M}\times\mathcal{H}\times\mathcal{W}$ denote the individual state space. Next, let λ be a probability measure on ($\mathcal{S}$, $\mathcal{B}_s$), where $\mathcal{B}_s$ is the Borel $\sigma-$algebra. For every Borel set $B \in \mathcal{B}_s$, let $\lambda(B)$ indicate the mass of agents whose individual state vectors lie in B. Finally, define a transition function $P : \mathcal{S}\times \mathcal{B}_s\rightarrow [0, 1]$ so that $P(x, B)$ defines the probability that a household with state x will have an individual state vector lying in B next period.

Definition (Stationary Equilibrium): A stationary equilibrium is a collection of value functions $v(x)$, a household policy $\{c(x), s(x), d'(x), m'(x), h'(x)\}$, probability measure, λ, and price vector (q, ρ) such that:

1. $c(x), s(x), d'(x), m'(x)$, and $h'(x)$ are optimal decision rules to the households' decision problem from Section 2.5, given prices q and ρ.

2. Markets clear:

   (a) Housing market clearing: $\int_{\mathcal{S}} h'(x)d\lambda = H$, where H is fixed;

   (b) Rental market clearing: $\int_{\mathcal{S}}(h'(x) - s(x))d\lambda = 0$;

   where $\mathcal{S}$=$\mathcal{D} \times \mathcal{M} \times \mathcal{H} \times \mathcal{W}$.

3. λ is a stationary probability measure: $\lambda(B) = \int_{\mathcal{S}} P(x, B)d\lambda$ for any Borel set $B \in\mathcal{B}_s$.

4 Calibration

The model is calibrated in two stages. In the first stage, values are assigned to parameters that can be determined from the data without the need to solve the model. In the second stage, the remaining parameters are estimated by the simulated method of moments (SMM). Table 1 summarizes the parameters determined in the first stage. These parameters were drawn from other studies or were calculated directly from the data. Table 2 contains the four remaining parameters that we estimate in the second stage based on moments constructed using the data from the American Housing Survey (AHS) and the Census Tables. These moments are listed in Table 3.

Table 1: Exogenous Parameters

Parameter	Value
Autocorrelation ρ_w	0.90
Standard Deviation σ_w	0.20
Risk Aversion σ	2.00
Down Payment Requirement θ	0.20
Selling Cost τ^s	0.07
Buying Cost τ^b	0.025
Risk-free Interest Rate r	0.04
Spread κ	0.015
Depreciation Rate for Homeowner-Occupiers δ_0	0.025
Property Tax Rate τ^h	0.01
Mortgage Deductibility Rate τ^m	1.00
Deductibility Rate for Depreciation of Rental Property τ^{LL}	0.023
Income Tax τ^y	0.20

4.1 Demography and Labor Income

To calibrate the stochastic aging economy, we assume that households live, on average, 50 periods (e.g., $L = 50$). In terms of the process for household productivity, many papers in the quantitative macroeconomics literature adopt simple AR(1) specification to capture the earnings dynamics for working-age households that is characterized by the serial correlation coefficient, ρ_w, and the standard deviation of the innovation term, σ_w.[11] Using data from the Panel Study of Income Dynamics (PSID), work by Card (1994), Hubbard, Skinner, and Zeldes (1995) and Heathcote, Storesletten, and Violante (2010) indicates a ρ_w in the range 0.88 to 0.96, and a σ_w in the range 0.12 to 0.25. For the purposes of this paper, we set ρ_w and σ_w to 0.90 and 0.20, respectively, and follow Tauchen (1986) to approximate an otherwise continuous process with a discrete number (7) of states.

[11] Heathcote (2005) discusses alternatives to the AR(1) specification in a technical appendix which is available on the Review of Economic Studies web site.

4.2 Preferences

Following the literature on housing choice (see, for example, Díaz and Luengo-Prado (2008), Chatterjee and Eyigungor (2009), and Kiyotaki, Michaelides, and Nikolov (2011)), the preferences over the consumption of non-durable goods (c) and housing services (s) are modeled as non-separable of the form

$$U\left(c, s, h'\right) = (1 - \chi I^{h'>s}) \frac{\left(c^{\alpha} s^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}. \tag{12}$$

The binary variable $I^{h'>s}$ indicates that a homeowner is also a landlord. The risk aversion parameter, σ, is set to 2. The remaining parameters that characterize preferences are the weight on non-durable consumption of the Cobb-Douglas aggregator, α, the discount factor, β, and the landlord utility loss parameter, χ. These three parameters are estimated in the second stage. Section 4.5 discusses our strategy for identifying these parameters and explains the role of the landlord utility loss in the model.

4.3 Market Arrangements

Using data from the Consumer Expenditure Survey (CE), Gruber and Martin (2003) document that selling costs for housing are on average 7 percent, while buying costs are around 2.5 percent. We use the authors' estimates and set $\tau^b = 0.025$ and $\tau^s = 0.07$. In terms of the maintenance cost function $M(h', s)$ in Equation (2), Harding, Rosenthal, and Sirmans (2007) estimate that the depreciation rate for housing units used as shelter is between 2.5 and 3 percent. We thus set $\delta_0 = 0.025$. The depreciation rate of rental property, δ_r, is estimated in the second stage (see Section 4.5).

To calibrate the interest rates on deposits r, we use the interest rate on the 30-year constant maturity Treasury deflated by year-to-year headline CPI inflation. Using the data from the

Federal Reserve Statistical Release, the deflated Treasury rate averaged 3.8 percent for the period between 1977 and 2008.[12] We thus set the real interest rate to 4 percent so that $r = 0.04$. To calibrate the mortgage rate $r^m = r + \kappa$, we set the markup κ to represent the spread between the nominal interest rate on a 30-year fixed-rate conventional home mortgage and the interest rate on nominal 30-year constant maturity Treasury. The average spread between 1977 and 2008 is 1.5 percent, so κ is set to 0.015. In the baseline model, a minimum down payment of 20 percent is required to purchase a home.[13]

4.4 Taxes

Using data from the 2007 American Community Survey, Díaz and Luengo-Prado (2010) compute the median property tax rate for the median house value and report a housing property tax rate of 0.95 percent. Based on information from TAXSIM, they document that on average, 90 percent of mortgage interest payments are tax deductible. We thus set $\tau^h = 0.01$, and allow mortgages to be fully deductible so that $\tau^m = 1$. The U.S. tax code assumes that a rental structure depreciates over a 27.5 year horizon, which implies an annual depreciation rate of 3.63 percent. However, only structures are depreciable for tax purposes, and the value of a house in our model includes both the value of the structure and the land that the house is situated on. Davis and Heathcote (2007) find that on average, land accounts for 36 percent of the value of a house in the U.S. between 1975 and 2006. Based on their findings, we set the depreciation rate of rental property for tax purposes to $\tau^{LL} = (1 - .36) \times .0363 = .023$. Lastly, we follow Díaz and Luengo-Prado (2008) and Prescott (2004) and set the income tax rate, τ^y, to 0.20.

[12] See Federal Reserve Statistical Release, H15, Selected Interest Rates.

[13] Using the 1993 AHS data, Chambers, Garriga and Schlagenhauf document that the average down payment is approximately 20 percent.

4.5 Estimation

Based on the previous discussion, four structural parameters must be estimated: the Cobb-Douglas consumption share, α, the discount factor, β, the landlord utility loss, χ, and the depreciation rate of rental property, δ_r. Let $\Phi = \{\alpha, \beta, \chi, \delta_r\}$ represent the vector of parameters to be estimated. Let m_k represent the $k-$th moment in the data, and let $m_k(\Phi)$ represent the corresponding simulated moment generated by the model. The SMM estimate of the parameter vector is chosen to minimize the squared difference between the simulated and empirical moments,

$$\widehat{\Phi} = \arg\min_{\Phi} \sum_{k=1}^{4} (m_k - m_k(\Phi))^2. \tag{13}$$

Minimizing this function is computationally expensive because it requires numerically solving the agents' optimization problem and finding the equilibrium house price and rent for each trial value of the parameter vector.

The four moments targeted during estimation are the homeownership rate, the landlord rate, the imputed rent-to-wage ratio, and the fraction of homeowners who hold collateral debt. The remainder of this section details the data sources for the targeted moments and discusses how the parameters (Φ) impact the simulated moments. The share parameter α affects the allocation of income between non-durable consumption and shelter by agents in the model. This motivates our use of the imputed rent-to-wage ratio as a targeted moment. Using data from 1980, 1990, and 2000 Decennial Census of Housing, Davis and Ortalo-Magné (2010) estimate the share of expenditures on housing services by renters to be roughly 0.25, and find that the share has been constant across time and MSA regions. The discount factor, β, directly impacts the willingness of agents to borrow, so we attempt to match the fraction of owner-occupiers with collateral debt. According to data from the 1994-1998 AHS, approximately 65 percent of homeowners report collateral debt balances.[14]

[14] The discount pattern β governs household borrowing behavior in our model. Since deceased agents in our

The final two targeted moments are the homeownership rate and landlord rate. The homeownership rate averaged 0.66 in the United States between 1995 and 2005. Chambers, Garriga, and Schlagenhauf (2009a) use the AHS data to compute the fraction of homeowners who claim to receive rental income. The authors find that approximately 10 percent of the sampled homeowners receive rental income. Targeting the homeownership and landlord moments implies that we are also implicitly targeting the fraction of households who are renters (0.34) and owner-occupiers (0.56) because the landlord, renter, and owner-occupier categories are mutually exclusive and collectively exhaustive. The homeownership and landlord moments provide information about the magnitude of the landlord utility loss parameter (χ) and the depreciation rate of rental property (δ_r). Within the model, the parameters χ and δ_r both impact the decision to become a landlord, but they have different implications for household behavior. When the landlord utility loss parameter χ is greater than zero, a household will only become a landlord if rental income increases utility by enough to offset the fact that χ reduces the utility received by a landlord from all consumption of housing and shelter. Owners of large houses are able to rent out more space, and consequently are able to obtain more rental income than owners of small houses, so they are more likely to find it optimal to pay the landlord utility cost χ in order to obtain rental income. In this sense, χ operates much as a fixed cost of being a landlord would operate in the model. In contrast, an increase in δ_r, holding ρ fixed, reduces the profitability of renting out a unit of housing for all households, and is effectively an increase in the marginal cost of being a landlord.

Estimated Parameters ($\mathbf{\Phi}$): Table 2 shows the estimated parameters, and Table 3 demonstrates that the model matches the empirical moments used in estimation well. The estimate of the discount factor, 0.959, appears reasonable. To put the estimate of δ_r in context, recall that we assume that owner occupied housing depreciates at rate $\delta_0 = 0.025$, so our estimate

model are replaced by newborn descendants who do not, however, inherit the asset positions of the dead, we calibrate β to ensure that households do not borrow excessively and to generate a realistic borrowing behavior of households in our model economy.

of δ_r indicates that the depreciation rate for rented property is 1.2 percentage points greater than the depreciation rate of owner occupied property. The estimate of the landlord utility loss parameter, χ, indicates that landlords incur only a 2.4 percent utility loss due to the burden of managing a rental property. The relatively small magnitude of χ indicates that the decision about whether or not to become a landlord is primarily determined by economic factors which impact the rate of return to investing in rental property, and credit constraints which limit the ability of households to purchase rental property.

Table 2: Estimated Parameters

Parameter	Value
Discount Factor β	0.959
Consumption Share α	0.720
Depreciation of Rental Property δ_r	0.037
Landlord Utility Loss χ	0.024

Table 3: Calibration Targets

Moment	Data	Model
Home-ownership rate	0.66	0.66
Landlord rate	0.10	0.10
Imputed rent-to-wage ratio	0.25	0.25
Fraction of homeowners with collateral debt	0.65	0.64

4.6 Moments not Targeted in the Estimation

As an external test of our model, Table 4 reports several other key statistics generated by the model that were not targeted in the estimation and compares them to statistics that are either drawn from other studies or computed from the 1998 Survey of Consumer Finances (SCF). Appendix D describes how we compute the SCF statistics. As can be seen in the table, the predictions of the model fall well within the range of estimates based on U.S. data. Overall, the ability of the model to replicate a number of key moments that were not targeted during

Table 4: Other Moments (average ratios)

Moment	Model	Data	Data Source
Loan to value ratio for homeowners	0.31	0.35	SCF 1998
Housing value to total income ratio for homeowners	4.02	4.43	SCF 1998
Loan to total income ratio for homeowners	1.34	1.28	SCF 1998
Net worth to total income ratio for homeowners	3.16	3.53	SCF 1998
House price-rent ratio	11.3	8 - 15.5	Various studies*

Notes*: The U.S. Department of Housing and Urban Development and the U.S. Census Bureau report a price-rent ratio of 10 in the 2001 Residential Finance Survey (chapter 4, Table 4-2). Garner and Verbrugge (2009), using Consumer Expenditure Survey (CE) data drawn from five cities over the years 1982-2002, report that the house price to rent ratio ranges from 8 to 15.5 with a mean of approximately 12. The cities included in this analysis are Chicago, Houston, Los Angeles, New York, and Philadelphia.

the calibration is encouraging.

4.7 Cross-sectional Implications of the Model

In our economy, renters are typically hand-to-mouth agents at the bottom of the wealth distribution who consume little housing. Nearly 68 percent of renters live in the smallest shelter space that is not available for sale, which we call a "room".[15] The rest of renters inhabit the smallest-sized house. In general, homeownership is preferred to renting, mostly due to the favorable tax-treatment of homeownership, so households who can afford a down payment on a house typically purchase one.[16]

Interestingly, the option to become a landlord exerts an important influence on agents' decisions in our model economy, and represents and an additional reason why ownership may be preferred to renting. First, rental units provide investment income. Second, the option to become a landlord is an important risk-management mechanism for homeowners facing adverse income shocks; exercising this option allows some to maintain higher consumption than would be otherwise possible, and to avoid sizable transactions costs associated with downsizing (see Appendix A for details on how transaction costs affect the rental market).

[15] The smallest-size shelter unit, which we call a "room," captures the idea that agents can also rent a very small living space that is not, however, available for sale. For example, a person can share a room with a roommate, or can rent a room while sharing a kitchen.

[16] Appendix B shows how the exclusion of imputed rental income from taxable income impacts the cost of housing for homeowners.

Table 5: Distribution of Landlords by Labor Income

Labor Income Group	% Landlords	% Total Rental Property
Group 1	3.32	1.7
Group 2	15.02	10.2
Group 3	33.85	20.7
Group 4	15.44	20.8
Group 5	14.47	20.8
Group 6	12.32	17.7
Group 7	5.58	7.8

Note: Labor income group refers to the seven discrete wage levels that are used to approximate the continuous wage process.

While only ten percent of agents are landlords in the baseline steady state, the varied motivations to become a landlord lead to a diverse landlord pool. Indeed, Table 5 indicates that landlords can be found among all income groups. While the majority of landlords are either middle or high income households owning large properties, nearly 20% of landlords are in the poorest two categories. These predictions are qualitatively consistent with the statistics reported by Chambers, Garriga and Schlagenhauf (2009c) who, using the 1996 Property Owners and Managers Survey, find that 25 percent of households receiving rental income are low-income households with annual earnings below $30,000, compared to 30 percent of high-income households with annual earnings over $100,000 (see their Table 2).

5 House Prices and Rents: Impact of Changed Fundamentals

The estimated model is employed to analyze the observed changes in house prices, rents, and the price-to-rent ratio between 1995 and 2005. The analysis is conducted in two steps. First, we study the model's predictions about the responsiveness of house prices and rents, and the price-rent ratio, to changes in interest rates, borrowing constraints, household incomes, and the combination of these macroeconomic factors in the steady-state housing market equilibrium. As such, all of the analysis in Section 5 is based on comparisons of different steady

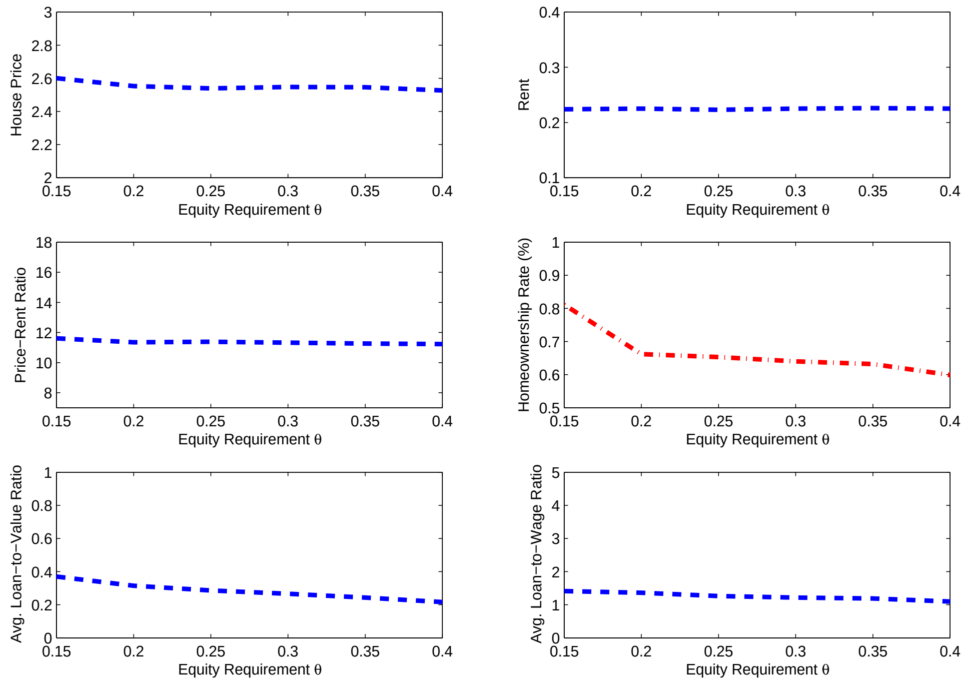


Figure 2: The Housing Market Equilibrium Under Different Equity Requirements

state economies. In the second step, presented in Section 6, we extend the model to include deterministic dynamics of house prices and rents. In this analysis, we study the effects of an unanticipated permanent change in the interest rate and required down payment along a transition path between two steady states. As a cross-check, we also study the model's implications for the homeownership rate, loan-to-income, and loan-to-value ratios.

5.1 Relaxation of Down Payment Requirements in a Steady-State Economy

Figure 2 illustrates the impact of changes in the minimum down payment requirement, θ, on equilibrium housing market outcomes. As θ is lowered from 40 percent to 15 percent, the equilibrium house price rises by 2.5 percent while the rent decreases by 1 percent, so the price-rent ratio increases by 3.4 percent. A reduction in θ more in line with the recent U.S. experience, from 0.20 to 0.15, leads to a 1.8 percent increase in the house price, and a trivial (0.04 percent)

Table 6: The Distribution of Owned Housing Under Different Downpayment Requirements

House Size	20% Downpayment		15% Downpayment	
	% Households	% Housing Stock	% Households	% Housing Stock
Renter	33.8	0.0	18.9	0.0
Small-size property	13.9	11.3	32.6	26.0
Medium-size property	48.5	70.8	47.0	67.0
Large-size property	3.84	17.7	1.51	7.1

Notes: For ease of exposition, this table aggregates the 11 house sizes in our model into the categories small, medium, and large.

decrease in rent.[17] Neither house prices nor rents respond strongly to θ, so a lessening of credit constraints cannot by itself explain the large run-up in house prices observed in recent years.

That said, the homeownership rate does respond strongly to changes in down payment requirements. When θ is lowered from 0.40 to 0.20, the homeownership rate rises from 60 to 66 percent; when θ falls further to 0.15, the homeownership rate increases to 81 percent. The loan-to-wage ratio and the fraction of homeowners in debt also rise as θ falls.

The key to understanding the large effect of θ on homeownership, but the small effect of θ on house prices, is the fact that housing market responses are primarily driven by households for whom the minimum down payment is a binding constraint: low-income, low-savings households. These households are numerous, but since these entrants are marginal in terms of income and wealth, their impact on house prices is modest. To demonstrate this, Table 6 shows changes in the steady state distribution of households across house sizes when θ falls. While the fraction of households who own small-size houses jumps up because of the influx of new homeowners into the housing market, the fraction of households owning the medium- and large-size properties actually declines, and so does the rental supply. Thus, the equilibrium response of landlords is effectively to sell housing to their tenants, and they are willing to do this because the house price has risen while the rent has remained virtually constant.[18]

[17]Chambers, Garriga and Schlagenhauf (2008) document decreases in the average down payment requirement for conventional mortgages since 1995, while Chomsisengphet and Pennington-Cross (2006) document similar trends in the subprime lending markets. In general, the fraction of households with a loan to value ratio greater than 90 percent rose from 10 percent in 1990 to 25 percent by 1995 before retracting slightly to 18 percent in 2005, according to the Federal Finance Board.

[18]This statement is not literally true, since the decrease in the downpayment discussed in this section is based

Table 7: The Partial and Equilibrium Effects of a Reduction in the Equity Requirement to 15%

	Baseline	15% Equity Requirement	
		Fixed Prices	Equilibrium Prices
	(1)	(2)	(3)
House Price	2.55	2.55	2.60
Rent	0.22	0.22	0.22
Share of Homeowners	0.66	0.81	0.81
Share of Renters	0.34	0.19	0.19
Share of Landlords	0.10	0.11	0.08
Share of Owner-Occupiers	0.56	0.70	0.73
Share of Homeowners in Debt	0.64	0.69	0.67

Table 7 highlights the important role of general equilibrium forces in generating housing market outcomes. Column (2) displays a partial equilibrium experiment, which counterfactually holds both house prices and rents fixed at the baseline market-clearing level while reducing θ from 20 to 15 percent. In contrast, Column (3) reports the general equilibrium impact of this decrease in θ; here both house prices and rents adjust to clear the housing and rental markets. As noted above, rent is nearly unresponsive to θ in general equilibrium; the initial rent level of 0.22 holds in both the partial and general equilibrium experiments. The share of renters (equivalently, the share of homeowners) is also essentially the same across experiments: a reduction in θ causes the share of renters to fall from 34 to 19 percent in either case. However, the share of landlords in the economy varies significantly across experiments. In the partial equilibrium experiment, the share of landlords in the economy rises from 10 to 11 percent because when the house price and rent are held constant, a decrease in θ allows a small fraction of households to purchase larger properties and become landlords. However, when prices are allowed to adjust, the share of landlords actually falls from 10 to 8 percent. This happens because the increase in the equilibrium house price and falling rent reduce the rate of return to being a landlord. As noted above, despite the large increase in homeownership, only a 2 percent increase in the house price is needed to clear the market and accommodate these new homeowners.

on a comparison of two different steady state economies.

Our results are consistent with several recent studies which document a strong relationship between the size of the down payment requirement and homeownership (e.g., Chambers, Garriga, and Schlagenhauf (2009a), Díaz and Luengo-Prado (2008), and Ortalo-Magné and Rady (2006)). These studies suggest that, while financial sector innovations have minimal impact for existing homeowners, lower down payment requirements strongly affect households for whom the high down payment is a binding constraint; the initially excluded households enter the housing market and the homeownership rate rises. Further support is found in the empirical findings of Ortalo-Magné and Rady (1999), Muellbauer and Murphy (1990), and Muellbauer and Murphy (1997), who document that decreases in the down payment requirements in England and Wales after the financial liberalization of the early 1980s were one of the two most important factors associated with unprecedented increases in young-household homeownership (the second factor being optimistic appreciation expectations).

In summary, the model clearly indicates that in the absence of changes in other factors, a relaxation of borrowing constraints is important for understanding homeownership rates but cannot by itself account for the magnitude of the recent increase in house prices. With this result in mind, the next sections of the paper examine the impact of changes in the interest rate and income on the equilibrium house prices and rents.

5.2 Changes in the Interest Rate in a Steady-State Economy

Figure 3 shows the evolution of the real contract and effective mortgage rates on conventional single-family mortgages in the United States between 1985 and 2005.[19] The real mortgage rate for residential property oscillated around the 5 percent mark between 1990 and 1997, but then started tending downwards, reaching 2.5 percent in 2005.

Figure 4 illustrates the impact of changes in the real risk-free rate, r, on equilibrium housing

[19] The effective rate represents the sum of the contract rate and the discounted initial fees and charges. The estimates are provided by the Federal Housing Financing Board. The nominal rate is deflated by year-to-year CPI inflation.

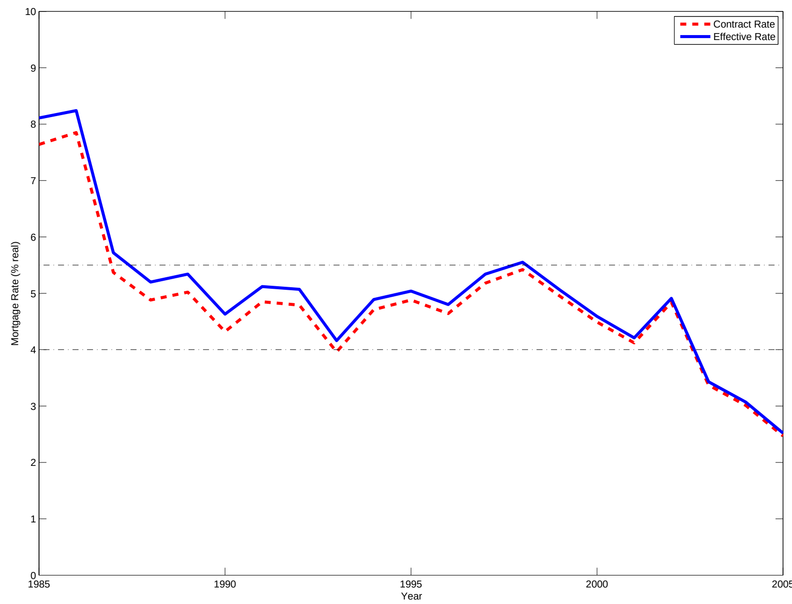


Figure 3: The Evolution of Real Mortgage Rates in the United States

market outcomes. Since the mortgage interest rate r^m is determined by a constant markup, κ, over r, changes in r directly translate into changes in r^m; hence changes in r affect both the cost of borrowing and the rate of return on saving.[20] As can be seen in the figure, interest rate changes have a large effect on house prices and rents. When r is lowered from 6 percent to 1 percent, the equilibrium house price increases by 33 percent, while the equilibrium rent decreases by 14 percent, leading to a 54 percent increase in the price-rent ratio (from 9.9 to 15.2). When r is lowered from 4 percent to 2 percent, a decrease broadly consistent with the actual decline between 1995 and 2005, the house price level rises by 16.4 percent, the rent falls by 2.6 percent, and the price-rent ratio rises by 20 percent from its initial level of 11.3.

Lower interest rates reduce the cost of household borrowing and reduce the rate of return on household savings. Both effects increase demand for owned housing. On the intensive margin, demand for housing services rises, both due to the lower opportunity cost and to the lower costs

[20]The mortgage spread, defined as the difference between the real mortgage rate on a 30-year conventioal fixed-rate mortgage and the interest rate on a 30-year constant maturity Treasury, fluctuated in a relatively narrow range between 1 and 2 percent since 1995, although the mark-up fell temporarily below one percent between 1991 and 1993.

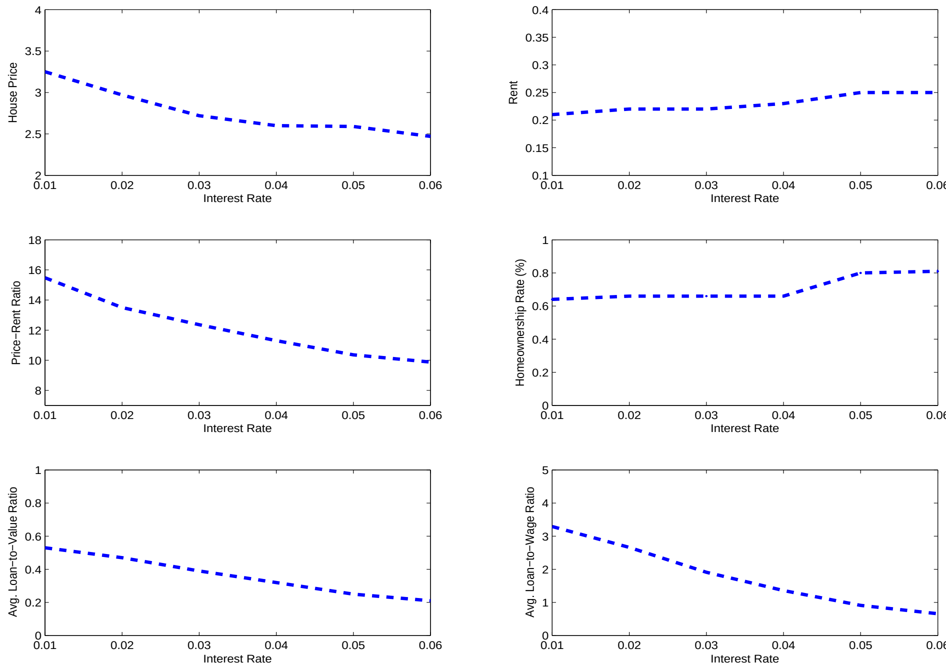


Figure 4: The Housing Market Equilibrium Under Different Interest Rates

of financing a given mortgage; thus, house prices rise, which prices out some of the less wealthy. At the same time, there is a portfolio shift: rental property investment becomes relatively more attractive as borrowing costs fall and as the return to the alternative investment, deposits, falls. Despite the rise in house prices, which ceteris paribus raises the cost of becoming a landlord, the net effect is that the supply of rental properties rises, and rents fall. For example, when the interest rate decreases from 4 to 2 percent, the aggregate supply of rental property increases by 4 percent, while the rent falls from 0.22 to 0.21.

Perhaps surprisingly, the 50 percent reduction in the interest rate from 4 percent to 2 percent has almost no impact on the homeownership rate (Figure 4). This is caused by general equilibrium price effects, which are illustrated in Table 8. As in Table 7, column (2) displays the partial equilibrium counterfactual, while column (3) displays the general equilibrium outcome. If house prices and rents did not adjust, the homeownership rate would rise from 66 percent to 81 percent, reflecting the lower cost of consuming owned housing services and the reduced

Table 8: The Partial and Equilibrium Effects of the Interest Rate Reduction to 2%

	Baseline ($r = 0.04$)	Reduction of Interest Rate to 2%	
		Fixed Prices	Equilibrium Prices
	(1)	(2)	(3)
House Price	2.55	2.55	2.97
Rent	0.22	0.22	0.21
Share of Homeowners	0.66	0.81	0.66
Share of Renters	0.34	0.19	0.34
Share of Landlords	0.10	0.49	0.10
Share of Owner-Occupiers	0.56	0.32	0.55
Share of Homeowners in Debt	0.64	0.94	0.84

attractiveness of saving relative to housing investment. At the same time, the fraction of landlords in the economy would rise from 10 percent to nearly 50 percent, because when q and ρ are held constant, a decrease in r increases the rate of return to being a landlord and decreases the rate of return to the alternative of holding deposits. In equilibrium, however, higher house prices increase the minimum down payment, and the lower interest rate makes it difficult for prospective homeowners to save up for it. Furthermore, equilibrium rent decreases from 0.22 to 0.21, despite the fact that house prices are rising, making renting relatively more attractive, and reducing the return obtained by landlords.

To provide some intuition for the inverse relationship between the price-rent ratio and interest rates in our model, it is useful to examine a simplified model that abstracts from credit constraints, discreteness, transactions costs, or utility loss for landlords; see Appendix B. In such an environment, an optimizing landlord holding a mortgage chooses h' in order to satisfy

$$\rho = \frac{q(1 + (1-\tau^y)\left(\tau^h + \delta_r\right) - \tau^y \tau^{LL} - \frac{1}{1+(1-\tau^y)r^m})}{(1-\tau^y)}. \quad (14)$$

The steady state price-rent ratio (q/ρ) is inversely related to the interest rate. Of course, equation 14 will not hold exactly in our full model with discreteness and frictions. Furthermore, it cannot on its own predict the individual responses of house prices and rents to a change in the interest rate; determining those responses requires solving for the the pair of market prices which jointly clear the markets for shelter and owned housing.

It is also useful to place our results in the context of the widely-used representative rental firm model of the housing market developed by Gervais (2002). In this model, the price-rent ratio is determined by the arbitrage condition

$$\frac{q}{\rho} = \frac{1}{r + \delta_r},$$

which equates the returns between housing and deposits. As in our model, there is an inverse relationship between q/ρ and r, and the sensitivity of the price-rent ratio to the interest rate depends upon the housing depreciation rate. However, in this class of model, there is no theory of the individual responses of house prices and rents to a change in the interest rate; the price of housing (q) is normalized to one due to the assumption of costless conversion between the consumption good and the housing good, and the above relationship only determines a price-rent ratio. Our model is distinctive in providing a quantitative theory of how both equilibrium house prices and rents respond to changes in various parameters, given the existence of important frictions and tax wedges. In our view, this is a significant distinction, particularly when viewed within the context of the recent housing boom and bust.

5.3 Changes in Income in a Steady-State Economy

A large body of empirical literature identifies the level and growth rate of income as an important determinant of house price dynamics (see, for example, Poterba (1991), Englund and Ioannides (1997), Muellbauer and Murphy (1997), Malpezzi (1999), and Sutton (2002)). In the United States, real hourly wages increased by 9.4 percent between 1995 and 2005.[21]

Figure 5 summarizes the impact of changes in income on the housing market equilibrium. In

[21] This calculation is based on the BLS Current Employment Statistics (CES) real wage data, series ID CES0500000032.

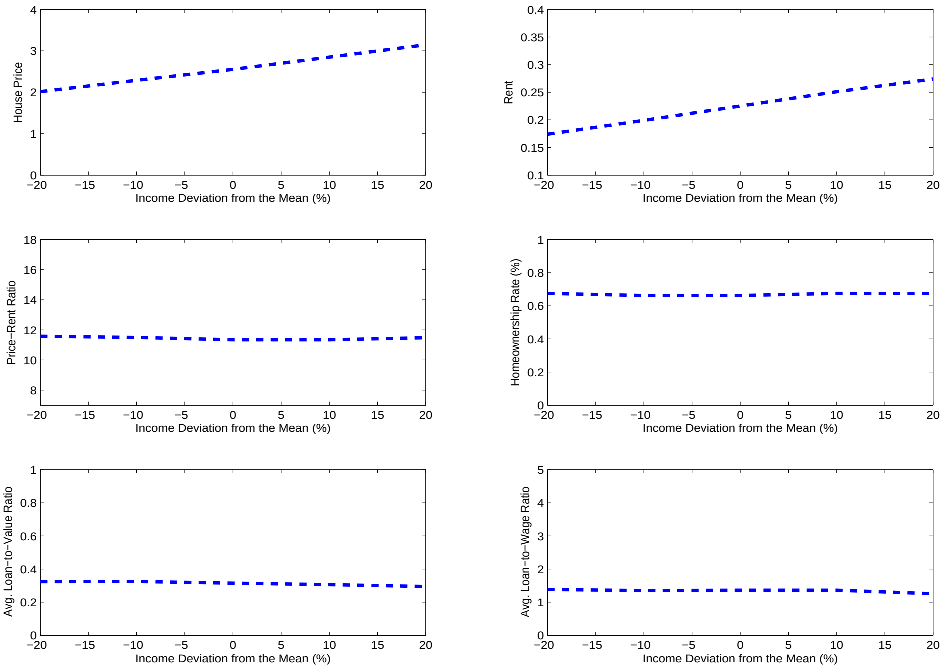


Figure 5: The Housing Market Equilibrium Under Different Income Levels

our experiment, we assume that household wages rise at the same rate across all wage groups. The model suggests that both house prices and rents increase at about the same rate as wages.[22] For example, when the wage level increases by 10 percent relative to the benchmark economy, both the equilibrium house price and the rent rise by approximately 11 percent, so the house price-rent ratio stays approximately constant. Since the relative price of obtaining housing services via homeownership versus renting remains unchanged, symmetric changes in income of the sort examined here have no effect on the homeownership and landlord rates.

Once again, equilibrium price effects play a central role, as illustrated in Table 9. As in Tables 7 and 8, column (2) displays the partial equilibrium counterfactual, while column (3) displays the general equilibrium outcome. Where house prices and rents not allowed to adjust,

[22] The actual changes in the income levels were not, however, symmetric. Heathcote, Perri, and Violante (2010) document the changes in the U.S. earnings inequality between 1967 and 2006. Using the Current Population Survey data, the authors find that the real earnings of the bottom decile of the earnings distribution did not, on average, grow between 1985 and 2000, although the earnings of the top earnings distribution grew steadily over the sample period (see their Figure 7). The authors also find that the wage dynamics of the bottom decile of the earnings distribution are very similar to those for the median workers. Given our stylized income process, we leave the exploration of asymmetric income changes to further work.

Table 9: The Partial and Equilibrium Effects of a 10% Increase in Income

	Baseline	10% Increase in Income	
		Fixed Prices	Equilibrium Prices
	(1)	(2)	(3)
House Price	2.55	2.55	2.85
Rent	0.22	0.22	0.25
Share of Homeowners	0.66	0.92	0.67
Share of Renters	0.34	0.21	0.33
Share of Landlords	0.10	0.08	0.11
Share of Owner-Occupiers	0.56	0.71	0.56
Share of Homeowners in Debt	0.64	0.70	0.64

rising income would have a substantial impact on the housing market, with the homeownership rate increasing from 66 to 92 percent, reflecting the fact that more households are able to afford the down payment and mortgage payments required to purchase a house. In addition, many households would stop renting out their units as they could more easily cover their mortgage payments: the share of owner-occupied housing would increase from 0.56 to 0.71. But in equilibrium, house prices and rents rise proportionally, leaving the proportions of renters, homeowners, and landlords unchanged.

5.4 Combined Changes in Market Fundamentals

As discussed in the preceding sections, neither declines in the real interest rate, nor relaxation of borrowing constraints, nor rising incomes can on their own account for the increases in the price-rent ratio, homeownership rate, and household debt between 1995 and 2005. This section examines the combined effects of changes in these fundamentals on equilibrium housing market outcomes. Figure 6 depicts the percentage deviation of the steady state price-rent ratio from the baseline economy for a range of interest rates and required down payments. Point A represents the calibrated baseline economy with an interest rate on deposits, r, of 4 percent and a required down payment, θ, of 20 percent. As illustrated in the figure, the price-rent ratio rises with a falling interest rate and lower down payment requirement. In fact, the price-rent ratio

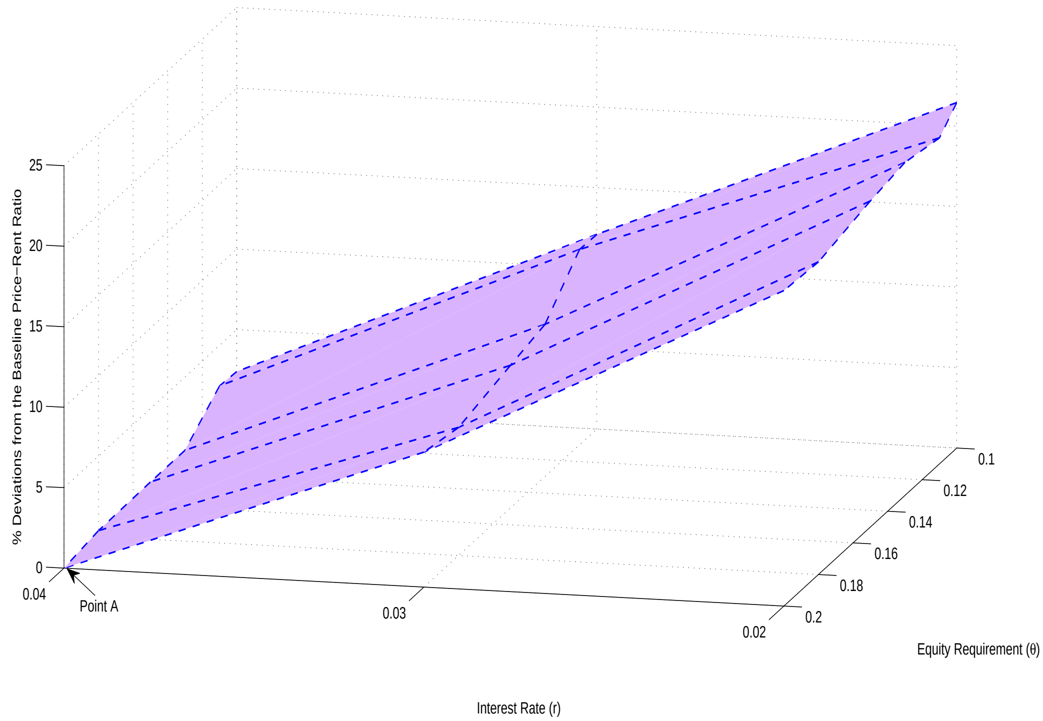


Figure 6: Percentage Deviations of the House Price-Rent Ratio from the Baseline (Point A) Under Different Interest Rates and Required Downpayment

increases by 20 percent over its baseline value for a reasonable representation of the recent U.S. experience; i.e., a reduction in the interest rate from 4 to 2 percent and in down payment from 20 to 15 percent. This represents a sizable portion of the actual increase: the U.S. price-rent ratio increased by 36 percent from 1995 to 2005, and by 26 percent between 2000 and 2005.[23]

Table 10 provides a more comprehensive analysis of the simulated effects by showing the percentage deviations in house prices, rents, and the price-rent ratio from their baseline values (column (1)). To facilitate a comparison of the model's predictions to the data, columns (4) and (5) show recent changes in the U.S. Column (2) shows that when income is held constant, lowering θ and r raises house prices, lowers rents, and consequently increases the price-rent ratio. In addition, the homeownership rate rises from 66 percent to 67 percent. Column (3) of Table 10 shows that increasing wages by 10 percent while decreasing θ and r does not change

[23]The U.S. price-rent ratio is constructed using the FHFA house price index and the BLS Rent of Primary Residence Index.

Table 10: The Combined Effects of Interest Rate, Required Downpayment, and Income Changes

	Baseline	Changes in r and θ (%Δ from Baseline)		U.S. Data (%Δ)	
	r=0.04	r=0.02	r=0.02	1995-2005	2000-2005
	θ=0.2	$\theta = 0.15$	$\theta = 0.15$		
		$\Delta w = 0\%$	$\Delta w = 10\%$		
	(1)	(2)	(3)	(4)	(5)
house price	2.55	16.1%	28.1%	46.3%	31.1%
rental price	0.22	-3.5%	7.1%	7.5%	4.2%
price-rent ratio	11.3	20.0%	20.0%	36.1%	25.9%

Notes: Columns (2) - (3) show percent changes in the equilibrium value of each variable from the baseline model shown in column (1). Columns (4) and (5) show the actual percent changes observed in the U.S. over two different time periods.

the price-rent ratio compared to the scenarios where income is held constant.[24] However, the model also predicts that higher income will cause a small increase in rents that is quite close to the growth in rents observed in the data. As noted above, the actual increase in the house price-rent ratio from 1995 to 2005 was about 36 percent, so a plausible calibration of the model can account for over one-half of the observed increase. These results suggest that the changes in the interest rate and required down payment observed in the United States had a substantial impact on the price-rent ratio.

In our model, holding house prices and rents constant, when the mortgage interest rate and required down payment fall, the demand curve for rental property shifts inward because households switch from renting to owning as homeownership becomes more affordable. At the same time, the supply curve for rental property shifts to the right because when θ and r decrease, more households are able to afford down payments and mortgage payments on rental properties. In addition, since both the mortgage rate and rate of return on deposits fall when interest rates decrease, investing in rental property becomes more attractive relative to the alternative of holding bank deposits. The net result of the declining demand and increasing supply in the rental market is a decrease in the equilibrium rent. At the same time, the demand for housing (or homeownership) increases when the interest rate and the required down payment

[24] A 10 percent increase in real wages is approximately what was observed in the U.S. between 1995 and 2005.

decrease because more households can afford to purchase homes, and existing homeowners can afford larger homes. Given that the supply of housing is fixed, the equilibrium house price rises. It follows that the price-rent ratio increases as the house price increases and rent falls in response to the change in fundamentals.

Finally, we examine the robustness of these results to relaxing the assumption of a fixed stock of housing. A 5-percent increase in the supply of housing, which is roughly in line with the data, combined with a lowered interest rate (from 4 to 2 percent) and down payment requirement (from 20 to 15 percent) leads to a proportional decrease in both the house price and rent, and does not affect the sensitivity of the price-rent ratio to changes in θ and r. Regardless of whether the housing stock is held fixed or allowed to increase by 5 percent, the price rent ratio increases by 20 percent when interest rate is lowered and down payment requirement is relaxed.

6 Transitional Dynamics

Up to this point, we have confined our analysis to comparisons of different steady state economies. This section studies the transitional dynamics of the housing market between two steady states. We assume that the economy is initially in a steady state that corresponds to the baseline calibration of the model, where the interest rate is 4 percent and the required down payment is 20 percent. Starting from this initial steady state, the interest rate and required down payment unexpectedly and permanently fall to $r = 0.02$ and $\theta = 0.15$. We solve for equilibrium movements of house prices and rents along the perfect foresight transition path that ends at the new steady state. Along the transition path, all agents correctly forecast the sequence of equilibrium house prices and rents which leads to the new steady state, and the housing market clears in each time period.[25]

Figure 7 shows the transition path for the house price, rent, and price-rent ratio. In the

[25] Appendix C describes the solution of the model along the transition path in more detail.

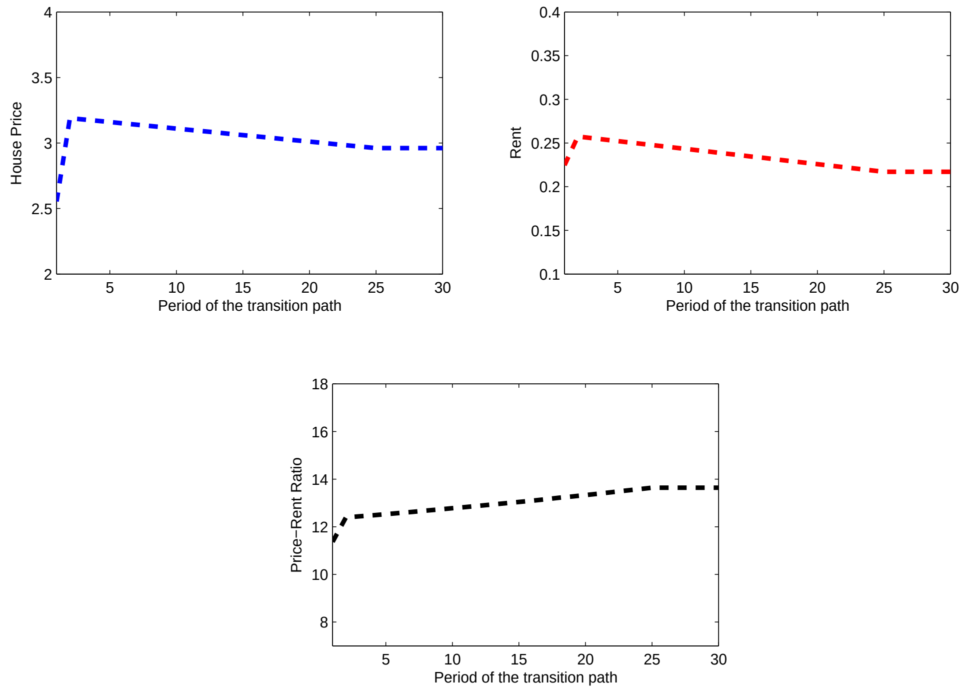


Figure 7: Transition Paths: Simultaneous Decrease in Interest Rate and Minimum Downpayment

first period of the transition, both the house price and rent overshoot their long-run steady state values: the house price increases by 25 percent while the rent increases by 15 percent. Although the price-rent ratio does not overshoot, it jumps by 9 percent on impact. After the initial spike in the house price and rent, the equilibrium prices decline gradually over time, and the price-rent ratio steadily increases to its new steady state level.

Why do house price and rent overshoot? Several mechanisms are jointly operative. The first mechanism is a portfolio reallocation between deposits and housing by households. The initial steady state features a relatively large amount of financial wealth (deposits), owing to the high (4 percent) rate of return and relatively high (20 percent) down payment requirement. An unexpected decline in the interest rate and the down payment requirement give households an incentive to shift their portfolios from deposits into housing, since the rate of return to deposits falls, the mortgage cost of financing falls, and housing consumption becomes more attractive relative to saving. In conjunction with this, a capital gains mechanism is operative. The initial increase in the house price allows existing homeowners to capitalize gains, trade-up, and move to larger homes. This mechanism operates in the same manner as the one discussed in Ortalo-Magné and Rady (2006), where a fixed supply of start-up homes bolsters the overshooting relative to our model. As a result, housing demand rises sharply, but there is a fixed housing supply, so the surge of funds into housing drives up the house price. Turning to the rental market, two forces jointly operate: on one hand, the lower interest rate reduces the cost of rental investment; on the other, the higher house price increases it. Initially, the house price effect dominates, and rent increases to compensate landlords for the increased cost of rental space. Initially, the homeownership rate stays roughly constant, in part due to higher shelter cost in both the homeownership and rental markets.

The initial spikes in the house price and rent are not sustainable as a long run equilibrium, because they are fueled by the large amount of financial wealth that households accumulated in the high interest rate steady state. Over time, the house price and rent decrease as households draw down their financial wealth, and live for more time periods with the low interest rate. Moreover, as the overshooting in prices fades away, more renters shift into homeownership, the homeownership rate rises to its new long run equilibrium level, and rents fall because of the reduced demand for rental space.

7 Conclusion

This paper develops a dynamic equilibrium model of the housing market in which both house prices and rents, not merely their ratio, are determined endogenously. We use the model to study the effect of changes in fundamentals such as the interest rate, required down payment, and income on equilibrium in the housing market. This analysis is motivated by the fact that although house prices, rents, and their ratio are widely used economic indicators, there is no existing quantitative model of how these objects are simultaneously determined by the clearing of markets for owned and rented housing. Without an understanding of this theoretical relationship, it is not possible to determine whether observed changes in the relationship between house prices and rents reflect changing fundamentals or an asset price bubble.

Building on existing models of the housing market, our model incorporates uninsurable earnings risk, incomplete markets, preferential tax treatment of homeowners and landlords, and lumpy transactions costs. Households choose between renting and owning housing as a means of securing shelter services, and also have the option of investing in housing to earn rental income. A key feature of our model is that both the house price and rent are determined in equilibrium by the interactions of households who have heterogeneous levels of income and wealth. When fundamentals change, shifts in the willingness of households to supply property on the rental market have a large impact on the equilibrium house price and rent. This aspect of the model is empirically supported by the observed expansion of the buy-to-lease segment of the housing market during the housing boom.

We document that the interest rate and minimum required down payment reached very low levels by historical standards during the housing boom of 1995-2005, and use our model as a tool to quantitatively evaluate the effects of changes in these fundamentals on house prices and rents. The model predicts that the combination of low interest rates and reduced down payment requirements leads to a large increase in the rational expectations equilibrium house price, but rents remain approximately constant. As a result, the house price-rent ratio increases. In fact, changes in these fundamentals are capable of explaining approximately one-half of the 36 percent increase in the price-rent ratio observed between 1995 and 2005. At the same time, changes in fundamentals generate increases in homeownership and household debt that are consistent with recent U.S. experience.

Throughout our analysis, we have maintained the assumption of rational expectations about future house prices and rents. Piazzesi and Schneider (2009) examine household beliefs during the housing boom, and develop a tractable search model where optimistic traders can push up house prices. Consistent with our focus on credit conditions, these authors find that in the 2003 Michigan Survey of Consumers, the primary reason reported by households for believing that it was a good time to buy a house was favorable credit conditions. Later in the boom, they find that the proportion of households who believed that house prices would continue to increase reached a 25-year high. It is well recognized that incorporating this type of information about expectations into a model such as ours raises many difficult conceptual and practical questions. As such, we leave this extension for future work.

Our model of the housing and rental markets illustrates that large increases in house prices accompanied by comparatively constant rents are consistent with the rational expectations, equilibrium response of the housing market to low interest rates and relaxed down payment requirements. However, quantitatively, changes in fundamentals designed to mimic the U.S. economy between 1995 and 2005 are able to account for only one-half of the observed increase in the price-rent ratio.

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8 Appendix A: Sensitivity Analysis (not for publication)

This section examines the effects of transaction costs and differential depreciation of rented and owner-occupied properties on the level of the price-rent ratio and its responsiveness to interest rate changes.[26] Our results highlight the important role that these features of the model play in determining demand for housing and rental properties, as well as rental property supply.

Our first experiment eliminates transaction costs in the housing market. When the costs of buying (τ^b) and selling (τ^s) a house are reduced from their benchmark values of 7.5 and 2.5 percent to zero, the equilibrium house price increases by 32 percent, the equilibrium rent increases by 24 percent, and the price-rent ratio increases by approximately 7 percent. These effects arise because eliminating transactions costs simultaneously increases the demand for housing and decreases the supply of rental property. The increase in the housing demand

[26] We focus on the interest rate in this section because changes in taxes and depreciation have little impact on the responsiveness of the price-rent ratio to changes in the required downpayment.

is partly caused by an income effect: agents are wealthier, all else constant, in an economy without transactions costs. At the same time, the rental supply declines because in the absence of transactions costs, households choose to downsize instead of renting out their home in the rental market when hit by an adverse income shock. Overall, the fraction of households that trade their house in each period (i.e., $h' \neq h$) rises from 2 to 4 percent when transaction costs are eliminated.

Transactions costs also impact the responsiveness of the price-rent ratio to changes in the interest rate. In the economy without transactions costs, when the interest rate falls from 4 percent to 2 percent, the price-rent ratio increases by 16 percent versus 22 percent in the economy with transaction costs. These results demonstrate the importance of accounting for the lumpy transactions costs incurred during home sales and purchases when modeling the housing market. Ignoring these transactions costs would lead to an understatement of the responsiveness of the price-rent ratio to changes in interest rates and would lead to an overstatement of the level of the price-rent ratio.

To highlight the important effect of differential depreciation of rental and owner occupied properties on the price-rent ratio, we compare the baseline economy where rental properties depreciate at a higher rate ($\delta_r = 0.037$) than owner-occupied space ($\delta_0 = 0.025$) to a counterfactual economy where rented space depreciates at the same rate as owner-occupied space ($\delta_0 = \delta_r = 0.025$). When the depreciation rate of rental property is reduced, the cost of rental investment declines, leading to an increased demand for investment properties. Demand for housing rises, as does rental supply. The equilibrium results are an 8.6 percent increase in the house price, a 6 percent decrease in the rent, and a 15.7 percent increase in the price-rent ratio. At the same time, a reduction in the depreciation rate of rental properties increases the responsiveness of the price-rent ratio to interest rate changes. When r falls from 4 to 2 percent, the price-rent ratio increases by 26 percent compared to 20 percent in the baseline economy where rental properties depreciate at a higher rate than owner-occupied homes.

9 Appendix B: Frictionless Analytical Results (not for publication)

Consider a problem of a homeowner who consumes all housing services yielded by the owned property (e.g., $s = h'$) but also chooses how much to invest into a rental property, h'_r. For simplicity, we assume that mortgage interest payments are fully tax deductible ($\tau^m = 1$), and that there are no borrowing constraints, buying and selling costs, income uncertainty, or landlord utility penalty. The homeowner thus chooses (c, h', h'_r, m', d') to optimally solve:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c, h')$$

subject to initial conditions and

$$\begin{aligned} & c + d' - m' + qh' + qh'_r \\ & \leq w \; + (1+r)d - (1+r^m)\, m + \rho h'_r + qh + qh_r - \tau^y \widetilde{y} - \tau^h qh' - \tau^h qh'_r - \delta_0 qh' - \delta_r qh'_r, \end{aligned}$$

where

$$\widetilde{y} = w + rd + \rho h'_r - \left[r^m m + \tau^h qh' + \tau^h qh'_r + \delta_r qh'_r + \tau^{LL} qh'_r \right].$$

The corresponding first order conditions are:

$$
\begin{aligned}
c &: u_c(c,h') - \lambda = 0,\\
h' &: u_h(c,h') + \lambda(-\tau^y \frac{\partial \widetilde{y}}{\partial h'} - \tau^h q - \delta_0 q - q) + \lambda' q' = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}}{\partial h'} = -\tau^y \tau^h q,\\
h'_r &: \lambda(\rho - \tau^y \frac{\partial \widetilde{y}}{\partial h'_r} - \tau^h q - \delta_r q - q) + \lambda' q' = 0 \text{ where } \tau^y \frac{\partial \widetilde{y}}{\partial h'_r} = \tau^y(\rho - \tau^h q - \delta_r q - \tau^{LL} q),\\
d' &: -\lambda + \lambda'(-\tau^y \frac{\partial \widetilde{y'}}{\partial d'} + (1+r)) = 0 \text{ where } \tau^y \frac{\partial \widetilde{y'}}{\partial d'} = \tau^y r,\\
m' &: \lambda + \lambda'(-\tau^y \frac{\partial \widetilde{y'}}{\partial m'} - (1+r^m)) = 0 \text{ where } \tau^y \frac{\partial \widetilde{y'}}{\partial m'} = -\tau^y r^m.
\end{aligned}
$$

Combining the first order conditions with respect to c and h', we obtain the expression representing the user cost of a homeowner,

$$\frac{u_h(c_t,h')}{u_c(c_t,h')} = q(1 + (1-\tau^y)\tau^h + \delta_0) - \frac{\lambda'}{\lambda} q'. \tag{15}$$

Similarly, the first order condition with respect to h'_r gives the asset pricing equation for a landlord in this frictionless economy:

$$\rho = \frac{q(1 + (1-\tau^y)\tau^h + (1-\tau^y)\delta_r - \tau^y \tau^{LL}) - \frac{\lambda'}{\lambda} q'}{(1-\tau^y)}. \tag{16}$$

Equations 15 and 16 can be used to compare the cost of housing of a renter to that of a homeowner. Landlords can access deductions not available to homeowners, such as physical depreciation of the rental property and maintenance costs. However, rental property depreciates at a higher rate, and rental income (unlike user-occupied space) is taxable. Letting $C := 1 + (1-\tau^y)\tau^h - \frac{\lambda'}{\lambda}$, then in steady-state, equations 15 and 16 become

$$
\begin{aligned}
\frac{u_h(.)}{u_c(.)} &= q(C + \delta_0)\\
(1-\tau^y)\rho &= q(C + (1-\tau^y)\delta_r - \tau^y \tau^{LL})
\end{aligned}
$$

Clearly, the fact that imputed rental income from owner-occupied shelter is excluded from taxable income is of central importance when examining the decision of a homeowner to supply rental property. At our calibrated parameter values, $\frac{u_h(.)}{u_c(.)} < \rho$, primarily due to the tax treatment of rental income – a result consistent with Díaz and Luengo-Prado (2008). Moreover, Díaz and Luengo-Prado (2008) show that when there is a spread between the return on deposits and the mortgage rate (as in here), then households do not simultaneously hold deposits and debt; see their Proposition 2. As a result, using the first order conditions with d' and m', the user cost and the landlord asset pricing equations above can be further simplified by substituting $\frac{\lambda'}{\lambda} = \frac{1}{1+(1-\tau^y)r}$ if the homeowner holds deposits, or $\frac{\lambda'}{\lambda} = \frac{1}{1+(1-\tau^y)r^m}$ if the homeowner holds a mortgage loan.

10 Appendix C: Solving the Model (not for publication)

10.1 Finding Equilibrium in the Housing and Rental Markets

Equilibrium in the housing and rental markets is formally defined by the conditions presented in Section 3. In practice, the market clearing rent (ρ^*) and house price (q^*) are found by finding the (q^*, ρ^*) pair that simultaneously clear both the housing and shelter markets in a simulated economy. The market clearing conditions for a simulated cross section of N agents are

$$\sum_{i=1}^{N} h_i'(q^*, \rho^*|x) = H \tag{17}$$

$$\sum_{i=1}^{N} s_i'(q^*, \rho^*|x) = H. \tag{18}$$

The optimal housing and shelter demands for each agent are functions of the market clearing steady state prices and the agents other state variables (x). Solving for the equilibrium of the housing market is a time consuming process because it involves repeatedly re-solving the optimization problem at potential equilibrium prices and simulating data to check for market clearing until the equilibrium prices are found. The algorithm outlined in the following section exploits theoretical properties of the model such as downward sloping demand when searching for market clearing prices. Taking advantage of these properties dramatically decreases the amount of time required to find the equilibrium relative to a more naive search algorithm.

10.2 The Algorithm

Let q_k represent the kth guess of the market clearing house price, let ρ_k represent a guess of the equilibrium rent, and let $\rho_k(q_k)$ represent the rent that clears the market for housing conditional on house price q_k. The algorithm that searches for equilibrium is based on the following excess demand functions

$$ED_k^h(q_k, \rho_k) = \sum_{i=1}^{N} h_i'(q_k, \rho_k | x) - H \tag{19}$$

$$ED_k^s(q_k, \rho_k) = \sum_{i=1}^{N} s_i'(q_k, \rho_k | x) - H. \tag{20}$$

The equilibrium prices q^* and ρ^* simultaneously clear the markets for housing and shelter, so

$$ED_k^h(q^*, \rho^*) = 0 \tag{21}$$

$$ED_k^s(q^*, \rho^*) = 0. \tag{22}$$

The following algorithm is used to find the market clearing house price and rent.

1. Make an initial guess of the market clearing house price q_k.

2. Search for the rent $\rho_k(q_k)$ which clears the market for owned housing conditional on the current guess of the equilibrium house price, q_k. The problem is to find the value of $\rho_k(q_k)$ such that $ED_k^h(q_k, \rho_k(q_k)) = 0$. This step of the algorithm requires re-solving the

agents' optimization problem at each trial value of $\rho_k(q_k)$, simulating data using the policy functions, and checking for market clearing in the simulated data. One useful property of the excess demand function $ED_k^h(q_k, \rho_k(q_k))$ is that conditional on q_k, it is a strictly decreasing function of ρ_k. Based on this property, $\rho_k(q_k)$ can be found efficiently using bisection.

3. Given that the *housing* market clears at prices $(q_k, \rho_k(q_k))$, check if this pair of prices also clears the market for *shelter* by evaluating $ED_k^s(q_k, \rho_k(q_k))$.

    (a) If $ED_k^s(q_k, \rho_k(q_k)) < 0$ and $k = 1$, the initial guess q_1 is too high, so set $q_{k+1} = q_k - \varepsilon$ and go to step (2). This initial house price guess q_1 is too high if $ED_k^s(q_k, \rho_k(q_k)) < 0$ because $ED_k^s(q_k, \rho_k(q_k))$ is decreasing in q_k.

    (b) If $ED_k^s(q_k, \rho_k(q_k)) > 0$ set $k = k + 1$ and $q_{k+1} = q_k + \varepsilon$ and go to step (2).

    (c) If $ED_k^s(q_k, \rho_k(q_k)) = 0$, the equilibrium prices are $q^* = q_k$, $\rho^* = \rho_k(q_k)$, so stop.

10.3 Solving for the Transition Path

This appendix describes the solution of the model along the perfect foresight transition path between two steady states. In the first time period, the economy is in the initial, high interest rate, high down payment steady state. In time period $t = 2$, the interest rate and minimum down payment unexpectedly, and permanently, decline. Let T represent the number of time periods that it takes for the economy to converge to the final steady state.[27] Let (q^*, ρ^*) and (q^{**}, ρ^{**}) represent the initial and final steady state equilibrium house price and rent. The transition path is a sequence of prices, $\{q_t, \rho_t\}_{t=1}^{T}$, along which the optimal decisions of households clear both the markets for shelter and housing. Solving the household optimization problem along the transition path requires adding time to the state variables listed in the steady

[27] In practice, we set $T = 30$, but find that the economy converges to the new steady state after 25 periods. The computed equiibrium is unchanged by extending the horizon to $T > 30$.

state problem described in section 3 because both current-period prices and future prices affect households' optimal decisions. Given a sequence of prices $\{q_t, \rho_t\}_{t=1}^{T}$, the dynamic programming problem is solved recursively, moving backwards in time from time period T.

The algorithm begins by setting the market clearing prices in periods $t = 1$ and $t = T$ equal to their initial and final steady state values, so $q_1 = q^*$, $\rho_1 = \rho^*, q_T = q^{**}$, $\rho_T = \rho^{**}$. Next, a guess is made for the remaining prices along the transition path, $\{q_t, \rho_t\}_{t=2}^{T-1}$. The transition path is found using the following algorithm:

1. Solve the household problem recursively, moving backward from period T, taking the sequence of prices $\{q_t, \rho_t\}_{t=1}^{T}$ as given.

2. Use the optimal decision rules to simulate data from the model for each period along the transition path.

3. Check for market clearing in each time period using the conditions listed in section 3. If markets clear in all time periods, stop because the transition path has been found. If markets do not clear, make a new guess of the transition path and go back to step 1.

11 Appendix D: SCF Data (not for publication)

The Survey of Consumer Finances (SCF) 2007 is used to construct the moments summarized in Table 4. The SCF is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of U.S. families. The total housing wealth is constructed as the total sum of all residential real estate owned by a household, and is taken to represent the housing wealth qh' in the model. Secured debt (i.e., debt secured by primary or other residence) is used as a model analog of the collateralized debt, m'. The model analogue of the total net worth (i.e., $d' + qh' - m'$) is constructed as the sum of household's deposits in the transaction accounts and the housing wealth (as defined above), net of the secured debt. The total household income reported in the SCF is taken to represent the total household income defined in the model as $y = w + rd' + I^{h'>s}[\rho(h' - s)]$. Both data and the SAS code are available at request, or can be found at the official Survey of Consumer Finances website.