Constant	Description	Units
ρ	Density	kg / m^3
p	Pressure	Pa = kg / m / s^2
T	Temperature	K
α	Specific volume ($= 1/\rho$)	m^3 / kg
$\mathbf{u}$	Velocity vector	m / s
u	Zonal component of the velocity vector	m / s
v	Meridional component of the velocity vector	m / s
w	Vertical component of the velocity vector	m / s
λ	Longitude	rad
ϕ	Latitude	rad
f	Coriolis parameter ($= 2\Omega \sin\phi$)	1 / s
θ	Potential temperature	K

Constant	Description	Value
a	Radius of the Earth	6.37122×10^6 m
g	Acceleration due to gravity at Earth's surface	~ 9.80616 m s^{-2}
G	Gravitational constant	6.673×10^{-11} N m^2 / kg^2
Ω	Rotational velocity of the Earth	7.292×10^{-5} rad s^{-1}
R	Ideal gas constant for dry air	287.0 J / kg / K
c_p	Specific heat capacity of dry air at constant pressure	1004.5 J / kg / K
c_v	Specific heat capacity of dry air at constant volume	717.5 J / kg / K
ν	Kinematic viscosity coefficient	$\sim 1.5 \times 10^{-5}$ m^2 / s

Momentum Equations (Lagrangian Frame)

$$\frac{Du}{Dt} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + \nu\nabla^2 u$$
$$\frac{Dv}{Dt} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + 2\Omega u\sin\phi + \nu\nabla^2 v$$
$$\frac{Dw}{Dt} - \frac{u^2+v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - 2\Omega u\cos\phi + \nu\nabla^2 w$$

Momentum Equations (Eulerian Frame)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - \frac{uv\tan\phi}{a} + \frac{uw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega v\sin\phi - 2\Omega w\cos\phi + \nu\nabla^2 u$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{u^2\tan\phi}{a} + \frac{vw}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + 2\Omega u\sin\phi + \nu\nabla^2 v$$
$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} - \frac{u^2+v^2}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g - 2\Omega u\cos\phi + \nu\nabla^2 w$$
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Continuity Equation (Lagrangian Frame)

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{u}$$

Continuity Equation (Eulerian Frame)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Thermodynamic Equation

$$c_v \frac{DT}{Dt} + p\frac{D}{Dt}\left(\frac{1}{\rho}\right) = J$$

Potential Temperature

$$\theta = T\left(\frac{p_s}{p}\right)^{R/c_p}$$

Ideal Gas Law

$$p = \rho RT$$

Material Derivative

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla$$

Conservation Laws

$$\frac{\partial \phi}{\partial t} + \nabla\cdot\mathbf{F} = \Psi.$$

Hydrostatic Equation

$$dp = -\rho g dz$$

Dry Adiabatic Lapse Rate

$$\Gamma_d = -\frac{\partial T}{\partial z} = \frac{g}{c_p}$$

Geostrophic Wind

$$\begin{aligned} u_g &= -\frac{1}{f\rho}\left(\frac{\partial p}{\partial y}\right)_z = -\frac{1}{f}\left(\frac{\partial \Phi}{\partial y}\right)_p \\ v_g &= \frac{1}{f\rho}\left(\frac{\partial p}{\partial x}\right)_z = \frac{1}{f}\left(\frac{\partial \Phi}{\partial x}\right)_p \end{aligned}$$

Gradient Wind Balance

$$\frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n}$$

Beta-plane Approximation

$$f = f_0 + \beta y, \qquad f_0 = 2\Omega\sin\theta_0, \qquad \beta = \frac{2\Omega\cos\theta_0}{a}$$

Scaling (large-scale mid-latitude systems)

$U \approx 10$ m s^{-1}	$\Delta P \approx 10$ hPa
$W \approx 0.01$ m s^{-1}	$\rho \approx 1$ kg m^{-3}
$L \approx 10^6$ m	$\Delta\rho \approx 10^{-2}$ kg m^{-3}
$H \approx 10^4$ m	$f_0 \approx 10^{-4}$ s^{-1}
$L/U \approx 10^5$ s	$\beta = \frac{\partial f}{\partial y} \approx 10^{-11}$ m^{-1} s^{-1}
$a \approx 10^7$ m	$\nu \approx 10^{-5}$ m^2 s^{-1}

Brunt-Väisälä Frequency

$$\mathcal{N}^2 = \frac{g}{\theta}\frac{\partial \theta}{\partial z}$$

Static Stability Parameter

$$S_p = \left(\frac{R_d T}{p c_p} - \frac{\partial T}{\partial p}\right) = -\frac{T}{\Theta}\frac{\partial \Theta}{\partial p} = \frac{\Gamma_d - \Gamma}{\rho g}$$

Hypsometric Equation

$$z_T = (z_2 - z_1) = \frac{R_d}{g}\int_{p_2}^{p_1}\frac{T}{p}dp$$

Equations of Motion (no curvature)

$$\begin{aligned} \frac{D\rho}{Dt} &= -\rho\nabla\cdot\mathbf{u} \\ \frac{D\mathbf{u}}{Dt} &= -\frac{1}{\rho}\nabla p - g\mathbf{k} + fv\mathbf{i} - fu\mathbf{j} + \nu\nabla^2\mathbf{u}. \end{aligned}$$

Equations of Motion (pressure coordinates)

$$\frac{D\mathbf{v}_h}{Dt} + f\mathbf{k}\times\mathbf{v}_h = -\nabla_p\Phi$$

$$\frac{\partial \Phi}{\partial p} = -\frac{R_d T}{p}$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_p + \frac{\partial \omega}{\partial p} = 0$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} - S_p\omega = \frac{J}{c_p}$$

$$p = \rho R_d T$$

Shallow Water Equations

h	fluid depth
h_s	topography height (above $z = 0$)
H	fluid surface height (above $z = 0$)

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -g\frac{\partial H}{\partial x} + fv \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -g\frac{\partial H}{\partial y} - fu \\ \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) &= 0 \\ H &= h + h_s \end{aligned}$$

Stability Parameter

$$S_p \equiv \frac{RT}{c_p p} - \frac{\partial T}{\partial p} = -\frac{T}{\theta}\frac{\partial \theta}{\partial p} = \frac{(\Gamma_d - \Gamma)}{\rho g}$$

Thermal Wind (Layer Mean Temperature)

$$u_T = u_2 - u_1 = -\frac{R}{f}\left(\frac{\partial \langle T\rangle}{\partial y}\right)_p \ln\left(\frac{p_1}{p_2}\right)$$
$$v_T = v_2 - v_1 = +\frac{R}{f}\left(\frac{\partial \langle T\rangle}{\partial x}\right)_p \ln\left(\frac{p_1}{p_2}\right)$$

Thermal Wind (Layer Thickness)

$$u_T = u_2 - u_1 = -\frac{1}{f}\frac{\partial}{\partial y}(\Phi_2 - \Phi_1)$$
$$v_T = v_2 - v_1 = +\frac{1}{f}\frac{\partial}{\partial x}(\Phi_2 - \Phi_1)$$

Thermal Wind (Vertical Wind Shear)

$$\frac{\partial \mathbf{v}_g}{\partial \ln p} = -\frac{R}{f}\mathbf{k} \times \nabla_p T.$$

$$p\frac{\partial u_g}{\partial p} = +\frac{R}{f}\left(\frac{\partial T}{\partial y}\right)_p,$$
$$p\frac{\partial v_g}{\partial p} = -\frac{R}{f}\left(\frac{\partial T}{\partial x}\right)_p.$$

Vertical Motion (Kinematic Method)

$$\omega(p) = \omega(p_s) - (p_1 - p_2)\left(\frac{\partial \langle u\rangle}{\partial x} + \frac{\partial \langle v\rangle}{\partial y}\right)_p$$

Vertical Motion (Adiabatic Method)

$$\omega(p) = \frac{1}{S_p}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

Surface Pressure Tendency

$$\frac{\partial p_s}{\partial t} \approx \omega(p_s) = -\int_0^{p_s} (\nabla \cdot \mathbf{v})dp$$

Vorticity

$$\boldsymbol{\omega}_a \equiv \nabla \times \mathbf{u}_a, \quad \boldsymbol{\omega} \equiv \nabla \times \mathbf{u}$$

$$\boldsymbol{\omega} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$
$$\zeta = \mathbf{k}\cdot(\nabla \times \mathbf{u}), \quad \eta = \mathbf{k}\cdot(\nabla \times \mathbf{u}_a), \quad \eta = \zeta + f$$

Stokes' Theorem

$$\oint_{\partial A} \mathbf{v}\cdot d\mathbf{l} = \iint_A (\nabla \times \mathbf{v})\cdot \mathbf{k} dA$$

Barotropic Potential Vorticity

$$\frac{D_h}{Dt}\left(\frac{\zeta + f}{h}\right) = 0$$

Vorticity Equation (Lagrangian Form)

$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$
$$-\left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

Quasi-Geostrophic Equations

$$\frac{D_g \mathbf{v}_g}{Dt} = -f_0 \mathbf{k} \times \mathbf{v}_a - \beta y \mathbf{k} \times \mathbf{v}_g$$
$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$
$$\left(\frac{\partial}{\partial t} + \mathbf{v}_g \cdot \nabla\right)\left(-\frac{\partial \Phi}{\partial p}\right) - \sigma\omega = \frac{\kappa J}{p}$$
$$\sigma \equiv -\frac{RT_0}{p}\frac{d \ln \theta_0}{dp}$$

Quasi-Geostrophic Vorticity Equation

$$\frac{D_g \zeta_g}{Dt} = -f_0\left(\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y}\right) - \beta v_g$$

Geopotential Tendency Equation

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\chi = -f_0 \mathbf{v}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)$$
$$-\frac{\partial}{\partial p}\left[-\frac{f_0^2}{\sigma}\mathbf{v}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right]$$

Omega Equation

$$\left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\mathbf{v}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right)\right]$$
$$+\frac{1}{\sigma}\nabla^2\left[\mathbf{v}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p}\right)\right] - \frac{\kappa}{\sigma p}\nabla^2 J$$

Approximate Omega Equation (Adiabatic)

$$\left(\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\omega \approx \frac{f_0}{\sigma}\left[\frac{\partial \mathbf{v}_g}{\partial p}\cdot \nabla\left(\frac{2}{f_0}\nabla^2\Phi + f\right)\right]$$