MATH 186: Homework 5
Winter 2008

Due: Wednesday, Mar. 5 before class

> "Imagination is more important than knowledge."
>
> –Albert Einstein

1. Let $f\colon [\alpha,\beta] \to [0,\infty)$ be a continuously differentiable function and $\alpha < \beta < \alpha + 2\pi$. Show that the curve (given in polar coordinates) $r = f(\theta)$, $\alpha \le \theta \le \beta$ has the length

$$\int_\alpha^\beta \sqrt{f'(t)^2 + f(t)^2}\,\mathrm{d}t.$$

(*Hint:* Write the curve in (x,y)-coordinates.)

From the textbook:

The problems in parentheses are warm-up problems. You won't turn these in, but they are good practice and can help with the graded problems. The problems in **bold** are the graded problems that you will hand in.

§7.6: (1–14,27,29) **30**

§7.8: (2) **3**

§7.14: (1–13,26(b,e)) **25, 26(g)**

The problems below are "extra": if you do only the textbook problems in bold above (and do them correctly) you will receive full credit for this homework. You should only work on these after you've completed the textbook problems.

Outside the box:

1. Give an example of a continuously differentiable function $f\colon [1,\infty) \to (0,\infty)$ such that the solid of revolution of the graph around the x-axis has finite volume but the surface area of the solid is infinite. Here the volume and the area are understood as improper integrals.
2. Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is such a function that $|f(x) - f(y)| \le 3|x-y|^2$ for every x and y in $\mathbb{R}$. Show that f is a constant function. (*Hint:* Subdivide.)
3. Consider the surface of a round cylinder of radius 1 and height 3 between base and top. Does this surface have a smallest area among all surfaces joining boundary circles? (*Hint:* No. But why not? Give an example.)