Modeling Consideration Set Substitution

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JOB MARKET PAPER

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Version: June, 2015

Invited for resubmission to Marketing Science

ABSTRACT: Consumers purchasing from a large set of alternatives often evaluate only a subset—a consideration set—in order to balance the expected benefits from search (e.g., finding a high-quality product) with costs (e.g., time). If the marginal expected benefit from search decreases in the number of considered alternatives, marketing actions that encourage consideration of one alternative may discourage consideration of another. This paper develops a model of consideration set formation that can account for this "consideration set substitution." Using stated consideration set and observed purchase data from the automotive industry, we measure the impact of marketing actions (vehicle redesigns) and market events (Toyota vehicle recalls, Tōhoku tsunami) on consideration and purchase. We benchmark our model against one that is commonly used in the literature but that does not account for consideration set substitution. We show the benchmark model misestimates the impact of studied marketing actions and events on market share by as much as 13%. Further, it underestimates the frequency with which a gained consideration is converted to a sale. Lastly, although the benchmark model often appears to "fit" the data well, its failure to account for the role search costs play in consideration set formation causes it to infer critical quantities incorrectly, such as the distribution of consideration set sizes and price elasticities, the latter of which are underestimated by nearly 10%.

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The authors wish to thank Anocha Aribarg, Avery Haviv, Elisabeth Honka, Ryan Kellogg, Peter Lenk, Kanishka Misra, Yesim Orhun, Eric Schwartz, Stephan Seiler, and Srinivasaraghavan Sriram for their assistance and suggestions. We also wish to thank the participants of the ISMS 36th Marketing Science Conference at the Goizueta Business School (Emory University) and the 2015 Haring Symposium at the Kelley School of Business (Indiana University). We are grateful to Ford Motor Company for providing us with the New Vehicle Customer Survey data.

1. Introduction

Consideration sets can be thought of as an intermediate outcome of a consumer's search process (Roberts and Lattin 1991, Mehta et al. 2003, Honka 2014). In categories with broad competitive landscapes, consumers may not expend the effort necessary to learn about all alternatives. They may instead construct a consideration set of alternatives to search over in an effort to balance expected benefits (e.g., finding a high quality product or low price) with costs (e.g., time needed to collect information or the mental cost of evaluating alternatives).

One consequence of consumers' limited willingness (or ability) to search is that marketing actions that increase consideration of one product may in turn decrease consideration of others. For example, if a consumer sees a commercial advertising the Ford Fusion, this consumer might (a) consider the Fusion in addition to any other vehicles s/he would have considered had the commercial not been seen (increasing consideration set size), or (b) consider the Fusion *instead of* another vehicle which s/he would have considered (keeping consideration set size constant). We refer to the latter of these possibilities as "consideration set substitution."

A wealth of literature has documented the importance of accounting for variations in which alternatives consumers consider when modeling demand. Failing to do so will lead to biased estimates of demand determinants such as brand valuations (Draganska and Klapper 2011) and price sensitivity (Mehta et al. 2003, Van Nierop et al. 2010). Moreover, optimal marketing strategies generated by models that do and do not account for consideration can differ substantially (Van Nierop et al. 2010). Consideration set substitution is a potentially important element of the consumer choice process, yet consideration set models in the literature often enact implicit assumptions about the frequency with which it occurs—at the extremes, either never (e.g., Goeree 2008, Terui et al. 2011) or always (e.g., Feinberg and Huber's 1996 'quota' model).

This paper develops a new two-stage, consideration and choice model that can flexibly measure consideration set substitution. Importantly for applications, the model admits a closed form solution for which the number of calculations does not increase exponentially in the number of available alternatives, so estimation is not impaired by the curse of dimensionality. This makes it an attractive alternative to models with simplifying assumptions that alleviate the curse of dimensionality but also restrict the degree to which consideration set substitution can (be modeled to) occur. The model is also a useful alternative to more structural models that do not admit a closed form solution.

A primary objective of this paper is to demonstrate the importance of flexibly and accurately accounting for consideration set substitution when modeling consumer demand. To this end, we model consideration and choice in the automotive industry. This industry provides a particularly appropriate empirical setting to examine consideration set substitution, as previous research has shown that automotive consumers engage in fairly limited search, typically considering only a small fraction of the (several hundred) available alternatives (Hauser et al. 2010). Our data consists of 634,539 responses to the New Vehicle Customer Survey from 2009 through 2011. The NVCS is an industry standard regularly utilized by Ford and other vehicle manufacturers to gauge consumer preferences, measure the effectiveness of past marketing actions, and make decisions about future ones. The dataset contains respondents’ stated consideration sets, observed purchases, demographics, and purchase history information.

We use our model to estimate how consumer demand for automobiles changed in response to marketing actions (vehicle redesigns) and market events (the 2009-2010 Toyota vehicle recalls and the 2011 Tōhoku earthquake and tsunami). We compare these estimates to those from a restricted version of the model that artificially constrains consideration set substitution. The parametric restrictions employed reduce the model to one with implicit assumptions about consideration set substitution that precisely mirror those of the model previously used in Van Nierop et al. (2010). The differences between the models' estimates highlight two consequences of not accounting for consideration set substitution. First, the restricted model underestimates the frequency with which a gained consideration leads to a sale, because it underestimates how often a gained consideration kicks a competing alternative out of a consumer's consideration set. Second, the restricted model misestimates the considerations gained or lost due to a marketing action or event. For example, we find that Toyota compact and mid-sized cars ("C" and "CD" vehicle classes) lost 4.9 considerations and 2.3 purchases per 100 consumers in our sample due to the recalls, while Japanese C and CD cars lost 9.4 considerations and 5.1 purchases. The restricted model overestimates these losses by as much as 14%.

Additionally, the restricted model's failure to account for search cost leads to an interesting result—though it accurately estimates the average consideration set size, it misestimates the distribution of set sizes. It consequently misestimates important quantities such as price elasticities, which are heavily dependent upon a consumer's set size. The restricted model underestimated the own-price elasticity of six redesigned vehicles by 9.9%, on average.

The remainder of this paper is organized as follows. In section two we review literature related to consumer search and consideration set formation. Section three describes the data and details our sampling approach. The model is introduced in section four. Section five discusses identification and describes how we conduct our analyses. Results are presented in section six. Limitations and potential avenues for extensions to our work are discussed in section seven. We conclude in section eight.

2. Related Literature

We discuss two streams of research to which the present paper hopes to contribute. The first concerns methodological approaches to modeling consumer search and consideration set formation; the second, the substantive literature on the role of search in the automotive industry.

2.1. Consumer Search and Consideration Set Formation Models

Consumers in many categories do not consider all alternatives, instead choosing from a small subset—a consideration set (Hauser and Wernerfelt 1990, Roberts and Lattin 1991). How consumers construct this set has primarily been modeled in two ways. One stream of literature models consideration sets as the outcome of an optimal search process (Stigler 1961, Weitzman 1979). Empirical papers are typically structural and model the consumer's search process to be either simultaneous (e.g., Mehta et al. 2003, Seiler 2013, Honka 2014, Honka et al. 2014) or sequential (e.g., Hortaçsu and Syverson 2004, Kim et al. 2010, Kim et al. 2014), though some work has addressed which assumption is more appropriate for a given context (de los Santos et al. 2012, Honka and Chintagunta 2014). Consumers in structural search models are typically modeled as though they know a portion of their utility for a product and that the population distribution of the unknown portion (e.g., normal with known mean and variance, as in Kim et al. 2010) is known to each of them. In *simultaneous* structural search models, consumers are modeled as though they construct a consideration set to maximize the expected utility from their choice from the set, less search costs. In *sequential* structural search models, consumers are modeled as though they search alternatives one by one (revealing the utility of each) and stop when the expected value of continued search no longer exceeds a reservation utility.

A second stream of literature models the probability of a product being considered without specifying a search process through which this probability is generated. One common approach is to model the probability of observing each possible consideration set (as in Swait's seminal 1984 dissertation). The probability of an alternative being considered is then a summation of the probabilities associated with each possible set containing that alternative. This approach can quickly become infeasible as the number of available alternatives rises, though. A common alternate approach is to model consideration as an alternative-specific construct (e.g., Goeree 2008, Van Nierop et al. 2010, Terui et al. 2011). Consideration of each alternative is modeled with a latent variable and alternatives enter a consideration set if this variable exceeds some level (often set to zero for identification). We refer to these as 'level' models (as per Feinberg and Huber 1996). Models that do not specify a search process have typically not accounted for consideration set substitution, as they (implicitly) assume that consideration of one alternative does not affect consideration probabilities of others.

Both model types can accommodate markets with a large number of alternatives, but each sacrifices something to do so. The (non-structural) consideration set models abstract away from the cost-benefit trade-off associated with search, losing the ability to measure consideration set substitution. A common limitation of structural search models is that the probability of an observed consideration set being optimal typically does not have an analytic solution, and numerical integration is needed to calculate the likelihood function. An exception is the model used by Kim et al. (2010), but that model cannot be estimated using individual-level data.

Our model mostly follows in the tradition of the (non-structural) consideration set formation models, in that it necessitates neither an account of which attribute(s) a consumer is searching for information about nor an explicit model of consumer expectations. However, it does accommodate the possibility that the marginal expected benefit from search may be decreasing, and can thereby measure consideration set substitution.

2.2. Search and Substitution in the Automotive Industry

Other research has explored the role that search plays in the automotive industry. For example, Ratchford et al. (2003) find that the availability of internet-based sources of information leads to less search (fewer alternatives are considered), Zettelmeyer et al. (2006) find that online information search helps consumers negotiate lower prices at dealerships, and Singh et al. (2014) find that four information sources (dealer visits, print advertising, dealer websites, and resale websites) serve as complements, rather than substitutes. Moraga-González et al. (2015) show that price elasticity estimates are lower when limited search is accounted for than when full information assumptions are applied. This is in line with past research in other categories finding that full information models produce biased price sensitivity estimates (e.g., Mehta et al. 2003, Goeree 2008, Koulayev 2009, Van Nierop et al. 2010, Draganska and Klapper 2011). Sudhir (2001) builds a structural model of competitive pricing behavior to better estimate how pricing decisions affect market share. Berry et al. (2004) use stated second choice data to improve measurement of substitution patterns in the automotive industry. Here, we intend to complement and enhance this line of work—specifically, to improve measurement of marketing mix effects (as well as the impact of market events not controlled by the manufacturer) by accounting for consideration set substitution.

3. Data

We avail of cross-sectional data consisting of 634,539 responses to the New Vehicle Customer Survey from 2009 through 2011. Each quarter, the survey is mailed by Maritz Research, Inc. to consumers who purchased a new (unused) vehicle in the United States during the previous quarter. The purchased vehicle is recorded by dealerships at the time of sale. Buyers of small-share vehicles are oversampled, and each observation is given a weight to ensure that the total proportion of weight assigned to a vehicle matches that vehicle's market share for a given quarter.

Survey respondents are asked to list any vehicles they considered buying other than the one purchased. The vehicle make, model, and class are collected for both the considered and purchased vehicles. 43.4% of respondents in our sample had a consideration set consisting of a single vehicle, with 33.7%, 16.1%, and 6.9% having consideration sets of two, three, and four.

The survey also asks respondents to list all other vehicles currently owned or leased and to identify whether one of those vehicles is being replaced by the purchased vehicle. We use these purchase history variables in our model. We also use a few sets of importance rating and product use questions: those used to identify how important fuel economy and brand loyalty are to a respondent and those that catalogue what a consumer uses their vehicle for (e.g., taking children to school or off-roading). The NVCS also collects demographic information. We use the respondent’s age, sex, and household income, which were the demographics most highly correlated with vehicle class preference.

3.1. Sampling

Several screening criteria are used to reduce the dataset’s size. We focus on compact and mid-sized cars (C and CD car vehicle classes)—by far the two most frequently purchased classes, accounting for 33.7% of new vehicle sales (and 82% of non-premium car sales) between 2009 and 2011. We therefore include only consumers who considered at least one car, as C/CD cars were unlikely to be co-considered with trucks, vans, or utilities (Table 2). We also remove erroneous or incomplete survey responses and exclude a few respondents who purchased vehicles with exceptionally small market share. We then sampled 9,000 respondents for estimation. A detailed description of our sampling approach can be found in Web Appendix B.

3.2. Summary Statistics: Vehicle Redesigns

Measuring the impact of vehicle redesigns serves as a natural means of demonstrating the importance of accounting for consideration set substitution, as redesigns are strategic marketing actions undertaken by firms. Vehicles are redesigned on a cycle, with smaller "refreshes" (e.g., changes to the lights or seat fabric) occurring annually or bi-annually and major redesigns occurring approximately every five years (an industry benchmark, according to Ford). A major redesign marks the beginning of a new "generation" for a nameplate (e.g., the Toyota Camry). Several vehicles transitioned to a new generation between the years of 2009 and 2011, with the Hyundai Elantra, VW Jetta, Subaru Legacy, Kia Optima, Subaru Outback, and Hyundai Sonata being a few for which the new generation saw particularly large growth in market share. The new generations of these vehicles garnered 95.1% more considerations and 86.1% more purchases per 100 consumers in our data sample (Table 3) than the previous generation during the three year period under study.[1]

The new generation of a nameplate often features superior styling as well as quality and technological improvements. Manufacturers make these improvements with the expectation that demand will increase, but the aim is not merely to increase market share—often manufacturers intend to charge a higher price commensurate with greater demand. This can be seen in the data: for the six redesigned vehicles of interest, the new generation saw an average inflation-adjusted increase in price paid of approximately 9.9% (Table 3) relative to the previous generation.

3.3. Summary Statistics: Market Events

Since marketing actions like redesigns are intended to increase vehicle demand, we also measure the impact of two market events that decreased vehicle demand in order to provide a comprehensive overview of how consideration set substitution shapes substitution patterns.

[1] The number of considerations and purchases a vehicle received per 100 consumers in our data sample are the primary measures of consideration and choice that we report. Both are weighted measures—a respondent with a weight of two will be counted twice. For ease of exposition, we will sometimes simply refer to "considerations" or "purchases," omitting the explicit reference to the scale and data sample.

Toyota recalled vehicles due to safety concerns on three separate occasions during 2009 and 2010, with the final recall conducted in the first quarter of 2010. In the second quarter of 2010, consideration of Toyota C and CD cars dropped sharply, from 28.4 considerations per 100 consumers (Q1 2009 through Q1 2010) to only 23.8 over the next year (a decline of 16.2%). Purchases also dropped by 12.0%, from 15.1 to 13.3 (Table 4).

In 2011, the Tōhoku earthquake and tsunami disrupted the ability of Japanese manufacturers to produce vehicles and deliver them to the United States. Though this disruption served as a supply shock, it also affected demand through consideration. From the second quarter of 2010 to the first quarter of 2011, Japanese C and CD cars received 68.4 considerations per 100 consumers. After the tsunami (Q2 through Q4 of 2011), these vehicles received only 61.9 considerations per 100 consumers, a decline of 9.5% (Table 4). Honda and Toyota took the brunt of the damage—considerations for their vehicles declined by 16.8%, while considerations for other Japanese manufacturers remained virtually unchanged. Though explaining why consumers considered Japanese vehicles less often post-tsunami is not the objective of this paper, there are reasonable explanations. For example, consumers may have decided not to risk expending effort to search for information about vehicles that they might not be able to purchase.

Figures 1 and 2 make clear that the drop in considerations and purchases for Toyota C and CD cars post-recall and for Japanese C and CD cars post-tsunami are not merely the product of long-term trends. There are stark drops coinciding with the occurrence of each market event.

4. Model Development

In this section we develop a model that accounts for consideration set substitution. Each consumer i ($i = 1, 2, \dots, I$) receives utility u_{ij} from purchasing alternative j ($j = 1, 2, \dots, J$), but this utility is not fully known to the consumer. Consumer i will conduct a simultaneous search for information about all alternatives in his or her consideration set, S_i, revealing u_{ij} for $j \in S_i$. From that set, consumer i will then purchase the alternative offering the highest utility.

Our model is agnostic to which attribute(s) a consumer is searching for more information about. Consumers may be searching for information about prices (e.g., Kim et al. 2010, Honka 2014), product quality (e.g., Kim et al. 2010), specific product attributes (e.g., Koulayev 2014), among others. Instead of explicating consumer expectations over attributes, we model the probability that alternative j will be considered by consumer i in a manner that accounts for search costs and allows for the net marginal expected benefit from search to be decreasing.

4.1. A Simple Consideration Set Formation Model

Consumer i's latent preference for information about alternative j is given by ω_{ij}. ω_{ij} (which we refer to as "consideration propensity") consists of a deterministic (w_{ij}) and stochastic component (ϵ_{ij}, observed by the consumer but not the researcher):[2]

$$[1] \qquad \omega_{ij} = w_{ij} + \epsilon_{ij}$$

Recall that a consumer will learn his or her utility for an alternative j, u_{ij}, if j is included in S_i. A consumer's relative preference to learn his or her utility for any two alternatives a and b (that is, to reveal u_{ia} or u_{ib}) is represented by the levels of ω_{ia} and ω_{ib}. If $\omega_{ia} > \omega_{ib}$, then consumer i prefers to know u_{ia}, and alternative b will only be included in S_i if alternative a is included. More generally, the fact that consumer i has decided to search only for more information about the alternatives in S_i implies the following inequality:

$$[2] \qquad \omega_{ij^+} > \omega_{ij^-} \; \forall \, j^+ \in S_i, \; j^- \notin S_i$$

[2] Consumer i is searching to reveal u_{ij}, not ϵ_{ij} (which s/he already observes). w_{ij} does not represent the portion of u_{ij} observable to the consumer, nor does ϵ_{ij} represent the unobservable portion. w_{ij} is a collection of covariates that are predictive of consumer i's propensity to consider alternative j, and ϵ_{ij} represents the portion of this propensity that cannot be explained by the covariates available to the researcher.

Equation 2 does not provide insight as to *why* consumer i only considers the alternatives in S_i and no others. It only reflects one half of the cost-benefit trade-off underlying search and consideration set formation. However, if searching for information about alternative j (including it in S_i) incurs cost c_j, then the consumer's objective becomes clear: search for information about any alternative j for which $\omega_{ij} > c_j$.

In practice, alternative-specific search costs (c_j) will not be separately identifiable from consideration propensities (ω_{ij}). Instead, researchers commonly model consideration as an alternative-specific construct (e.g., Feinberg and Huber 1996, Van Nierop et al. 2010), where j enters S_i if his or her consideration propensity for that alternative exceeds some level (often set to zero for identification). The probability S_i is optimal is then given by:

$$[3] \qquad \Pr(S^* = S_i) = \prod_{j^+ \in S_i} \Pr(\omega_{ij^+} > 0) \prod_{j^- \notin S_i} \Pr(\omega_{ij^-} < 0)$$

The "level" model in equation 3 has a particularly attractive feature: estimation of the model circumvents the curse of dimensionality. However, a notable limitation is that it cannot account for consideration set substitution. The determination of whether alternative j is worth considering depends only on ω_{ij}. Thus, a change to the deterministic portion of consumer i's consideration propensity for any one alternative will not affect the consideration propensity of any other. If, for example, consumer i sees a commercial advertising alternative k, increasing the deterministic portion of consideration w_{ik} and (correspondingly) the probability that alternative k is considered, the probability of other alternatives being considered will not change.

4.2. Consideration Set Substitution

If the cost of searching for information about alternative j is c_j, then consideration set substitution can only occur if the net expected benefit from search ($\Lambda_{ij} = \omega_{ij} - c_j$) is decreasing in the number of other alternatives considered. To see why, consider an example: under a hypothetical set of market conditions, consumer i will consider alternatives x and y but not z ($S_i = \{x, y\}$). However, this consumer will consider z if s/he sees a commercial advertising it. If Λ_{ix} is decreasing in the number of alternatives considered, then:

[4] $$\left\{\omega_{ix} - c_x \mid S_i = \{x, y, z\}\right\} < \left\{\omega_{ix} - c_x \mid S_i = \{x, y\}\right\}$$

It may be the case, then, that $\omega_{ix} < c_x$ when alternative z is added to the consideration set, even though $\omega_{ix} > c_x$ without z in the set. Alternative x may therefore be removed from the set as a direct consequence of z being added to it.

If Λ_{ij} is decreasing in the number of alternatives considered, then either ω_{ij} is decreasing or c_j is increasing (or both). That the marginal expected benefit from search (ω_{ij}) might be decreasing is fairly intuitive, and this effect has been referenced or modeled in past work (e.g., Roberts and Lattin 1991, Kim et al. 2010, Honka 2014). The more alternatives a consumer considers, the more likely it is that at least one will have a particularly low price, be of very high quality, or otherwise be an especially good fit for the consumer. For that reason, the expected benefit from adding alternative j to a set of size n is likely to be lower than the expected benefit from adding it to a set of size $n - 1$. It is perhaps less intuitive why search costs (c_j) might be increasing, but situations where this might be the case are hardly uncommon. For example, choice overload might cause a consumer's psychological costs to increase superlinearly in the length of search or number of alternatives considered.

We cannot separately identify whether the expected benefit from search is decreasing, costs are increasing, or both. We can, however, account for the net effect—that Λ_{ij} may be decreasing in consideration set size. Since alternative-specific search costs are not identified, we instead model the marginal cost of considering an n^{th} alternative:

$$[5] \qquad C_{N_{S_i}} = c + \sum_{n=2}^{N_{S_i}} \tilde{c}_n \qquad \tilde{c}_n = c + \psi_n \qquad \psi_n \geq 0 \; \forall \, n \qquad \psi_{n+1} \geq \psi_n \; \forall \, n$$

In equation 5, N_{S_i} is the size of consideration set S_i and ψ_n represents the degree to which Λ_{ij} is decreasing in n (ψ_n is consequently non-decreasing in n). c and ψ_1 must be normalized for identification (so $\tilde{c}_n = \psi_n$), but the parameters ψ_n are identified for all $n > 1$. Consumer i will add alternatives to S_i in decreasing order of consideration propensity, ω_{ij}, until no remaining alternative has a consideration propensity exceeding the marginal cost of adding it to the set.

4.3. Likelihood Function

With the net expected benefit from search decreasing in the number of alternatives considered, the probability that an observed consideration set S_i is optimal can no longer be cleanly expressed as a product of alternative-specific probabilities (as in equation 3). Instead, we identify a set of conditions under which S_i is optimal and calculate the probability that these conditions hold. S_i is optimal if consumer i cannot be made better off by perturbing the alternatives in the set. More specifically, consumer i cannot (1) remove an alternative from S_i, (2) add an alternative to S_i, or (3) swap an alternative in S_i with one excluded from S_i and be better off. These three conditions can be formally defined as follows:

[6] No removal: $\omega_{ij^+} > \tilde{c}_{N_{S_i}} \; \forall \, j^+ \in S_i$

[7] No additions: $\omega_{ij^-} < \tilde{c}_{N_{S_i}+1} \; \forall \, j^- \notin S_i$

[8] No swaps: $\omega_{ij^+} > \omega_{ij^-} \; \forall \, j^+ \in S_i \, , j^- \notin S_i$

In other words: (1) consumer i's propensity to consider each alternative in S_i is greater than the marginal cost of including an $N_{S_i}^{th}$ alternative in the set, (2) consumer i's propensity to consider each alternative excluded from S_i must be lower than the marginal cost of adding it to S_i ($\tilde{c}_{N_{S_i}+1}$), and (3) consumer i's propensity to consider each alternative in S_i must exceed his or her propensity to consider each alternative excluded from S_i. The probability that these conditions hold for an observed set S_i is consumer i's likelihood function:

$$[9] \quad \pi(S_i) = \Pr\left[\{\omega_{ij}\}_{j\in S_i} > \{\omega_{ij}\}_{j\notin S_i},\ \{\omega_{ij}\}_{j\in S_i} > \tilde{c}_{N_{S_i}},\ \{\omega_{ij}\}_{j\notin S_i} < \tilde{c}_{N_{S_i}+1}\right]$$

When ϵ_{ij} is distributed i.i.d. Gumbel, the model in equation 9 generalizes the exploded logit model in two specific ways—it incorporates partial rank orderings (we know only that alternatives in S_i are of higher rank than those excluded from S_i) and includes a means of estimating the truncation point of the consideration set (via the search cost variables, $\tilde{c}_n$).

Equation 9 has a closed form solution when ϵ_{ij} is distributed i.i.d. Gumbel (somewhat surprisingly, given the non-trivial additions it makes to the exploded logit model). The closed form solution is provided in equation 10. A proof of this solution can be found in Appendix A. Web Appendix F also contains the results of simulation studies run to demonstrate that the parameters β_i and $\tilde{c}_n$ can be accurately retrieved in an estimation algorithm.

$$[10] \quad \pi(S_i) = \prod_{j^+\in S_i}\left[1-\exp\left(-\exp(w_{ij^+})\exp\left(-\tilde{c}_{N_{S_i}}\right)\right)\right]\exp\left(-a\exp\left(-\tilde{c}_{N_{S_i}}\right)\right)$$
$$+\exp\left(-a\exp\left(-\tilde{c}_{N_{S_i}+1}\right)\right)-\exp\left(-a\exp\left(-\tilde{c}_{N_{S_i}}\right)\right)$$
$$+\sum_{t=1}^{t=k}(-1)^t\sum_{q=1}^{q=\binom{N_{S_i}}{t}}\frac{a}{a+b_{qt}}\begin{bmatrix}\exp\left(-(a+b_{qt})\exp\left(-\tilde{c}_{N_{S_i}+1}\right)\right)- \\ \exp\left(-(a+b_{qt})\exp\left(-\tilde{c}_{N_{S_i}}\right)\right)\end{bmatrix}$$

Here $a=\sum_{j^-\notin S_i}\exp(w_{ij^-})$, $b_{qt}=\sum_{j\in G_{qt}}\exp(w_{ij^+})$, and G_{qt} is the qth subset of alternatives $j^+\in S_i$ of size n (of which there are $N_{S_i}!/\left(t!\left(N_{S_i}-t\right)!\right)$ in total).

Note that the solution, while complex, actually consists of only four unique parts: the exponentials of (1) the consideration propensities for all alternatives $j^+ \in S_i$, $\exp\left(w_{ij^+}\right)$, (2) the consideration propensities for all alternatives $j^- \notin S_i$, $\exp\left(w_{ij^-}\right)$, (3) the (negative of the) marginal search cost for the $N_{S_i}^{th}$ considered alternative $\exp\left(-\tilde{c}_{N_{S_i}}\right)$, and (4) the (negative of the) marginal search cost for the $(N_{S_i}+1)^{th}$ considered alternative, $\exp\left(-\tilde{c}_{N_{S_i}+1}\right)$.

Critically, the number of calculations in the likelihood statement does not increase exponentially in the number of available alternatives. Estimation is therefore not impeded the curse of dimensionality. Though the third line of equation 20 contains $2^{N_{S_i}}-1$ calculations (and is thus exponentially increasing in the size of the consideration set, N_{S_i}), these calculations are merely $2^{N_{S_i}}-1$ combinations of N_{S_i} terms; they are easily constructed in an estimation algorithm, and take little time to compute, even for relatively large values of N_{S_i}.[3]

4.4. Relationship to the Level and Exploded Logit Models

The model presented in equations 9 and 10 is directly linked to two commonly used models in the literature—the level and exploded logit models. If marginal search costs $\tilde{c}_n$ are estimated to be zero for all n, our model reduces to the level model in equation 3. Our model therefore generalizes the level model, and can be formally tested against it to see if the data support the hypothesis that consumers engage in consideration set substitution.

Alternatively, if search costs are removed from equation 9 altogether (not merely set to zero), equation 9 becomes a simple rank order preference model:

[11] $\Pr\left[\{\omega_{ij}\}_{j\in S_i} > \{\omega_{ij}\}_{j\notin S_i}\right]$

[3] This formulation works well in our empirical context, where set sizes range from one to four. There will be some limit to the size of consideration sets that can be modeled. That limit will depend on computing power and data set size.

Because the stochastic component of consideration propensity, ϵ_{ij}, is distributed i.i.d. Gumbel, the probability statement in equation 11 becomes a simple summation of exploded logit probabilities (Chapman and Staelin 1982). Specifically:

$$[12] \quad \Pr\left[\{\omega_{ij}\}_{j\in S_i} > \{\omega_{ij}\}_{j\notin S_i}\right] = \sum_{r=1}^{N_{S_i}!} \Pr[R_r]$$

In equation 12, R_r is the r^{th} rank ordering of all alternatives in S_i, and $\Pr[R_r]$ is the exploded logit probability associated with this rank ordering. For example, if $S_i = \{a, b\}$ and A is the set of all alternatives, then there are two possible rank orders ($R_1 = (a, b)$, $R_2 = (b, a)$), and:

$$[13] \quad \sum_{r=1}^{N_{S_i}!} \Pr[R_r] = \frac{\exp(w_{ia})}{\sum_{j\in A}\exp(w_{ij})}\frac{\exp(w_{ib})}{\sum_{j\in\{A-a\}}\exp(w_{ij})} + \frac{\exp(w_{ib})}{\sum_{j\in A}\exp(w_{ij})}\frac{\exp(w_{ia})}{\sum_{j\in\{A-b\}}\exp(w_{ij})}$$

5. Analyses

We estimate the impact of vehicle redesigns, the Toyota recalls, and the Tōhoku earthquake and tsunami using our model. These estimates are compared to those from a restricted version of the model that artificially constrains consideration set substitution (our "benchmark" model). Differences between the two models' estimates serve as a barometer for the importance of accounting for consideration set substitution.

5.1. Benchmark Model

The level model is an attractive option for a benchmark, since it is commonly used, is properly nested in the proposed model, and does not account for consideration set substitution. Because our dataset consists only of consumers who considered at least one vehicle, we can enact parametric restrictions to both the full and benchmark model (in a manner similar to Van Nierop et al. 2010) to accommodate this feature. Specifically, the restriction $\tilde{c}_1 = -99$ imposes that consumers will always consider at least one alternative. For the benchmark model, we further restrict $\tilde{c}_n = 0 \; \forall \; n > 1$, which imposes that the first restriction is the only source of consideration set substitution. In other words, the restricted model implicitly assumes that consideration set substitution only occurs when a consumer with a consideration set of size one is induced to remove the lone considered alternative from his or her set.

5.2. Alternative Space

We model 54 C and CD cars available from 2009-2011. For vehicles in other classes, we (mostly) model class-and-continent-specific "outside options." One exception is that we model Toyota and Honda vehicles from the B Car (super compact) and DE Car (full size) classes as individual alternatives to facilitate more accurate estimates of recall and tsunami effects (which primarily affected Toyota and Honda). For estimation speed and stability, we do not model a few C and CD cars, nor the European B Car and Truck outside options. These alternatives received exceptionally small market share. In total, we model 76 alternatives that consumers can consider and purchase. Details are provided in Web Appendix C.

5.3. Empirical Specification of Consideration Propensity

For both the full and restricted model, we take a Hierarchical Bayesian approach to modeling consumer i's propensity to consider alternative j from vehicle class l at time t:

$$[14] \qquad \omega_{ijt} = \alpha_{ij} + \gamma D_{jt} + \delta R_{jt} + \lambda E_{jt} + \beta X_{ij} + \eta U_{jt} + \epsilon_{ijt} \qquad \epsilon_{ijt} \sim \text{i.i.d. Gumbel}$$

$$\alpha_{ij} = \alpha_j + \xi_{il} \qquad \begin{pmatrix} \xi_{i,C} \\ \xi_{i,CD} \end{pmatrix} \sim MVN(0, \Sigma_\xi) \qquad \Sigma_\xi = \begin{bmatrix} \sigma_C^2 & \sigma_{C,CD} \\ \sigma_{C,CD} & \sigma_{CD}^2 \end{bmatrix}$$

It is not feasible to estimate 76 fully heterogeneous alternative-specific constants (α_{ij}) using only one observed consideration set per consumer (Andrews, Ainslie, and Currim, 2002), so we instead estimate heterogeneous preferences for our focal vehicle classes, C and CD Cars ($\xi_{i,C}$ and $\xi_{i,CD}$). A larger covariance matrix with more classes (e.g., super compact "B" cars) was not empirically identifiable. We therefore rely on the covariates in our model to explain heterogeneous preferences for other vehicle classes. σ_C^2 is fixed to one for identification.

Redesigns are modeled using the set of variables D_{jt}. We include a dummy variable for each redesigned vehicle. For any redesign variable, the variable is equal to one if alternative j is the associated redesigned vehicle and if consumer i purchased his or her vehicle during any quarter in which the redesigned generation of the vehicle was available.

R_{jt} is the Toyota recall variable. For any respondent that purchased during or after the second quarter of 2010 (after which all Toyota recalls had been announced), R_{jt} is equal to one if vehicle j was (a C- or CD-car) manufactured by Toyota. E_{jt} is a set of five separate manufacturer-and-class-specific variables that account for the effect of the earthquake and tsunami on (1) Toyota C/CD cars, (2) Toyota vehicles from other classes, (3) Honda C/CD cars, (4) Honda vehicles from other classes, and (5) C/CD cars from all other Japanese manufacturers (bundled together they have approximately the same market share as Toyota or Honda do individually). For any respondent that purchased during or after the second quarter of 2011, a tsunami variable is equal to one if vehicle j was made by the corresponding manufacturer(s) and comes from the corresponding vehicle classes.

X_{ij} consists of 69 consumer × vehicle class interaction terms designed to control for heterogeneous preferences. Consumer purchase history, demographic, importance rating, and product use variables are interacted with vehicle class dummy variables. U_{jt} is a "partial unavailability" variable, and can be thought of as a nuisance variable used to control for abnormally low consideration or purchase for a vehicle in the first quarter of its availability or at the tail end of its availability. Details about X_{ij} and U_{jt} can be found in Web Appendix C.

Consistent with past research (Bronnenberg and Vanhonacker 1996), Ford Motor

Company has stated that vehicle price tiers, rather than price, drive consideration decisions. For example, a low income household may consider several compact cars (knowing they are priced economically) but never think to consider premium SUVs. More granular price differences between vehicles are factored in by consumers at the choice stage. Since vehicles within a class are similarly priced, X_{ij} includes vehicle class × income interaction terms, while the specification for choice utility (see section 5.6) includes price × income interaction terms.

5.4. Empirical Specification of Search Costs

Since we only observe consumers with consideration sets of sizes one through four, $\tilde{c}_1$ and $\tilde{c}_n$ for $n > 4$ are not identified. Recall that $\tilde{c}_1$ is set to -99. We set $\tilde{c}_5$ similarly to 99 (i.e. to values suitably close to $-\infty$ and ∞ on the logit scale). We must fix one more parameter for identification, and so set $\tilde{c}_2$ equal to zero.[4] Lastly, we set $\tilde{c}_3 \geq \tilde{c}_2$ and $\tilde{c}_4 \geq \tilde{c}_3$, in line with our model's assumption that net marginal expected benefit from search is non-increasing in consideration set size:

$$[15] \qquad \tilde{c}_3 = \exp(\theta_3) \qquad\qquad \tilde{c}_4 = \tilde{c}_3 + \exp(\theta_4)$$

Heterogeneity (observable or unobservable) in search costs and consideration propensity are not separately identifiable. We include heterogeneity only in consideration propensities because doing so allows for greater flexibility. Variables intended to capture heterogeneous preferences for vehicle attributes like vehicle class can absorb the impact of heterogeneous search costs fairly easily (e.g., by estimating higher or lower consideration propensities across all vehicle types), but the reverse is not so.

5.5. Identification of Marginal Search Costs

Because point estimates for marginal search costs are (a) at least in part identified by

[4] We could instead fix one alternative-specific constant from ω_{ijt}, but doing so leads to a great deal of autocorrelation in our Markov chain.

functional form assumptions placed upon the unobserved (to the researcher) component of consideration propensity, ϵ_{ijt}, and (b) the sole driver of consideration set substitution patterns in our model, exogenous variation in the attractiveness or number of available alternatives is critical. We have five primary shifters of alternative attractiveness or availability: the Toyota recalls, the tsunami, vehicle redesigns, vehicle discontinuations, and the launch of new vehicles.

If consideration set substitution does not occur in a market, then changes in the attractiveness or availability of one alternative should not affect the consideration frequencies of other alternatives, and $\tilde{c}_3$ and $\tilde{c}_4$ will be estimated to be zero. However, if consideration set substitution does occur, then $\tilde{c}_3$ and $\tilde{c}_4$ must necessarily be greater than zero. Consider the Toyota recalls: If consideration set substitution does not occur in this market, then when Toyota vehicles lost considerations after the recalls, consideration of other alternatives should not have changed (if there were no other changes in the market at the same time). $\tilde{c}_3$ and $\tilde{c}_4$ are identified (in part) by the degree to which non-Toyota alternatives saw their consideration frequency rise in the aftermath of the recalls (and in part by consumer response to other changes in the market).

5.6. Empirical Specification of Choice Utility

Consumer i derives utility from the purchase of vehicle j, given by:

$$[16] \quad u_{ijt} = \alpha_j^c + \gamma^c D_{jt} + \delta^c R_{jt} + \lambda^c E_{jt} + \beta^c X_{ij}^c + \eta^c U_{jt} + \zeta^c P_{jt} + \varepsilon_{ijt}$$

$$\varepsilon_{ijt} \sim \mathrm{i.i.d.\,Gumbel}$$

The choice utility specification includes all redesign, recall, and earthquake covariates that were included in consideration propensity, allowing for the identification of which affect choice, consideration, or both. There are three primary differences between the specification of consideration propensity and choice utility. First, alternative-specific constants are modeled as homogeneous in choice utility. Unobservable heterogeneity in vehicle class preference is not empirically identifiable in choice utility due to the small sizes of consideration sets. Second, X_{ij}^{c} (in choice utility) only includes a subset of the covariates from X_{ij} (from consideration propensity), because some of the interaction terms were not empirically identifiable at the choice stage. Third, a set of four price × income interactions (P_{jt}) are included in choice utility. Dummy variables for each of four household income groups ($0 - $24,999, $25,000 - $49,999, $50,000 - $79,999, and $80,000+) are interacted with a vehicle price variable. The price variable is defined as the (inflation-adjusted) average price paid by respondents during quarter t for vehicle j.

5.7. Bayesian Estimation

We estimate the parameters of model using a Metropolis-Hastings algorithm with a normal random-walk proposal. The parameters of Σ_{ξ} are sampled over the unidentified space using Gibbs steps, and posterior distributions for the identified parameters are obtained by dividing the draw for Σ_{ξ} by the draw for σ_{C}^{2} (McCulloch and Rossi 1994). We draw 8,000 values for each parameter and dispose of the first 3,000 (burn in).[5] The remaining 5,000 are used to generate parameter estimates and credible intervals (Appendix B). The consideration and choice levels of the model were estimated separately. Note also that the choice level of the model does not differ across the full and restricted models. Trace plots were checked for convergence. Our estimation algorithm can be found in Web Appendix A.

5.8. Simulation Studies

The objectives of these analyses are (1) to measure how vehicle redesigns, the Toyota recalls, and the tsunami affected consideration and choice, and (2) examine whether and how estimates from the proposed model (which accounts for consideration set substitution) differ from those from the restricted model. For each model, we run a baseline simulation (using the

[5] We use parameter estimates from a logistic regression run on our data sample as starting values for our model's homogeneous parameters. This allows for fairly quick convergence.

model's parameter estimates) and counterfactual simulations wherein an effect of interest (e.g., the recalls) is "removed" by setting the associated parameter(s) to zero. For the redesign counterfactuals we also reduce the price of the redesigned vehicles to pre-redesign levels. If consideration is higher (lower) for a counterfactual simulation than the baseline, the event associated with the counterfactual had a negative (positive) impact on consideration. The same is true for choice. Web Appendix D contains a detailed description of our simulation approach.

In table 4, we compare the simulated considerations and purchases (by vehicle type) from the baseline simulations to the summary statistics from our data sample. The simulated results strongly mirror the data sample's summary statistics, giving us confidence that our counterfactual analyses have an accurate baseline off of which to work from.

6. Results

6.1. Parameter Estimates

The full model's parameter estimates (Appendix B) for marginal search costs are greater than zero ($\tilde{c}_3$ and $\tilde{c}_4$ are estimated to be .218 and .399, respectively), supporting the hypothesis that consideration set substitution does occur in this market. The 95% credible intervals for $\tilde{c}_3$ and $\tilde{c}_4$ are (0.189, 0.246) and ($\tilde{c}_3$ + 0.149, $\tilde{c}_3$ + 0.218), and the lowest draw (of 5,000) from the posterior distribution of $\tilde{c}_3$ was 0.175—suitably far from zero. If consideration set substitution did not occur in this market, one would expect a mass of points at zero in the posterior distribution for these parameters.[6]

For the consideration stage of the model, the redesign parameter estimates for the consideration level of the model were significant and positive for five of the six vehicles under

[6] In practice, we constrain the marginal search cost parameters below at zero by making these parameters exponential terms. Thus, we would not expect values of precisely zero for $\tilde{c}_3$ and $\tilde{c}_4$ if consideration set substitution did not occur, but values very close to zero (e.g., zero to the fourth or fifth decimal), and would expect that the posterior distribution for these two parameters would not converge in a region so far from zero.

study (all but the Subaru Legacy). The Toyota C/CD car recall parameter (-0.242) was also significant, as expected given the sharp drop in Toyota considerations post-recall. We find that the redesign parameters and the recall parameter were *ns* for the choice level of the model.

The tsunami parameter estimates were also mostly significant at the consideration stage. We have evidence that the tsunami hurt consideration of Toyota and Honda, but not other Japanese manufacturers. One could have reasonably hypothesized that the tsunami would have primarily had supply-side consequences for these manufacturers, possibly manifesting as lower conditional choice probabilities. We do find this to be the case for Japanese manufacturers other than Honda and Toyota—consideration probabilities remained the same post-tsunami (the tsunami parameter was not significant in the consideration stage of the model), but conditional choice probabilities were lower post-tsunami (the tsunami parameter was significant and negative in the choice stage). Interestingly, however, we find the opposite for Toyota and Honda vehicles—consideration probabilities for these vehicles are lower post-tsunami, while conditional choice probabilities are unchanged. This result is striking. While we cannot draw a conclusion as to why consumers reacted this way, the result is consistent with at least one story: perhaps consumers anticipated limited supply, and adjusted their consideration sets accordingly.

We find that wealthier consumers are (unsurprisingly) more likely to consider expensive vehicle classes (Web Appendix E) and that at the choice stage, higher income consumers are less price sensitive than consumers with lower levels of income (Appendix C).

The parameter estimates for the various controls we've included in the model can be found in Web Appendix E. Our estimates for the parameters intended to measure heterogeneity through past purchase history are mostly positive and significant for the consideration level of the model, indicating that (as would be expected) consumers who previously purchased a vehicle type are likely to consider it again in the future. For example, consumers who are buying a new vehicle to replace a previously purchased one are very likely to consider the same nameplate again—the parameter estimate for this variable is 1.171, a huge value on the logit scale. Some of these parameters are also significant for the choice stage of the model, but not as many. The parameter estimates for demographic and other interaction variables are fairly intuitive: consumers to whom fuel efficiency matters are more likely to consider smaller vehicles; men are more likely to consider premium cars and trucks than are women; consumers who rate themselves as brand loyal are more likely than others to again consider a previously purchased manufacturer; and consumers who use their vehicle for towing, hauling, and off-roading are more likely to consider larger vehicles such as vans, utilities, and trucks.

6.2. Full versus Restricted Model

Recall that we benchmark the performance of the proposed model against a restricted version of that model. The restriction employed—that the marginal search costs for all set sizes except a size of one are equal to zero—reduce the proposed model to a level model similar to the one used in Van Nierop, et al. (2010). The restricted model therefore can only account for consideration set substitution in one specific circumstance—when the lone alternative in a set of size one is removed and another is added in its place.

The restricted model's inability to fully account for consideration set substitution has two primary consequences. First, the restricted model underestimates the conversion rate of considerations gained by marketing actions or market events (the probability that a gained consideration will translate to a purchase). The restricted model cannot account for the fact that a consumer who adds an alternative to his or her consideration set (e.g., due to advertising) may in turn kick out a competing alternative from that set. The restricted model therefore overestimates the consideration set size of these consumers and underestimates the purchase probability for the added alternative.

Second, the restricted model misestimates the number of considerations gained (or lost) due to a marketing action (or any other effect of interest). This occurs because the restricted model cannot take into account that some of an alternative's period-to-period gains are substitutions coming from another alternative's losses (or vice-versa). For example, some of the considerations that Toyota lost after the recalls were stolen by redesigned vehicles, and would have been lost even if Toyota had not been harmed by the recall crisis.

Lastly, a third consequence of the restricted model stems from its failure to incorporate search costs: the parameters of consideration propensity must be fit to both (1) the distribution of consideration set sizes observed in the data and (2) the relative frequency with which each alternative is considered. The model may fit one or both poorly without the explanatory power of search costs (which truncate a consumer's consideration set). In our empirical context, the restricted model fits the distribution of consideration set sizes poorly, noticeably overestimating the proportion of sets of size one (54.9% in the restricted model's baseline simulation versus 45.8% in the data and the full model's baseline simulation) and of size four or greater (6.5% in the data and 6.8% for the full model versus 8.7% for the restricted model; Table 1). This issue is likely to be less problematic if fairly granular unobservable heterogeneity can be accounted for (e.g., fully heterogeneous alternative-specific constants for consideration propensity).

Because consideration set sizes are not well captured by the restricted model, predicting how the two models' estimates will differ is not straightforward. An additional complicating factor is that, because the restricted model lacks the explanatory power of consideration set substitution, other parameters will be used to "fill the gap." If consideration of Toyota vehicles is lower post-recall than pre-recall, the restricted model is forced to attribute increases in consideration to other vehicles post-recall to heterogeneous differences across quarters. Consequently, the parameters of Σ_{ξ} are the only ones for which the two models' 95% credible intervals do not overlap.

The remainder of section six is devoted to illustrating these consequences in the context of measuring the impact of vehicle redesigns, the Toyota recalls, and the Tōhoku tsunami.

6.3. Vehicle Redesigns

We find that the average redesigned vehicle gained 1.9 considerations and 1.0 purchases per 100 consumers due to the redesign. The estimated consideration gains were statistically significant for five of the six vehicles. The purchase gains were significant for all six (Table 5).

The restricted model underestimates the frequency with which considerations gained due to the redesign lead to purchase (the conversion rate). Table 6 illustrates this. Among all consumers with a consideration set of size 2 in the full model's no-redesign counterfactuals (averaged across all six vehicle counterfactuals), those who added the redesigned vehicle to their consideration set in the baseline simulation saw their consideration set rise to (on average) 2.70 (rather than 3.0, indicating that 30% of those additions supplanted a competing alternative). By contrast, in the restricted model's simulation, their consideration sets rose to 3.0 (because no consideration set substitution occurs). The full model has a correspondingly higher conversion rate, with 47.6% of considerations added to a set size of two translating to a purchase (versus 42.4% for the restricted model). Similar patterns are observed with larger set sizes. The one exception here is consumers who begin with a set size of one in the counterfactual simulations—both models allow for consideration set substitution for set sizes of one (because each consumer must consider at least one alternative).

Table 6 also highlights that the restricted model overestimates the proportion of consideration sets of size one (and greater than four). This can be seen in the last two columns: the proportion of consideration sets of size one is 44.4% in the full model counterfactual simulations and 51.0% in the restricted model's simulations. Because of this, the two models actually predict similar degrees of consideration set substitution overall, and similar gains in purchase due to the redesigns. The difference in predicted purchase gains by the two models is particularly small for the Outback, which was predicted to have gained 0.83 purchases by the full model and 0.84 by the restricted model (Table 5). In this particular example, two wrongs actually do seem to make a right. But we should not conclude that the restricted model is "good enough" for this empirical setting. The poor fit of consideration set sizes, while balancing the scales for our redesign effect estimates, leads to unambiguous mismeasurement of price sensitivity. We run a counterfactual (using both models) to estimate how sales would have differed had the six vehicle redesigns been priced $1,000 lower. Table 7 provides the results. For any given consideration set size, the change (induced by the lower price point at the choice stage of the model) in conversion rate of a consideration is the same for both models (as expected—the choice level of both models is the same). However, the total (across all set sizes) conversion rate is higher for the full model, because the restricted model overestimates the proportion of singleton consideration sets. Consumers with a set of size one will, of course, not be swayed by any choice stage variables, including price. The restricted model consequently underestimates own-price elasticity for the redesigned vehicles by an average of 9.9%. This can be seen in Table 8, which (for each redesigned vehicle) lists its average post-redesign paid price, the percentage change that a $1,000 price decrease represents, the forecasted percentage change in market share due to this $1,000 price decrease (for both models), and the corresponding own-price elasticity estimates (for both models).

6.4. Market Events

We find that Toyota C and CD cars lost about 4.9 considerations and 2.3 purchases per 100 consumers as a consequence of the recalls (Table 9), declines of about 17.9% and 19.6% respectively. In total, 40.3% of Toyota's lost considerations were absorbed by other vehicles. By contrast, the rate of consideration set substitution for losses due to the Tōhoku earthquake and tsunami was only 36.3%. This is because the Tsunami affected many close substitutes—all Japanese vehicles. Of the considerations lost by Toyota due to the recalls, 12.2% were absorbed by other Japanese C or CD cars (28.1% by other vehicles). By contrast, only 5.4% of the losses due to the tsunami were absorbed by other Japanese C or CD cars (30.9% by other vehicles). With fewer attractive substitutes available post-tsunami, consumers were driven to reduce their consideration set size more frequently. We estimate that the tsunami lost Japanese C and CD cars 9.4 considerations and 5.1 purchases per 100 consumers (Table 9), declines of 13.2% and 13.6%.

The restricted model overestimated the considerations and purchases lost by Toyota due to the recall by 11.2% and 12.9%. It overestimated Japanese manufacturers' consideration losses due to the tsunami by 14.4%. Interestingly, even though lost considerations were noticeably overestimated in the restricted model's tsunami counterfactuals, lost purchases were only slightly overestimated (2.5%). This occurred because the restricted model underestimated the rate of consideration set substitution that occurred in response to the tsunami. More specifically, the restricted model underestimated the proportion of considerations lost by Japanese manufacturers that were replaced by considerations of North American, European, or Korean vehicles. For intuition, consider this example: the restricted model may have correctly estimated that some hypothetical consumer, X, did not consider the Toyota Corolla as a consequence of the tsunami, but failed to recognize that consumer X replaced the Corolla with the Ford Focus in his or her consideration set. If this hypothetical consumer also considered the Honda Accord, then the restricted model would predict a consideration set consisting only of the Accord (rather than a set of both the Accord and Focus). The restricted model would therefore overestimate the probability that consumer X would purchase a Japanese vehicle. This logic, applied in aggregate, explains how the restricted model might greatly overestimate the number of considerations lost by Japanese manufacturers while only slightly overestimating the number of purchases lost.

7. Limitations and Potential Extensions

One limitation of our model is that consideration set substitution is a function only of consideration set size, and not consideration set composition. By contrast, consideration set substitution in the structural search cost model used by Honka (2014) is driven by consideration set composition. That model can therefore account for some substitution patterns that our model cannot. For example, if a consumer has a consideration set $S_i = \{a, b, c\}$, and w_{ia} is increased due to some marketing action by the manufacturer of alternative a, Honka's model can account for the possibility that either b or c may no longer be worth considering (because the benefit from including either alternative is decreasing in w_{ia}).

Our empirical analysis is limited by at least two factors. First, NVCS survey respondents were given the option to list up to three vehicles they considered in addition to the one they purchased. Thus, the respondents who provided consideration sets of four vehicles may have considered more than four. We cannot know whether these consumers considered exactly four vehicles or more, and so use these stated consideration sets as given. We note also that there is a steep decline from the proportion of respondents who considered three vehicles (16.1%) to the proportion that considered (at least) four (6.9%), so it is not unreasonable to think that few (if any) respondents may have considered more than four vehicles.

Second, we excluded from our sample consumers who did not consider at least one car. Consequently, we cannot measure the degree to which market events caused consumers who would have considered at least one car to stop considering cars altogether (or the reverse). We decided that including a larger number of consumers who considered the focal vehicle classes (C and CD cars) in the sample was more critical to accurately assessing substitution patterns in these classes than including consumers who exclusively considered trucks and utilities. That cars were rarely co-considered with these other vehicle types made this decision easier (Table 2).

Lastly, one potentially valuable extension would be to model search over a specific attribute (e.g., price, as in Honka 2014). The proposed model is flexible in that it does not *require* this, but incorporating such behavior could nonetheless be useful (e.g., to disentangle the relative importance of price and quality search).

8. Conclusion

Ample work has illustrated that accounting for consideration is critical to properly modeling demand. In spite of this, there is a paucity of econometric models in the literature that can both account for the cost-benefit trade-off that underlies consideration set formation and be estimated for markets with a large number of alternatives. One especially under-discussed element of the consumer's cost-benefit trade-off is consideration set substitution.

This paper develops a new model of consideration and choice that accounts for the fact that an increase in consideration of one alternative may decrease consideration of others. Moreover, it documents two primary consequences of failing to account for consideration set substitution: (1) misestimation of the impact of marketing actions and market events, and (2) underestimation of the rate at which gained considerations are converted to sales. Additionally, this paper shows that models that do not account for search costs may poorly capture the distribution of consideration set sizes, and consequently misestimate important quantities such as price elasticities.

The model developed in this paper has several features (above and beyond its ability to account for consideration set substitution) that make it an attractive option for researchers and marketing managers. First, the model is directly related to two commonly used models—it serves as an extension of the exploded logit model and generalizes the level model. Moreover, it can be formally tested against the level model to determine whether the researcher’s data provides evidence that consumers engage in consideration set substitution. Finally, and perhaps most critically, estimation of the model is not impeded by the curse of dimensionality. The model is therefore especially useful for modeling markets with many alternatives—the very markets for which consumers are most incentivized to engage in limited search.

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Tables and Figures

Figure 1: Toyota Considerations and Purchases

Recall period begins after purple line, tsunami period after grey line.

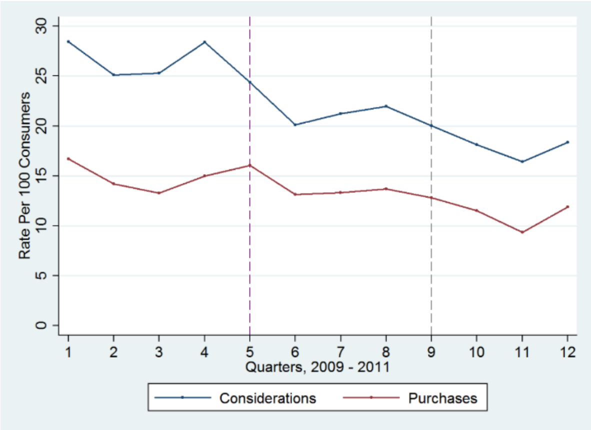

Figure 2: Japanese Considerations and Purchases*

*Excluding Toyota. Tsunami period begins after grey line.

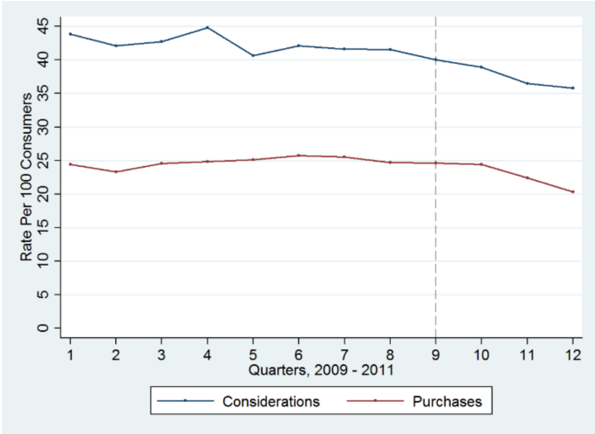

Table 1: Distribution of Consideration Set Sizes

Consideration Set Size	1	2	3	4+
Data Sample	45.8%	32.5%	15.2%	6.5%
Full Model Simulations	45.8%	32.1%	15.3%	6.8%
Restricted Model Simulations	54.9%	23.9%	12.5%	8.7%

Table 2: Co-consideration of vehicle classes

	C/CD cars	B/D cars	Premium Cars	Small Utilities	Med/Lg Utilities	Trucks	Vans
Respondent considered C	75.4%	8.9%	6.0%	5.9%	2.4%	0.9%	0.6%
Respondent considered CD	70.5%	8.1%	9.8%	5.7%	4.8%	0.8%	0.3%

Table 2 provides, for all consideration sets containing at least one C (row 1) or CD (row 2) car, the (weighted) proportion of all vehicles belonging to a class. E.g., 0.6% of vehicles in sets containing a C Car were vans.

Table 3: Considerations, Purchases, and Price for Redesigned Vehicles (Pre- and Post-Redesign)

		Considerations / 100			Purchases / 100			Average Price Paid		
Vehicle	RD Date	Pre-	Post-	Change	Pre-	Post-	Change	Pre-	Post-	Change
Elantra	10-Q4	2.3	4.7	93%	0.9	1.6	83%	$17.7	$18.9	6.5%
Jetta	10-Q4	3.2	4.3	37%	1.2	1.6	29%	$24.1	$23.4	-3.0%
Legacy	09-Q4	0.7	1.1	44%	0.3	0.4	41%	$24.9	$26.8	7.9%
Optima	11-Q1	0.6	1.8	137%	0.3	0.6	121%	$19.4	$23.6	21.7%
Outback	09-Q4	1.3	2.8	124%	0.4	0.9	120%	$27.2	$29.7	9.3%
Sonata	10-Q2	2.5	6.1	138%	0.9	2.0	123%	$20.5	$23.9	16.7%

Price Pre-RD and Post-Rd are the average of prices reported by buyers of a vehicle pre- and post-redesign during the three year period under study. Prices are adjusted for inflation, with Q1 2009 as the reference point.

Table 4: Considerations and Purchases per 100 consumers, by period

Considerations per 100 Consumers									
	Data Sample			Baseline Sim - Full Model			Baseline Sim - Restricted		
Period →	1	2	3	1	2	3	1	2	3
C and CD Cars	115.9	111.5	116.1	114.2	116.8	112.9	112.3	115.3	110.0
Toyota	28.4	23.8	20.8	28.4	23.8	20.9	29.0	24.1	21.1
Honda	25.6	22.7	18.4	23.9	25.1	18.4	24.5	25.8	18.6
Other Japanese	22.1	22.9	22.7	21.6	23.7	22.7	20.8	22.9	21.5
Other Countries	39.8	42.2	54.3	40.3	44.1	50.9	38.1	42.5	48.8
Other Vehicle Classes	69.3	66.5	67.6	68.8	67.6	68.1	68.0	67.0	67.5
Average Set Size	1.85	1.78	1.84	1.83	1.84	1.81	1.80	1.82	1.78

Purchases per 100 Consumers									
	Data Sample			Baseline Sim - Full Model			Baseline Sim - Restricted		
Period →	1	2	3	1	2	3	1	2	3
C and CD Cars	60.6	61.9	61.6	61.0	62.4	60.7	60.9	62.3	60.3
Toyota	15.1	13.3	10.9	14.4	12.2	10.5	15.2	12.6	11.0
Honda	12.5	11.8	9.5	12.9	13.6	9.8	13.8	14.4	10.3
Other Japanese	12.0	13.4	12.7	12.3	13.5	12.4	11.8	12.9	11.8
Other Countries	21.1	23.4	28.4	21.3	23.2	27.9	20.2	22.5	27.2
Other Vehicle Classes	39.4	38.1	38.4	39.0	37.6	39.3	39.1	37.7	39.7

Period 1 is Q1 '09 – Q1 '10 (Pre-recall); Period 2 is Q2 '10 – Q1 '11 (Post-Recall, Pre-Tsunami); Period 3 is Q2 '11 – Q4 '11 (Post-Tsunami).

Table 5: Simulated Considerations and Purchases Per 100 Consumers for Redesigned Vehicles

	Elantra	Jetta	Legacy	Optima	Outback	Sonata
Consideration Gain						
Full Model Estimate	2.54	1.24	0.33	1.56	1.51	4.06
Restricted Model Est.	2.48	1.16	0.21	1.57	1.54	4.20
Difference (Pct)	-2.4%	-6.9%	-35.6%	0.8%	2.4%	3.4%
Full Model 95% CI	1.59	0.41	-0.16	0.90	0.87	3.07
	3.56	2.24	0.83	2.33	2.16	5.08
Purchase Gain						
Full Model Estimate	1.45	0.65	0.34	0.81	0.83	2.00
Restricted Model Est.	1.39	0.61	0.25	0.84	0.84	2.17
Difference (Pct)	-3.8%	-6.3%	-27.9%	4.1%	1.9%	8.4%
Full Model 95% CI	0.76	0.05	0.04	0.36	0.41	1.26
	2.24	1.36	0.65	1.34	1.29	2.76
Consideration Set Substitution Rate						
Full Model Estimate	43.8%	43.8%	42.6%	43.6%	43.2%	42.7%
Restricted Model Est.	26.9%	28.7%	23.1%	24.8%	24.5%	26.5%
Difference (Pct)	-38.6%	-34.5%	-45.9%	-43.1%	-43.2%	-38.1%
Full Model 95% CI	33.1%	27.6%	0.0%	28.5%	32.7%	35.6%
	54.6%	60.7%	75.7%	59.1%	53.7%	49.8%

For each vehicle, we list considerations and purchases gained due to the redesign and the rate of consideration set substitution estimated using both the full and restricted model. The restricted model's percentage over- or under-estimate is also provided ('Difference').

Table 6: Conversion Rate of Considerations Gained by Redesigns

Set size w/o RD	Set size w/ RD		Conversion Rate		Rest/Full	Proportion of Sets	
	Full	Rest.	Full	Rest.	Ratio	Full	Rest.
1	1.55	1.49	78.4%	80.5%	2.6%	44.4%	51.0%
2	2.70	3.00	47.6%	42.4%	-11.0%	30.4%	21.6%
3	3.65	4.00	35.5%	32.3%	-8.9%	16.9%	13.6%
4+	4.00	5.87	31.7%	22.8%	-28.2%	8.3%	13.7%
Weighted Avg:	2.46	2.76	57.9%	57.8%	-	-	-

For consumers who considered any redesigned vehicle in the baseline simulation, but did not consider that vehicle in the corresponding "no redesign" counterfactual, we report: (column 1) consideration set size in the "no redesign" counterfactual, (2,3) average set size in the baseline simulation for the full and restricted model, (4,5) conversion rate of the gained considerations (gained purchases divided by considerations), (6) % difference of rate between models (a negative value indicates underestimation by restricted model), and (7,8) proportion of consumers that began with a set of the size indicated in column 1.

Table 7: Change in Conversion Rate of Considerations $1K Price Reduction

	Full Model			Restricted Model		
	Set Size	Δ Convert	% of Sets	Set Size	Δ Convert	% of Sets
	1	0.0%	20.2%	1	0.0%	25.8%
	2	1.8%	33.0%	2	1.8%	24.8%
	3	1.9%	27.3%	3	1.9%	21.5%
	4	1.8%	19.5%	4	1.7%	13.6%
	-	-	-	> 4	1.5%	14.2%
Wt Avg	2.46	1.5%		2.80	1.3%	-

For consumers who considered a redesigned vehicle, we report: (1,4) consideration set size for full and restricted model (2,5) the change in conversion rate for considerations in sets of that size due to the $1,000 price reduction, and (3,6) the proportion of consumers who had consideration sets of that size.

Table 8: Own-Price Elasticities for Redesigned Vehicles

	Price	% chg	Purch gain/loss		% chg		Elasticity		Elasticity
			Full	Rest.	Full	Rest.	Full	Rest.	Difference
Elantra	$18,895	-5.3%	0.076	0.067	2.4%	2.2%	-0.45	-0.42	-7.2%
Jetta	$23,402	-4.3%	0.070	0.059	2.3%	2.1%	-0.54	-0.49	-10.5%
Legacy	$26,819	-3.7%	0.020	0.016	2.7%	2.5%	-0.73	-0.66	-8.4%
Optima	$23,568	-4.2%	0.033	0.028	2.5%	2.3%	-0.60	-0.54	-9.0%
Outback	$29,718	-3.4%	0.044	0.037	2.8%	2.5%	-0.84	-0.73	-12.7%
Sonata	$23,937	-4.2%	0.099	0.088	2.6%	2.3%	-0.61	-0.54	-11.5%
AVERAGE		-4.2%	0.057	0.049	2.6%	2.3%	-0.63	-0.56	-9.9%

Table 8 provides the (avg quarterly, post-RD) price for each vehicle (column 2), the % change a $1K price decrease represents (3), the estimated % change in purchases resulting from this decrease (6,7), corresponding elasticity estimates (7,8), the restricted model's % underestimation (9).

Table 9: Considerations and Purchases Lost due to Toyota Recalls and Tōhoku tsunami

	Full Model			Restricted Model			Difference
Recalls - Toyota C/CD	Est.	95% CI		Est.	95% CI		
Lost Considerations	-4.92	-7.21	-2.52	-5.47	-8.12	-3.25	11.2%
Lost Purchases	-2.25	-3.66	-0.88	-2.54	-3.98	-1.11	12.9%
CSET Substitution	40.3%	34.0%	47.0%	24.9%	19.6%	30.6%	-38.2%
Tsunami - Japan C/CD	Est.	95% CI		Est.	95% CI		
Lost Considerations	-9.43	-13.37	-5.52	-10.79	-15.06	-6.39	14.4%
Lost Purchases	-5.14	-7.31	-3.04	-5.27	-7.39	-3.10	2.5%
CSET Substitution	36.3%	29. 5%	45.5%	22.78%	17.2%	30.5%	-37.3%

Appendix A: Derivation of Likelihood Statement

There exists a set of alternatives $A = \{1, \dots, K\}$. Consumer i has a consideration set S_i consisting of alternatives $1, \dots, k$ (but not alternatives $k+1, \dots, K$). The consumer also faces marginal search costs of considering the nth alternative in their set $\tilde{c}_n$. The probability that S_i is given by:

(1) $\Pr\left[\{w_j + \epsilon_j\}_{j \le k} > \{w_j + \epsilon_j\}_{j > k}, \{w_j + \epsilon_j\}_{j \le k} > \tilde{c}_k, \{w_j + \epsilon_j\}_{j > k} < \tilde{c}_{k+1}\right]$

(2) $= \int_{m=-\infty}^{m=\infty} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > \min(\tilde{c}_{k+1}, max(\tilde{c}_k, m))\right] \Pr\left[\min(m, \tilde{c}_{k+1}) = max\{w_j + \epsilon_j\}_{j > k}\right] dm$

Note that $\{w_j + \epsilon_j\}_{j \le k}$ must always be greater than $\tilde{c}_k$ and $max\{w_j + \epsilon_j\}_{j > k}$. Since $max\{w_j + \epsilon_j\}_{j > k}$ must always be less than $\tilde{c}_{k+1}$, we have $\{w_j + \epsilon_j\}_{j \le k} > \min(\tilde{c}_{k+1}, max(\tilde{c}_k, m))$.

Given (a) $\frac{d}{dm}\left(\Pr\left[\min(m, \tilde{c}_{k+1}) > max\{w_j + \epsilon_j\}_{j > k}\right]\right) = \Pr\left[\min(m, \tilde{c}_{k+1}) = max\{w_j + \epsilon_j\}_{j > k}\right]$

(3) $= \int_{m=-\infty}^{m=\infty} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > \min(\tilde{c}_{k+1}, max(\tilde{c}_k, m))\right] \frac{d}{dm}\left(\Pr\left[\min(m, \tilde{c}_{k+1}) > max\{w_j + \epsilon_j\}_{j > k}\right]\right) dm$

(4) $= \int_{m=-\infty}^{m=\tilde{c}_k} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > \tilde{c}_k\right] \frac{d}{dm}\left[\Pr\left(m > max\{w_j + \epsilon_j\}_{j > k}\right)\right] dm$

$+ \int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > m\right] \frac{d}{dm}\left[\Pr\left(m > max\{w_j + \epsilon_j\}_{j > k}\right)\right] dm$

$+ \int_{m=\tilde{c}_{k+1}}^{m=\infty} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > \tilde{c}_{k+1}\right] \frac{d}{dm}\left[\Pr\left(\tilde{c}_{k+1} > max\{w_j + \epsilon_j\}_{j > k}\right)\right] dm$

Since m does not appear in the integral from (C_{k+1}, ∞):

(5) $= \int_{m=-\infty}^{m=\tilde{c}_k} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > \tilde{c}_k\right] \frac{d}{dm}\left[\Pr\left(m > max\{w_j + \epsilon_j\}_{j > k}\right)\right] dm$

$+ \int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}} \Pr\left[\{w_j + \epsilon_j\}_{j \le k} > m\right] \frac{d}{dm}\left[\Pr\left(m > max\{w_j + \epsilon_j\}_{j > k}\right)\right] dm$

(6) $= \int_{m=-\infty}^{m=\tilde{c}_k} \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-\tilde{c}_k)\right)\right] \frac{d}{dm}\left[\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-m)\right)\right] dm$

$+ \int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}} \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-m)\right)\right] \frac{d}{dm}\left[\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-m)\right)\right] dm$

(7) $= \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-\tilde{c}_k)\right)\right] \int_{m=-\infty}^{m=\tilde{c}_k} \frac{d}{dm}\left[\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-m)\right)\right] dm$

$+ \int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}} \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-m)\right)\right] \frac{d}{dm}\left[\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-m)\right)\right] dm$

Since (b) $\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-(-\infty))\right) = 0$

Then (c) $\int_{m=-\infty}^{m=\tilde{c}_k} \frac{d}{dm}\left[\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-m)\right)\right] dm = \exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-\tilde{c}_k)\right) - 0$

(8) $= \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-\tilde{c}_k)\right)\right] \exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-\tilde{c}_k)\right)$

$+ \int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}} \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-m)\right)\right] \frac{d}{dm}\left[\exp\left(-\sum_{j=k+1}^{j=K}\exp(w_j)\exp(-m)\right)\right] dm$

And if we (d) set $a = \sum_{j=k+1}^{j=K}\exp(w_j)$

(9) $= \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-\tilde{c}_k)\right)\right] \exp(-a\exp(-\tilde{c}_k))$

$+ \int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}} \prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-m)\right)\right] \frac{d}{dm}\left[\exp(-a\exp(-m))\right] dm$

(e) Set $\prod_{j=1}^{j=k}\left[1 - \exp\left(-\exp(w_j)\exp(-m)\right)\right] = \prod_{j=1}^{j=k}\left[1 - E_j\right] = 1 + \sum_{t=1}^{t=k}(-1)^t\,[T_t]$

Where (f) $E_j = \exp\left(-\exp\left(w_j\right)\exp(-m)\right)$

And (g) $T_t = \sum_{q=1}^{q=\frac{k!}{t!(k-t)!}} \prod_{j \in G_{qt}} \exp\left(-\exp\left(w_j\right)\exp(-m)\right)$

And (h) G_{qt} is the q^{th} subset of alternatives $j \in S_i$ $(j \leq k)$, of size t, of which there exist $\frac{K!}{t!(K-t)!}$

E.g., if $k = 3$, $T_1 = E_1 + E_2 + E_3$, $T_2 = E_1E_2 + E_1E_3 + E_2E_3$, and so on…

Therefore:

$$(10) = \prod_{j=1}^{j=k}\left[1-\exp\left(-\exp\left(w_j\right)\exp(-\tilde{c}_k)\right)\right]\exp(-a\exp(-\tilde{c}_k))$$
$$+\int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}}\left(1+\sum_{t=1}^{t=k}(-1)^t\left[\sum_{q=1}^{q=\frac{k!}{t!(k-t)!}}\prod_{j\in G_{qt}}\exp\left(-\exp\left(w_j\right)\exp(-m)\right)\right]\right)\frac{d}{dm}[\exp(-a\exp(-m))]dm$$

$$(11) = \prod_{j=1}^{j=k}\left[1-\exp\left(-\exp\left(w_j\right)\exp(-\tilde{c}_k)\right)\right]\exp(-a\exp(-\tilde{c}_k))$$
$$+\int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}}\frac{d}{dm}[\exp(-a\exp(-m))]dm$$
$$+\int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}}\left(\sum_{t=1}^{t=k}(-1)^t\left[\sum_{q=1}^{q=\frac{k!}{t!(k-t)!}}\prod_{j\in G_{qt}}\exp\left(-\exp\left(w_j\right)\exp(-m)\right)\right]\right)\frac{d}{dm}[\exp(-a\exp(-m))]dm$$

$$(12) = \prod_{j=1}^{j=k}\left[1-\exp\left(-\exp\left(w_j\right)\exp(-\tilde{c}_k)\right)\right]\exp(-a\exp(-\tilde{c}_k))$$
$$+\exp(-a\exp(-\tilde{c}_{k+1}))-\exp(-a\exp(-\tilde{c}_k))$$
$$+\int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}}\left(\sum_{t=1}^{t=k}(-1)^t\left[\sum_{q=1}^{q=\frac{k!}{t!(k-t)!}}\prod_{j\in G_{qt}}\exp\left(-\exp\left(w_j\right)\exp(-m)\right)\right]\right)\frac{d}{dm}[\exp(-a\exp(-m))]dm$$

(j) Set $b_{qt} = \sum_{j\in G_{qt}}\exp\left(w_j\right)$

Given (k) $\frac{d}{dm}\exp\left(-aexp(-m)\right) = a\exp(-m)\exp\left(-a\exp(-m)\right)$:

$$(13) = \prod_{j=1}^{j=k}\left[1-\exp\left(-\exp\left(w_j\right)\exp(-\tilde{c}_k)\right)\right]\exp(-a\exp(-\tilde{c}_k))$$
$$+\exp(-a\exp(-\tilde{c}_{k+1}))-\exp(-a\exp(-\tilde{c}_k))$$
$$+\sum_{t=1}^{t=k}(-1)^t\sum_{q=1}^{q=\frac{k!}{t!(k-t)!}}a\int_{m=\tilde{c}_k}^{m=\tilde{c}_{k+1}}\exp(-m)\exp\left(-(a+b_{qt})\exp(-m)\right)dm$$

Again using (k), $\exp(-m)\exp\left(-(a+b_{qt})\exp(-m)\right) = \frac{1}{a+b_{qt}}\frac{d}{dm}\exp\left(-(a+b_{qt})exp(-m)\right)$, and:

$$(14) = \prod_{j=1}^{j=k}\left[1-\exp\left(-\exp\left(w_j\right)\exp(-\tilde{c}_k)\right)\right]\exp(-a\exp(-\tilde{c}_k))$$
$$+\exp(-a\exp(-\tilde{c}_{k+1}))-\exp(-a\exp(-\tilde{c}_k))$$
$$+\sum_{t=1}^{t=k}(-1)^t\sum_{q=1}^{q=\frac{k!}{t!(k-t)!}}\frac{a}{a+b_{qt}}\left[\exp\left(-\left(a+b_{qt}\right)\exp(-\tilde{c}_{k+1})\right)-\exp\left(-\left(a+b_{qt}\right)\exp(-\tilde{c}_k)\right)\right]$$

This gives us equation 10 from the paper, although the notation in the paper is slightly different. Specifically, in the paper k is replaced with with N_{S_i} (the notation used for the size of consumer i's consideration set), and the alternatives in S_i are not presumed to be consecutive in number (e.g., a set size of three is not assumed to include alternatives 1, 2, and 3).

Appendix B: Primary Parameter Estimates

Parameter Estimates

Parameters	Consideration Stage - Full Model				Consideration Stage - Restricted				Choice Stage - Both Models			
Search Costs	Est.	95% CI			Est.	95% CI			Est.	95% CI		
θ_3	-1.522	-1.665	-1.402	*	-	-	-		-	-	-	
θ_4	-1.713	-1.902	-1.524	*	-	-	-		-	-	-	
$\tilde{c}_3$ $(\exp(\theta_3))$	0.218	0.189	0.246	*	0.000	-	-		-	-	-	
$\tilde{c}_4$ $(\tilde{c}_3 + \exp(\theta_4))$	0.399	$\tilde{c}_3$+0.149	$\tilde{c}_3$+0.218	*	0.000	-	-		-	-	-	
Heterogeneity Covariance Matrix	Est.	95% CI			Est.	95% CI			Est.	95% CI		
Variance - Preference for C Cars	1.000	-	-		1.000	-	-		-	-	-	
Covariance - C and CD Cars	-0.300	-0.422	-0.187	*	-0.508	-0.576	-0.435	*	-	-	-	
Variance - Preference for CD Cars	1.252	1.100	1.419	*	1.199	1.037	1.358	*	-	-	-	
Redesign	Est.	95% CI			Est.	95% CI			Est.	95% CI		
Hyundai Elantra	0.745	0.511	1.003	*	0.767	0.556	0.992	*	-0.232	-0.789	0.267	
VW Jetta	0.326	0.118	0.555	*	0.329	0.133	0.547	*	-0.179	-0.652	0.284	
Subaru Legacy	0.305	-0.130	0.740		0.319	-0.194	0.831		1.176	-0.009	2.332	
Kia Optima	1.272	0.847	1.640	*	1.241	0.829	1.661	*	-0.814	-1.882	0.176	
Subaru Outback	0.743	0.424	1.075	*	0.761	0.422	1.153	*	0.506	-0.377	1.599	
Hyundai Sonata	1.037	0.796	1.290	*	1.103	0.875	1.395	*	-0.511	-1.184	0.011	
Toyota Recall and Tsunami	Est.	95% CI			Est.	95% CI			Est.	95% CI		
Recall - Toyota C/CD Cars	-0.242	-0.344	-0.132	*	-0.241	-0.355	-0.132	*	0.020	-0.225	0.300	
Tsunami - Toyota C/CD Cars	-0.150	-0.274	-0.029	*	-0.144	-0.273	-0.023	*	-0.244	-0.580	0.084	
Tsunami - Toyota B/DE Cars	-0.769	-1.232	-0.345	*	-0.771	-1.192	-0.348	*	0.717	-0.869	2.494	
Tsunami - Honda C/CD Cars	-0.371	-0.497	-0.239	*	-0.379	-0.499	-0.264	*	-0.196	-0.535	0.138	
Tsunami - Honda B/DE Cars	-0.423	-0.817	-0.062	*	-0.433	-0.807	-0.061	*	0.220	-0.878	1.438	
Tsunami - Other Japanese C/CD	-0.098	-0.216	0.029		-0.087	-0.194	0.019		-0.366	-0.679	-0.074	*
Price	Est.	95% CI			Est.	95% CI			Est.	95% CI		
Price - Income Group 1	-	-	-		-	-	-		-0.156	-0.222	-0.093	*
Price - Income Group 2	-	-	-		-	-	-		-0.099	-0.139	-0.061	*
Price - Income Group 3	-	-	-		-	-	-		-0.097	-0.138	-0.061	*
Price - Income Group 4	-	-	-		-	-	-		-0.074	-0.112	-0.039	*

* = 95% credible interval does not contain zero