MATH 186: Homework 2
Winter 2008

Due: Wednesday, Jan. 23 before class

> "If a nonnegative quantity was so small that it is smaller than any given one, then it certainly could not be anything but zero. To those who ask what the infinitely small quantity in mathematics is, we answer that it is actually zero. Hence there are not so many mysteries hidden in this concept as they are usually believed to be. These supposed mysteries have rendered the calculus of the infinitely small quite suspect to many people. Those doubts that remain we shall thoroughly remove in the following pages, where we shall explain this calculus." –Leonhard Euler (1707 - 1783)

From the textbook:

The problems in parentheses are warm-up problems. You won't turn these in, but they are good practice and can help with the graded problems. The problems in **bold** are the graded problems that you will hand in.

§4.12: (1(ae), 2(ce), 3(ce)) **2(df), 3(df)**

§4.22: (3(ac)) **3(b), 5(c), 12**

§4.24: (1(bc)) **1(df)**

The problems below are "extra": if you do only the textbook problems in bold above (and do them correctly) you will receive full credit for this homework. You should only work on these after you've completed the textbook problems.

Outside the box:

1. Suppose that I is either a half open interval (that is $(a, b]$ or $[a, b)$) or a closed interval and suppose that $f\colon I \to \mathbb{R}$ is a continuously differentiable function on I (that is, the derivative of f is continuous. The Derivative is assumed to be one-sided at the endpoints of I). Show that there exist an open interval J and a continuously differentiable function $g\colon J \to \mathbb{R}$ so that $g(x) = f(x)$ for every $x \in I$. Does g satisfy $g'(x) = f'(x)$ for every $x \in I$?

2. Suppose you have a chocolate bar that is $m \times n$ squares (m and n are positive integers). You'd like to break the big bar down into its constituent squares. What is the minimum number of times that you have to break the bar (assuming that you only break it along the natural, straight, break lines)? What is the maximum number of times you can break the bar to get it into the small pieces?

   For example, if we have a 2×2 bar, there are only a few possibilities: On the first step, we can break it vertically, into two 2×1 bars, or horizontally, into two 1×2 bars. Say we break it into two 2×1 bars. From this point we have to break the smaller bars in two, and that requires two more breaks, for a total of three. If we had first done the horizontal break, we'd end up with the same thing. So in this case, with $m = n = 2$, the minimum and the maximum number of breaks is 3.