<table>
<thead>
<tr>
<th>Constant</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>kg / m$^3$</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>Pa = kg / m / s$^2$</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Specific volume ($= 1/\rho$)</td>
<td>m$^3$ / kg</td>
</tr>
<tr>
<td>$\mathbf{u}$</td>
<td>Velocity vector</td>
<td>m / s</td>
</tr>
<tr>
<td>$u$</td>
<td>Zonal component of the velocity vector</td>
<td>m / s</td>
</tr>
<tr>
<td>$v$</td>
<td>Meridional component of the velocity vector</td>
<td>m / s</td>
</tr>
<tr>
<td>$w$</td>
<td>Vertical component of the velocity vector</td>
<td>m / s</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Longitude</td>
<td>rad</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Latitude</td>
<td>rad</td>
</tr>
<tr>
<td>$f$</td>
<td>Coriolis parameter ($= 2\Omega \sin \phi$)</td>
<td>1 / s</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Potential temperature</td>
<td>K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constant</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Radius of the Earth</td>
<td>$6.37122 \times 10^6$ m</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity at Earth’s surface</td>
<td>$\sim 9.80616$ m s$^{-2}$</td>
</tr>
<tr>
<td>$G$</td>
<td>Gravitational constant</td>
<td>$6.673 \times 10^{-11}$ N m$^2$ / kg$^2$</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Rotational velocity of the Earth</td>
<td>$7.292 \times 10^{-5}$ rad s$^{-1}$</td>
</tr>
<tr>
<td>$R$</td>
<td>Ideal gas constant for dry air</td>
<td>287.0 J / kg / K</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat capacity of dry air at constant pressure</td>
<td>1004.5 J / kg / K</td>
</tr>
<tr>
<td>$c_v$</td>
<td>Specific heat capacity of dry air at constant volume</td>
<td>717.5 J / kg / K</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity coefficient</td>
<td>$\sim 1.5 \times 10^{-5}$ m$^2$ / s</td>
</tr>
</tbody>
</table>

**Momentum Equations (Lagrangian Frame)**

\[
\frac{Du}{Dt} - \frac{uv \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \phi - 2\Omega w \cos \phi + \nu \nabla^2 u
\]

\[
\frac{Dv}{Dt} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + 2\Omega u \sin \phi + \nu \nabla^2 v
\]

\[
\frac{ Dw}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g - 2\Omega u \cos \phi + \nu \nabla^2 w
\]

**Momentum Equations (Eulerian Frame)**

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - \frac{uv \tan \phi}{a} + \frac{uw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega v \sin \phi - 2\Omega w \cos \phi + \nu \nabla^2 u
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{u^2 \tan \phi}{a} + \frac{vw}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + 2\Omega u \sin \phi + \nu \nabla^2 v
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} - \frac{u^2 + v^2}{a} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g - 2\Omega u \cos \phi + \nu \nabla^2 w
\]

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0
\]

**Continuity Equation (Lagrangian Frame)**

\[
\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}
\]

**Continuity Equation (Eulerian Frame)**

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0
\]
Thermodynamic Equation
\[ c_v \frac{DT}{Dt} + p \frac{D}{Dt} \left( \frac{1}{\rho} \right) = J \]

Potential Temperature
\[ \theta = T \left( \frac{p_s}{p} \right)^{R/c_p} \]

Ideal Gas Law
\[ p = \rho RT \]

Material Derivative
\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \]

Conservation Laws
\[ \frac{\partial \phi}{\partial t} + \nabla \cdot \mathbf{F} = \Psi \]

Hydrostatic Equation
\[ dp = -\rho g dz \]

Dry Adiabatic Lapse Rate
\[ \Gamma_d = -\frac{\partial T}{\partial z} = -\frac{g}{c_p} \]

Geostrophic Wind
\[ u_g = -\frac{1}{f p} \left( \frac{\partial p}{\partial y} \right)_z = -\frac{1}{f} \left( \frac{\partial \Phi}{\partial y} \right)_p \]
\[ v_g = \frac{1}{f p} \left( \frac{\partial p}{\partial x} \right)_z = \frac{1}{f} \left( \frac{\partial \Phi}{\partial x} \right)_p \]

Gradient Wind Balance
\[ \frac{V^2}{R} + fV = -\frac{\partial \Phi}{\partial n} \]

Beta-plane Approximation
\[ f = f_0 + \beta y, \quad f_0 = 2\Omega \sin \theta_0, \quad \beta = \frac{2\Omega \cos \theta_0}{a} \]

Scaling (large-scale mid-latitude systems)
\[ U \approx 10 \text{ m s}^{-1}, \quad \Delta P \approx 10 \text{ hPa} \]
\[ W \approx 0.01 \text{ m s}^{-1}, \quad \rho \approx 1 \text{ kg m}^{-3} \]
\[ L \approx 10^6 \text{ m}, \quad \Delta \rho \approx 10^{-2} \text{ kg m}^{-3} \]
\[ H \approx 10^4 \text{ m}, \quad f_0 \approx 10^{-4} \text{ s}^{-1} \]
\[ L/U \approx 10^5 \text{ s}, \quad \beta = \frac{\partial f}{\partial y} \approx 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \]
\[ a \approx 10^7 \text{ m}, \quad \nu \approx 10^{-5} \text{ m}^2 \text{ s}^{-1} \]

Brunt-Väisälä Frequency
\[ N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z} \]

Static Stability Parameter
\[ S_p = \left( \frac{R_d T}{pc_p} - \frac{\partial T}{\partial p} \right) = -T \frac{\partial \Theta}{\partial p} = \frac{\Gamma_d - \Gamma}{\rho g} \]

Hypsometric Equation
\[ z_T = (z_2 - z_1) = R_d \int_{p_2}^{p_1} \frac{T}{p} dp \]

Equations of Motion (no curvature)
\[ \frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u} \]
\[ \frac{Du}{Dt} = -\frac{1}{\rho} \nabla p - g \mathbf{k} + f \mathbf{v} - f u \mathbf{j} + \nu \nabla^2 \mathbf{u} \]

Equations of Motion (pressure coordinates)
\[ \frac{Dv}{Dt} + f \mathbf{k} \times \mathbf{v} = -\nabla \Phi \]
\[ \frac{\partial \Phi}{\partial p} = -R_d T \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \]
\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - S_p \omega = \frac{J}{c_p} \]
\[ p = \rho R_d T \]

Shallow Water Equations
\[ \frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (hv) = 0 \]
\[ H = h + h_s \]
Stability Parameter

\[ S_p = \frac{RT}{c_p p} - \frac{\partial T}{\partial p} = -\frac{T}{\theta} \frac{\partial \theta}{\partial p} = \frac{(\Gamma_d - \Gamma)}{\rho g} \]

Thermal Wind (Layer Mean Temperature)

\[ u_T = u_2 - u_1 = -\frac{R}{f} \left( \frac{\partial (T)}{\partial y} \right)_p \ln \left( \frac{p_1}{p_2} \right) \]

\[ v_T = v_2 - v_1 = +\frac{R}{f} \left( \frac{\partial (T)}{\partial x} \right)_p \ln \left( \frac{p_1}{p_2} \right) \]

Thermal Wind (Layer Thickness)

\[ u_T = u_2 - u_1 = -\frac{1}{f} \frac{\partial}{\partial y} (\Phi_2 - \Phi_1) \]

\[ v_T = v_2 - v_1 = +\frac{1}{f} \frac{\partial}{\partial x} (\Phi_2 - \Phi_1) \]

Thermal Wind (Vertical Wind Shear)

\[ \frac{\partial v_g}{\partial \ln p} = -\frac{R}{f} k \times \nabla p T. \]

\[ \frac{\partial u_g}{\partial p} = +\frac{R}{f} \left( \frac{\partial (T)}{\partial y} \right)_p, \]

\[ \frac{\partial v_g}{\partial p} = -\frac{R}{f} \left( \frac{\partial (T)}{\partial x} \right)_p. \]

Vertical Motion (Kinematic Method)

\[ \omega(p) = \omega(p_s) - (p_1 - p_2) \left( \frac{\partial (u)}{\partial x} + \frac{\partial (v)}{\partial y} \right)_p \]

Vertical Motion (Adiabatic Method)

\[ \omega(p) = \frac{1}{S_p} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) \]

Surface Pressure Tendency

\[ \frac{\partial p_s}{\partial t} \approx \omega(p_s) = -\int_{p_s}^{p_1} (\nabla \cdot \mathbf{v}) dp \]

Vorticity

\[ \omega_a \equiv \nabla \times \mathbf{u}_a, \quad \omega \equiv \nabla \times \mathbf{u} \]

\[ \omega = \left( \begin{array}{ccc} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} & -\frac{\partial w}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial z} & \frac{\partial w}{\partial x} & -\frac{\partial u}{\partial z} \\ -\frac{\partial v}{\partial x} & \frac{\partial u}{\partial y} & 0 \end{array} \right) \]

\[ \zeta = \mathbf{k} \cdot (\nabla \times \mathbf{u}), \quad \eta = \mathbf{k} \cdot (\nabla \times \mathbf{u}_a), \quad \eta = \zeta + f \]

Stokes’ Theorem

\[ \oint_{\partial A} \mathbf{v} \cdot d\mathbf{l} = \iint_A (\nabla \times \mathbf{v}) \cdot d\mathbf{k} \]

Barotropic Potential Vorticity

\[ \frac{D_h}{Dt} \left( \frac{\zeta + f}{h} \right) = 0 \]

Vorticity Equation (Lagrangian Form)

\[ \frac{D}{Dt}(\zeta + f) = -\left( \zeta + f \right) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \]

\[ -\left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial y} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial x} \right) + \frac{1}{\rho^2} \left( \frac{\partial p}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial p}{\partial y} \frac{\partial p}{\partial x} \right) \]

Quasi-Geostrophic Equations

\[ \frac{D_\sigma v_g}{Dt} = -f_0 k \times v_a - \beta y k \times v_g \]

\[ \frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \]

\[ \left( \frac{\partial}{\partial t} + v_g \cdot \nabla \right) \left( \frac{\partial \Phi}{\partial p} \right) - \sigma \omega = \frac{\kappa J}{p} \]

\[ \sigma = -\frac{RT_0 d \ln \theta_0}{p} \frac{d p}{d p} \]

Quasi-Geostrophic Vorticity Equation

\[ \frac{D_\sigma \zeta_g}{Dt} = -f_0 \left( \frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} \right) - \beta v_g \]

Geopotential Tendency Equation

\[ \left[ \nabla^2 + \frac{\partial}{\partial p} \left( \frac{f_0^2}{\sigma} \frac{\partial \Phi}{\partial p} \right) \right] \chi = -f_0 v_g \cdot \nabla \left( \frac{1}{f_0} \nabla^2 \Phi + f \right) \]

\[ \frac{\partial}{\partial p} \left[ -\frac{f_0^2}{\sigma} v_g \cdot \nabla \left( \frac{\partial \Phi}{\partial p} \right) \right] \]

Omega Equation

\[ \left[ \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right] \omega = \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left[ v_g \cdot \nabla \left( \frac{1}{f_0} \nabla^2 \Phi + f \right) \right] \]

\[ + \frac{1}{\sigma} \nabla^2 \left[ v_g \cdot \nabla \left( \frac{\partial \Phi}{\partial p} \right) \right] - \frac{\kappa}{\sigma p} \nabla^2 J \]

Approximate Omega Equation (Adiabatic)

\[ \left( \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega \approx \frac{f_0}{\sigma} \left( \frac{\partial v_g}{\partial p} \cdot \nabla \left( \frac{2}{f_0} \nabla^2 \Phi + f \right) \right) \]