MATH 186: Homework 9
Winter 2008

Due: Friday, Apr. 11 before class

> "To Thales the primary question was not what do we know, but how do we know it."
> —Aristotle

From the textbook:

The problems in parentheses are warm-up problems. You won't turn these in, but they are good practice and can help with the graded problems. The problems in **bold** are the graded problems that you will hand in.

§8.13: (1) **2**

§8.16: (1,2a–e) **2f,2g**

§8.18: (4,5,6) **1,2b–e**

The problems below are "extra": if you do only the textbook problems in bold above (and do them correctly) you will receive full credit for this homework. You should only work on these after you've completed the textbook problems.

Outside the box:

1. 8.16, number 4.
2. Show that $\ln(2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$.

   (Hint 1: Write the power series expansion for $\frac{1}{1+x}$ around $c = 0$ and then integrate. Be careful about the domain of convergence!)

   (Hint 2: Check the series is continuous on its domain of convergence by showing that it converges *uniformly*; that is, you may assume the following lemma is true: uniform convergence of a sequence of functions implies continuity of the limit function. See the remark at the end of 8.14 for the definition of uniform convergence.)
3. (Jumping reals) Real numbers play the following game. Every real number is given two cards. The first card has a natural number in it and the second card has a positive real number in it. The game goes as follows. All reals start from the real axis (i.e., x starts from $(x, 0)$). In the nth round, every real that has number n on their first card must jump to the coordinate the written in the second card. Once a real number has jumped it will stay put for the rest of the game. For example, if x has 4 on its first card and $\pi/3$ on its second card, then x stays at $(x, 0)$ for rounds 1–3 and jumps to $(x, \pi/3)$ in the fourth round and stays there for rounds 5 and up.

   The winner is the real number that jumps last. There can be several winners.

   Let $f_k : \mathbb{R} \to \mathbb{R}$ to be the "y"-coordinate of the real number, that is, $(x, f_k(x))$ is the position of the real number x after the kth round. The numbers in the cards are given in such a way that functions f_k are continuous, $\lim_{k\to\infty} f_k$ is a positive function and f_1 is not a zero function.

   Your task is to show that the game is silly in this case, since it will last only one round and everybody wins.