MATH 186: Homework 4
Winter 2008

Due: Wednesday, Feb. 20 before class

> "It is more important that a proposition be interesting than that it be true... But of course a true proposition is more apt to be interesting than a false one."
>
> –Alfred North Whitehead

1. Draw a careful sketch for the graph of the function given by polar coordinates $r = \cos(2\theta)$, $0 \leq \theta \leq 2\pi$. Do the same with $r = \cos(4\pi)$, $0 \leq \theta \leq 2\pi$.

From the textbook:

The problems in parentheses are warm-up problems. You won't turn these in, but they are good practice and can help with the graded problems. The problems in **bold** are the graded problems that you will hand in.

§7.1: (1(ce),2(ce),3(a)) **3(b),5**

§7.2: (1(e),2(a)) **2(ab),3(b)**

§7.6: (11) **31**

The problems below are "extra": if you do only the textbook problems in bold above (and do them correctly) you will receive full credit for this homework. You should only work on these after you've completed the textbook problems.

Outside the box:

1. Starting from equation $\cos^2 x + \sin^2 x = 1$ find expressions for $\sin(\arctan t)$ and $\cos(\arctan t)$ as rational functions of t. Use the addtion formulas for sine and cosine to calculate $\sin(2\arctan t)$ and $\cos(2\arctan t)$. Are these formulas familiar?

2. Use the substitution $\tan(x/2) = t$ (and perhaps substitution $x = \tan(t)$) to calculute the integrals
$$\int \frac{1}{\sin x} dx \qquad \text{and} \qquad \int_{-\pi/2}^{\pi/2} \frac{1}{\sqrt{1+x^2}} dx.$$

3. Let $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, where for any positive integer n, $n! = n \cdot (n-1) \cdots 2 \cdot 1$, and $0! = 1$ by definition. The number $\binom{n}{k}$ can be interpreted as the number of k-element subsets of an n-element set. For example, $\binom{3}{2} = 3$, since there are three subsets of $\{1, 2, 3\}$ with exactly two elements: $\{1, 2\}, \{1, 3\}, \{2, 3\}$.

   For any positive integer n, find a simple expression (in terms of n only) for the following sum:
$$\sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}.$$