A Common Information-Based Multiple Access Protocol Achieving Full Throughput

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Abstract—We consider a multiple access communication system where multiple users share a common collision channel. In this system, coordination among users is essential to resolve collision issues. However, each system user can only observe its own local traffic and the feedback from the channel. Without a centralized controller, it is challenging to design an efficient coordination protocol.

We present a decentralized common information-based multiple access protocol (CIMA). We show that under CIMA collision is totally avoided without channel sensing and the full throughput region of the collision channel is achieved. In addition, simulation results indicate that the CIMA protocol incurs low transmission delay.

I. INTRODUCTION

Multiple access has played a crucial role in the operation of many communication systems, including satellite networks, radio networks, wired and wireless Local Area Networks (LANs). One important feature of multiple access is its decentralized information structure. Consequently, when multiple users share a common communication channel, coordination among them is essential to resolve collision issues. The design of efficient coordination mechanisms/protocols is a challenging problem.

Various multiple access protocols have been proposed to address coordination among users in multiple access systems. In this paper we present, due to space constraints, only a partial list of the references on multiple access. Important classes of multiple access protocols include fixed assignment protocols, contention based protocols and Carrier Sense Multiple Access (CSMA) type protocols (see [1, 2] and references therein).

In a fixed assignment protocol, coordination is done before transmission. The channel resource can be divided into transmission units, and each transmission unit is allocated to only one user to avoid collision. Since collision is avoided, a fixed assignment protocol can achieve full capacity of the channel when the arrival rates are symmetric. However, under asymmetric arrivals, fixed assignment protocols may create long delays and result in unstable systems [3]. To effectively utilize the channel in fixed assignment mechanisms, [3, 4] introduced adaptive time-division-multiple-access (TDMA) protocols. Adaptive TDMA protocols use the common channel feedback to adaptively coordinate users to avoid collision. Adaptation resolves the problems due to asymmetric arrivals, and collision avoidance allows higher throughput. However, there is no theoretical analysis for these protocols, and the full throughput region is not achieved [3, 4].

Another important class of multiple access protocols is contention based protocols. Due to the appearance of contention/collision, most contention based protocols or Aloha-type protocols cannot achieve the full throughput region. The authors of [5] prove that polynomial back-off protocols can achieve the full throughput region. However, polynomial back-off protocols have poor delay performance in simulation. In [6–8], the authors proposed a series of decentralized random access protocols that achieve the full throughput region. However, to implement those protocols each user needs to know additional information about the maximum queue length or the transmission results of all other users.

When channel sensing is allowed, various CSMA type protocols have been developed to achieve higher throughput. In [9, 10] rate-based CSMA approaches were considered; queue-based CSMA protocols were proposed in [11, 12]. Although CSMA can result in high throughput, any CSMA protocol cannot achieve the full throughput region of a collision channel due to the overhead for channel sensing.

Other models for multiple access have also been proposed in the literature. In [13], channel switching policies that achieve high throughput for multiple access have been considered within the context of the slotted Aloha protocol and the IEEE 802.11 WLANs protocol. The stability region of the multipacket reception multiple access channel has been investigated in [14]. Multiple access with noisy channels has been considered in [15, 16], and the stability region of policies with delayed shared information has been presented.

In this paper, we propose a common information (see [17]) based multiple access protocol (CIMA) that uses the common channel feedback to coordinate users. CIMA operates in a similar manner as adaptive TDMA protocols in [3, 4]. Specifically, we consider a typical slotted multiple access communication system where multiple users share a common
collision channel. Each user is equipped with an infinite size buffer and observes Bernoulli arrivals to its own queue. In addition to the local information, all users receive a common broadcast feedback from the channel. The feedback indicates whether the previous transmission was successful (exactly one user transmitted), or it was a collision (more than one users transmitted), or the channel was idle. In CIMA, each user constructs upper bounds on the lengths of the queues of all users, including itself, based on previous transmission strategies and the common feedback. Since the upper bounds are common knowledge, users can coordinate their transmission to avoid collision through the common upper bounds. We prove that without knowledge of any statistics, CIMA achieves the full throughput region of the collision channel. We also present simulation results indicating that the CIMA achieves the full throughput region of the collision channel. Each user constructs upper bounds on the lengths of the queues of all users, including itself, based on previous transmission strategies and the common feedback. Since the upper bounds are common knowledge, users can coordinate their transmission to avoid collision through the common upper bounds. We prove that without knowledge of any statistics, CIMA achieves the full throughput region of the collision channel.

The rest of the paper is organized as follows. In Section II we present the system model and formulate the problem under investigation. In Section III we present the CIMA protocol. We analyze the CIMA protocol in Section IV and prove its throughput optimality. We present simulation results and compare the delay of our protocol with the delay of other protocols that achieve full throughput in Section V. We conclude in Section VI. We present the proofs of the technical results in Appendices A-C.

Notation

Random variables are denoted by upper case letters, their realization by the corresponding lower case letter. In general, subscripts are used as time index while superscripts are used to index users. For time indices $t_1 \leq t_2$, $X_{t_1:t_2}$ is the short hand notation for $(X_{t_1}, X_{t_1+1},...,X_{t_2})$. For a policy/protocol $g$, we use $X^g$ to indicate that the random variable $X^g$ depends on the choice of policy $g$. $P(\cdot)$ is the probability of an event. For random variables $X, Y$ with realizations $x, y$, $P(x|y) := P(X = x | Y = y)$. For a policy $g$ and a parameter $\lambda$, $P^{\lambda,g}(\cdot)$ indicates that the probability depends on the choice of $g$ and the parameter $\lambda$.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider a slotted communication system, described by Fig. 1, where $N$ users, indexed by $1, 2, \ldots, N$, share a common collision channel. Each user $n$ is associated with an infinite size buffer with queue length $Q^n_t$ at the beginning of each time slot $t$. We assume that each queue is initially empty.

At each time slot $t$ each user can transmit one packet in its queue through the shared channel. If only one user transmits in a time slot, the transmission is successful and the transmitted packet is removed from the queue; if more than one users transmit simultaneously, a collision occurs and all packets involved in the collision remain in their queue. We consider Bernoulli arrivals to the system. Let $A^n_t$ denote the packet arrival to user $n$ at time $t$; $A^n_t = 1$ means that a packet arrives at queue $n$ right after the transmission at time $t$. We assume that the arrival $A^n_t$ is a Bernoulli random variable with parameter $\lambda^n$, and the arrival processes $\{A^n_t, t = 0, 1, \ldots\}, n = 1, \ldots, N$ are independent. Let $U^n_t$ denote the transmission decision of user $n$ at time slot $t$; $U^n_t = 1$ (resp. $0$) indicates that user $n$ transmits (resp. does not transmit) at time $t$. The dynamics of queues are given as follows.

$$Q^n_{t+1} = A^n_t + \left( Q^n_t - U^n_t \prod_{m \neq n} (1 - U^m_t) \right)^+,$$

(1)

where $(\cdot)^+ = \max(\cdot, 0)$. We assume that at the end of each time slot $t$, every user receives a feedback $F_t \in \{0, 1, e\}$ from the channel/receiver indicating whether no packets, one packet, or more than one packet (a collision) were transmitted, respectively, in this time slot. This communication system is decentralized: each user can only observe its own queue length, its arrivals and the common feedback. Moreover, the arrival rates $\lambda = (\lambda^1, \lambda^2, \ldots, \lambda^N)$ are not known to the users. Therefore, the users’ decisions according to any decentralized transmission policy/protocol $g = \{g^n_t, n = 1, 2, \ldots, N, t = 0, 1, \ldots\}$ are generated by

$$U^n_t = g^n_t(Q^n_{t+1} - A^n_{t+1}, U^n_{t+1}, F_{t+1}),$$

(2)

$n = 1, 2, \ldots, N, t = 0, 1, 2,\ldots$.

B. Stability and Throughput Optimality

For queueing systems that can be described by irreducible Markov chains, stability is usually defined to be positive recurrence of the corresponding Markov chains. However, in this problem, the users’ actions can generally depend on the whole history of information. When non-Markovian control policies are used, the resulting queue length processes are not Markov in general. Even within the class of Markovian policies, the corresponding Markov chain may not be irreducible under any Markovian policy.

To achieve higher throughput performance of the communication system, we consider general non-Markovian policies of the form given by (2). Therefore, a stability notion for general stochastic processes is essential for our analysis of the system. In this paper, we call a stochastic process
\{X_t, t = 0, 1 \ldots \} stable if for every \( \epsilon > 0 \) there exists a finite set \( K \) such that
\[
P(X_t^n \notin K) < \epsilon \text{ for all } t.
\] (3)

This stability concept is also used in [18–20], and it is called bounded in probability in [21]. Note that the stability criterion (3) is equivalent to positive recurrence for countable irreducible Markov chains [21, Proposition 18.3.1].

Given the arrival rates \( \lambda = (\lambda^1, \ldots, \lambda^N) \) to all queues, a policy/protocol \( g \) stabilizes the communication system if the resulting queue length process \( \{Q^n_t, t = 0, 1, \ldots \} \) for every user \( n = 1, \ldots, N \) is stable. The arrival rate \( \lambda \) is said to be supportable if there exist policies/protocols that can stabilize the communication system.

Since at most one packet can be transmitted through the collision channel at each time, we know that only \( \lambda \in \Lambda \) could be supportable, where
\[
\Lambda = \left\{ \lambda = (\lambda^1, \lambda^2, \ldots, \lambda^N) : \sum_{n=1}^{N} \lambda^n < 1 \right\}.
\]

Furthermore, any \( \lambda \in \Lambda \) is supportable by the time sharing policy which assigns \( \lambda^n \) portion of time slots to user \( n \). Therefore, arrival rates \( \lambda \) are supportable if and only if \( \lambda \in \Lambda \). We call \( \Lambda \) the throughput region of the multiple access communication system.

We call a decentralized policy/protocol throughput optimal if it can stabilize the communication system for any \( \lambda \in \Lambda \). Our objective is to determine a decentralized throughput optimal protocol for the system model of Section II-A.

III. THE COMMON INFORMATION-BASED MULTIPLE ACCESS (CIMA) PROTOCOL

A. Preliminaries

We first introduce common upper bounds for the queues. Let \( B^n_{t,g} := (B^n_{t,g}^1, B^n_{t,g}^2, \ldots, B^n_{t,g}^N) \), where \( B^n_{t,g} \) is the upper bound on \( Q^n_t \) at time slot \( t \) based on the transmission protocol \( g \) and the common information determined by the common feedback \( F_{0:t-1} \) at time slot \( t \). That is, when \( F_{0:t-1} = f_{0:t-1} \),
\[
b^n_{t,g} = \max\{q^n_{\lambda} : \exists \lambda \in \Lambda \text{ s.t. } P^{\lambda,g}(q^n_{\lambda} | f_{0:t-1}) > 0\}.
\]

Note that, \( B^n_{t,g} \) is a function of common information \( F_{0:t-1} \). We use \( B^n_{t} \) to denote that the common upper bounds depend explicitly on the transmission policy \( g \).

B. The CIMA Protocol

The CIMA protocol is defined as follows.
\[
U^n_t = \text{CIMA}^n_t (Q^n_{0:t}, A^n_{0:t-1}, U^n_{0:t-1}, F_{0:t-1})
\]
\[
= \begin{cases} 
1 & \text{if } v(B^n_t) = n \text{ and } Q^n_t > 0, \\
0 & \text{otherwise}
\end{cases}
\] (4)

where \( v(\cdot) \) is a function of common upper bounds \( B^n_{t} \) defined as
\[
v(B^n_t) = \min\{n : b^n_{t} = \max_{m=1,2,\ldots,N} b^m_{t} \},
\]

Note that \( v(B^n_t) \) is the user with the largest common upper bound. Since we want to avoid collision, if there are more than one users with the largest common upper bound, CIMA selects the user with the smallest index.

IV. PERFORMANCE ANALYSIS OF THE CIMA PROTOCOL

We prove that the CIMA protocol is throughput optimal (Theorem 1).

A. Preliminaries

In order to analyze the system dynamics under the CIMA protocol, we first provide the following result on the evolution of queue lengths and common upper bounds under CIMA.

Lemma 1. Under CIMA, the queue lengths evolve as
\[
Q^n_{t+1} = \begin{cases} 
A^n_t + Q^n_{t} & \text{if } n \neq v(B^n_t), \\
A^n_t + (Q^n_{t} - 1) & \text{if } n = v(B^n_t),
\end{cases}
\]

and the common upper bounds evolve according to
\[
B^n_{t+1} = \begin{cases} 
B^n_{t} & \text{if } n \neq v(B^n_t), \\
B^n_{t} + 1 & \text{if } n = v(B^n_t) \text{ and } F_t = 1, \\
1 & \text{if } n = v(B^n_t) \text{ and } F_t = 0.
\end{cases}
\]

Proof: See Appendix A.  \( \square \)

Using Lemma 1, the CIMA protocol can be easily implemented as described below.

Algorithm 1 The CIMA protocol for user \( n \in \{1, 2, \ldots, N\} \)

for \( k = 1 \) to \( N \) do
\[
B^k \leftarrow 0
\]
while user \( n \) is active do
\[
B^{\text{MAX}} \leftarrow \max_k (B^k)
\]
\[
v \leftarrow \min (k : B^k = B^{\text{MAX}})
\]
if \( n = v \) then
\[
\text{transmit if there is a packet to send}
\]
for \( k \neq v \) do
\[
B^k \leftarrow B^k + 1
\]
if the transmission was successful then
\[
B^v \leftarrow B^v
\]
else
\[
B^v \leftarrow 1
\]

B. Throughput Optimality

The main result of this paper is described by the following theorem.

Theorem 1. The CIMA protocol is throughput optimal. That is, for any arrival rates \( \lambda \in \Lambda \), the queue length processes under CIMA are stable.

To prove the theorem, we first show that under the CIMA protocol the queue lengths together with the upper bounds form a Markov chain.
Lemma 2. Let $Y^\text{CIMA}_t := (Q^\text{CIMA}_1, B^\text{CIMA}_1)$, where 
$$Q^\text{CIMA}_t := (Q^\text{CIMA}_1, Q^\text{CIMA}_{2,t}, \ldots, Q^\text{CIMA}_{N,t})$$
for every time slot $t = 0, 1, \ldots$. Then, $\{Y^\text{CIMA}_t, t = 0, 1, \ldots\}$ is a Markov chain.

Proof: See Appendix B. 

Since $\{Y^\text{CIMA}_t, t = 0, 1, \ldots\}$ is a Markov chain, we can use the Foster-Lyapunov theorem in the proof below to show that $\{Y^\text{CIMA}_t, t = 0, 1, \ldots\}$ is stable.

Proof of Theorem 1: Let $\epsilon = 1 - \sum_{n=1}^{N} \lambda^n$. Then $\epsilon > 0$ because $\lambda \in \Lambda$. Let $y := (q, b) = (q^1, q^2, \ldots, q^N, b^1, b^2, \ldots, b^N)$; define the Lyapunov function $h(y)$ by

$$h(y) = \sum_{n=1}^{N} (q^n + \alpha b^n),$$

where $\alpha = \frac{\epsilon}{2(N-1)}$. For $Y^\text{CIMA}_t = y$, let $v = v(b) = \min(n : b^n = \max_{k=1,\ldots,N}(b^k))$. Then from (5) and (6) in Lemma 1 we get

$$\mathbb{E} \left[ h(Y^\text{CIMA}_{t+1}) - h(Y^\text{CIMA}_t) \mid Y^\text{CIMA}_t = y \right]$$

$$\leq -\epsilon / 2 \quad \text{if } b^v \geq \frac{1}{\alpha} + 1. \quad (7)$$

(see Appendix C for a detailed derivation of (7))

Since $b^1 = \max_{k=1,\ldots,N}(b^k)$, $b^v \geq b^n$ and $b^v \geq q^n$ for all $n = 1, 2, \ldots, N$. Define

$$C = \{ y = (q, b) : q^n < \frac{1}{\alpha} + 1, b^n < \frac{1}{\alpha} + 1 \quad \forall n \}. $$

Then, (7) holds for every $y \notin C$. Since $C$ is a finite set, the Lyapunov-Foster drift criterion (Condition (DD2) in [22]) is satisfied. From [22, Theorem 4.5], $\{Y^\text{CIMA}_t, t = 0, 1, \ldots\}$ is bounded in probability (satisfies the stability condition (3)).

Therefore, for every $\epsilon > 0$ there exists a finite set $K$ such that

$$\mathbb{P}(Y^\text{CIMA}_t \notin K) < \epsilon \text{ for all } t.$$ 

Let $K^n = \{ q^n : \text{there exists } y = (q, b) \in K \text{ is the projection of } K \text{ on its } n\text{th component. Then,}$$

$$\mathbb{P}(Q^\text{CIMA}_t \notin K^n) \leq \mathbb{P}(Y^\text{CIMA}_t \notin K) < \epsilon \text{ for all } t.$$ 

Therefore, $\{Q^\text{CIMA}_t, t = 0, 1, \ldots\}$ also satisfies (3) and the stability of the communication system is established.

V. SIMULATION RESULTS

In this section, we use simulation results to show that in addition to throughput optimality, the CIMA protocol also has good delay performance. We compare CIMA with two other protocols that use exactly the same information as CIMA, namely, the basic TDMA and the quadratic back-off protocol, which is proved to be throughput optimal in [5]. In the quadratic back-off protocol, each user transmits a packet with probability $(c + 1)^{-2}$ and $c$ is the back-off counter.

In the numerical experiments, we have used different values of $N$ and $\lambda_{\text{total}} = \lambda^1 + \ldots + \lambda^N$ for each protocol. Arrival rates are asymmetric: half of the users have arrival rate $1.4\lambda_{\text{total}}/N$ and the other half of the users have arrival rate $0.6\lambda_{\text{total}}/N$. For each $N$ and $\lambda_{\text{total}}$, we run the simulation for $T = 10^5$ time steps.

Since the delay of a packet is proportional to the average total queue length of the system, we use $Q_{\text{total}} := \frac{1}{T} \sum_{t=0}^{T-1} \sum_{n=1}^{N} Q^n_t$, the average total queue length, as the delay performance metric.

Since there is no collision, we expect that the delay associated with the CIMA protocol will be (approximately) linear in the number of users. The simulation results in Fig. 2 confirm this intuition. In Fig. 3, we compare the delay performance of different protocols for a system of 4 users. Fig. 3 shows that the delay associated with the CIMA protocol is significantly smaller than that of the quadratic back-off protocol (that is also throughput optimal) and of the TDMA protocol (note that TDMA is unstable when $\lambda_{\text{total}} > 0.7$).
VI. Conclusion

We developed a transmission protocol that utilizes the common information of the system's users to achieve efficient/optimal coordination of their transmissions. The protocol is collisions free; thus, it is similar in spirit to TDMA (or adaptive TDMA), but it differs from TDMA in the way it selects the user to transmit at each time slot. We can think of the upper bound on the queue length of a user at time $t$ as the “index” of that user at $t$. The CIMA protocol selects the user with the largest index at time $t$ to transmit at $t$. The updates of indices at $t$ depend on the outcome of the transmission at $t$. Intuitively, we expect that the delay due to the CIMA protocol will increase (approximately) linearly with the number of users. The simulation results verify this intuition. Comparison of CIMA with other protocols that use different information (e.g. [9–12]) will be done in the extended version of this paper.

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References

APPENDIX A

To prove Lemma 1 we first derive the following result.

Lemma 3. Under the CIMA protocol, the queue lengths are independent conditional on the common feedback given any arrival rates \( \lambda \). Specifically, for any time \( t \) and realizations \( q_1^t, \ldots, q_n^t, f_0:t-1 \)

\[
P^\lambda,\text{CIMA}(q_t|f_0:t-1) = \frac{P^\lambda,\text{CIMA}(q_t^n|f_0:t-1)}{\prod_{n=1}^N P^\lambda,\text{CIMA}(q_t^n|f_0:t-1)}.
\]  

(8)

Moreover, the conditional probability can be updated as follows. For \( n \neq v(b_1^\text{CIMA}) \)

\[
P^\lambda,\text{CIMA}(q_{t+1}^n|f_0:t) = \lambda^n P^\lambda,\text{CIMA}(Q_t^n = q_t^n + 1|f_0:t-1) + (1 - \lambda^n) P^\lambda,\text{CIMA}(Q_t^n > q_t^n|f_0:t-1).
\]  

(9)

For \( n = v(b_1^\text{CIMA}) \) and \( f_1 = 1 \)

\[
P^\lambda,\text{CIMA}(q_{t+1}^n|f_0:t) = \frac{P^\lambda,\text{CIMA}(Q_t^n = q_t^n + 1|f_0:t-1) P^\lambda,\text{CIMA}(Q_t^n > 0|f_0:t-1)}{\lambda^n P^\lambda,\text{CIMA}(Q_t^n = q_t^n + 1|f_0:t-1)} + (1 - \lambda^n) P^\lambda,\text{CIMA}(Q_t^n > 0|f_0:t-1).
\]  

(10)

For \( n = v(b_1^\text{CIMA}) \) and \( f_1 = 0 \)

\[
P^\lambda,\text{CIMA}(q_{t+1}^n|f_0:t) = \begin{cases} 0 & \text{if } q_t^n + 1 = 2, \\ \lambda^n & \text{if } q_t^n + 1 = 1, \\ 1 - \lambda^n & \text{if } q_t^n + 1 = 0. \end{cases}
\]  

(11)

Proof of Lemma 3: The lemma is proved by induction. Equation (8) is true at \( t = 0 \) because all queues are initially empty. Suppose (8) is true at \( t = k \). At time \( t = k + 1 \) we have

\[
P^\lambda,\text{CIMA}(q_{k+1}|f_0:k) = \frac{P^\lambda,\text{CIMA}(q_{k+1}, f_k|f_0:k-1)}{\sum_{q_{k+1}} P^\lambda,\text{CIMA}(q_{k+1}, f_k|f_0:k-1)}.
\]  

(12)

Let \( v = v(b_1^\text{CIMA}) \). Consider the numerator in (12). There are two cases: \( f_k = 1 \) and \( f_k = 0 \).

When \( f_k = 1 \), we have

\[
P^\lambda,\text{CIMA}(q_{k+1}, f_k = 1|f_0:k-1) = \sum_{q_k} P^\lambda,\text{CIMA}(q_{k+1}, q_k, F_k = 1|f_0:k-1) = \sum_{q_k} P^\lambda,\text{CIMA}(q_{k+1}, q_k, Q_k^v > 0|f_0:k-1)
\]  

(13)

\[
= \sum_{q_k} P^\lambda,\text{CIMA}(q_k|f_0:k-1)
\]  

\[
P^\lambda(A_k^v = q_{k+1}^v - q_k^v + 1) \prod_{n \neq v} P^\lambda(A_k^n = q_{k+1}^n - q_k^n)
\]  

(14)

\[
= \sum_{q_{k+1}^v} \left\{ \prod_{n \neq v} \sum_{q_k^n} P^\lambda(A_k^n = q_{k+1}^n - q_k^n) P^\lambda,\text{CIMA}(q_k^n|f_0:k-1) \right\}
\]  

(15)

\[
= \sum_{q_{k+1}^v} \left\{ \prod_{n \neq v} \sum_{q_k^n} P^\lambda(A_k^n = q_{k+1}^n - q_k^n) P^\lambda,\text{CIMA}(q_k^n|f_0:k-1) \right\}
\]  

(16)

Equation (13) holds because \( F_k = 1 \) if and only if \( A_k^n Q_k^v > 0 \). Equation (14) is true because of the system dynamics (1) and the fact that \( A_k^n Q_k^v > 0 \) are independent and they are indepdendent of all variables before \( k \). Equation (15) follows from the induction hypothesis for (8). Equation (16) is true because each term in (15) depends only on each \( q_k^n \) for \( n = 1, 2, \ldots, N \).

Using (16), the denominator in (12) becomes

\[
= \sum_{q_k} \left\{ \prod_{n \neq v} \sum_{q_k^n} P^\lambda(A_k^n = q_{k+1}^n - q_k^n) P^\lambda,\text{CIMA}(q_k^n|f_0:k-1) \right\}
\]  

\[
= \sum_{q_k} P^\lambda(A_k^v = q_{k+1}^v - q_k^v + 1) P^\lambda,\text{CIMA}(q_k^v|f_0:k-1)
\]  

(17)

Equation (17) is true because all possible realizations except for \( q_k^n \) are summed out.
Substituting (16) and (17) back into (12) we obtain for $f_k = 1$

$$
P^\lambda,\text{CIMA}(q_{k+1}|f_{0:k}) = \prod_{n \neq v} \sum_{q_{k}^n} P^\lambda(A^v_k = q_{k+1}^v - q_{k}^v) P^\lambda,\text{CIMA}(q_{k}^n|f_{0:k-1})$$

$$= \lambda^n P^\lambda,\text{CIMA}(Q^n_k = q_{k+1}^n - 1|f_{0:k-1}) + (1 - \lambda^n) P^\lambda,\text{CIMA}(Q^n_k = q_{k+1}^n|f_{0:k-1}),$$

for $n \neq v$, by an argument similar to (19), we get

$$P^\lambda,\text{CIMA}(q_{k+1}|f_{0:k}) = \lambda^n P^\lambda,\text{CIMA}(Q^n_k = q_{k+1}^n - 1|f_{0:k-1}) + (1 - \lambda^n) P^\lambda,\text{CIMA}(Q^n_k = q_{k+1}^n|f_{0:k-1}),$$

for $n \neq v$, by an argument similar to (20), we obtain

$$P^\lambda,\text{CIMA}(q_{k+1}|f_{0:k}) = \lambda^n 1\{q_{k+1}^n = 1\} + (1 - \lambda^n) 1\{q_{k+1}^n = 0\}.$$  

Therefore, the induction step is completed and (8) holds for all $t$. Furthermore, (9) is established by (19) and (23); (10) is established by (20) and (11) is established by (24).

Using Lemma 3, we now prove Lemma 1.

**Proof of Lemma 1:** Equation (5) follows directly from (1), the queue length dynamics, and (4), the definition of the CIMA protocol.

For the common upper bounds, let $v = v(B^t,\text{CIMA})$, which is a function of $F_{0:t-1}$.

For $n \neq v$, form (9) in Lemma 3 we get $B^t,\text{CIMA} = B^t,\text{CIMA} + 1$.

For $n = v$ and $F_t = 1$, from (10) in Lemma 3 we obtain $B^t,\text{CIMA} = B^t,\text{CIMA}$.

For $n = v$ and $F_t = 0$, (11) in Lemma 3 gives $B^t,\text{CIMA} = 1$, and the proof of the lemma is complete.

**APPENDIX B**

**Proof of Lemma 2:** From (5) and (6) in Lemma 1 we know that $Q^t,\text{CIMA}$ and $P^t,\text{CIMA}$ are functions of $Q^t,\text{CIMA}$, $B^t,\text{CIMA}$, $A^t$, and $F_t$. From (4), the definition of the CIMA protocol, we know that

$$F_t = U^t,\text{CIMA} = 1\{Q^t,\text{CIMA}, Q^t > 0\}.$$  

Therefore, $F_t$ is a function of $Q^t,\text{CIMA}$ and $B^t,\text{CIMA}$. Consequently, $Y^t,\text{CIMA}$ is a function of $Q^t,\text{CIMA}$, $B^t,\text{CIMA}$ and $A^t$. Let $f(Y^t,\text{CIMA}, A^t) := Y^t,\text{CIMA}$, we have

$$P(Y^t,\text{CIMA} = y_{t+1}, A^t = y_k, k \leq t) = P(f(Y^t,\text{CIMA}, A^t) = y_{t+1}|Y^t,\text{CIMA} = y_k, k \leq t)$$

$$= P(f(y_t, A^t) = y_{t+1}|Y^t,\text{CIMA} = y_k, k \leq t)$$

$$= P(f(y_t, A^t) = y_{t+1}|Y^t,\text{CIMA} = y_t)$$

$$= P(Y^t,\text{CIMA} = y_{t+1}|Y^t,\text{CIMA} = y_t).$$

where (*) is true because $A^t$ is independent of $Q^t,\text{CIMA}$, $B^t,\text{CIMA}$ and all random variables before time slot $t$.

Therefore, $\{Y^t,\text{CIMA}, t = 0, 1, \ldots\}$ is a Markov chain.


\textbf{APPENDIX C}

\textit{Detailed derivation of (7) in the proof of Theorem 1:} Consider the conditional expected difference \( \mathbb{E} [h(Y_{t+1}^{\text{CIMA}}) - h(Y_0^{\text{CIMA}}) | Y_t^{\text{CIMA}} = y] \) of the Lyapunov function \( h(\cdot) \).

\[
\mathbb{E} [h(Y_{t+1}^{\text{CIMA}}) - h(Y_t^{\text{CIMA}}) | Y_t^{\text{CIMA}} = y] = \mathbb{E} \left[ \sum_{n=1}^{N} (Q_{t+1}^{n,\text{CIMA}} - Q_t^{n,\text{CIMA}}) | Y_t^{\text{CIMA}} = y \right] \\
+ \alpha \mathbb{E} \left[ \sum_{n=1}^{N} (B_{t+1}^{n,\text{CIMA}} - B_t^{n,\text{CIMA}}) | Y_t^{\text{CIMA}} = y \right]. \quad (25)
\]

For \( n \neq v \), from (5) and (6) in Lemma 1, we get

\[
\mathbb{E} \left[ (Q_{t+1}^{v,\text{CIMA}} - Q_t^{v,\text{CIMA}}) | Y_t^{\text{CIMA}} = y \right] \\
+ \alpha \mathbb{E} \left[ (B_{t+1}^{v,\text{CIMA}} - B_t^{v,\text{CIMA}}) | Y_t^{\text{CIMA}} = y \right] \\
= \mathbb{E} [A_t^v | Y_t^{\text{CIMA}} = y] + \alpha \mathbb{E} [1 | Y_t^{\text{CIMA}} = y] \\
= \lambda^v + \alpha, \quad (26)
\]

where the last equality in (26) holds because \( A_t^v \) is independent of \( Y_t^{\text{CIMA}} \).

For \( n = v \), from (5) and (6) in Lemma 1, we obtain

\[
\mathbb{E} \left[ (Q_{t+1}^{v,\text{CIMA}} - Q_t^{v,\text{CIMA}}) | Y_t^{\text{CIMA}} = y \right] \\
+ \alpha \mathbb{E} \left[ (B_{t+1}^{v,\text{CIMA}} - B_t^{v,\text{CIMA}}) | Y_t^{\text{CIMA}} = y \right] \\
= \mathbb{E} [A_t^v - 1 + 1_{\{Q_v = 1\}} | Y_t^{\text{CIMA}} = y] \\
+ \alpha \mathbb{E} [(1 - B_t^v) 1_{\{Q_v = 0\}} | Y_t^{\text{CIMA}} = y] \\
= \mathbb{E} [A_t^v - 1 + 1_{\{Y_v = 0\}} | Y_t^{\text{CIMA}} = y] \\
+ \alpha \mathbb{E} [(1 - b^v) 1_{\{y_v = 0\}} | Y_t^{\text{CIMA}} = y] \\
= \lambda^v - 1 + (1 + \alpha (1 - b^v)) 1_{\{y_v = 0\}}, \quad (27)
\]

where the last equality in (27) follows by the fact that \( A_t^v \) is also independent of \( Y_t^{\text{CIMA}} \).

Putting (26) and (27) back into (25) we get

\[
\mathbb{E} [h(Y_{t+1}^{\text{CIMA}}) - h(Y_t^{\text{CIMA}}) | Y_t^{\text{CIMA}} = y] \\
= \sum_{n \neq v} \lambda^n + \alpha (N - 1) \\
+ \lambda^v - 1 + (1 + \alpha (1 - b^v)) 1_{\{y_v = 0\}} \\
(a) = - \epsilon + \alpha (N - 1) + (1 + \alpha (1 - b^v)) 1_{\{y_v = 0\}} \\
(b) = - \epsilon/2 + (1 + \alpha (1 - b^v)) 1_{\{y_v = 0\}} \\
\leq - \epsilon/2 \quad \text{if} \quad b^v \geq \frac{1}{\alpha + 1}, \quad (28)
\]

where (a) in (28) is true because \( \sum_{n=1}^{N} \lambda^n = 1 - \epsilon \), and (b) in (28) is true because \( \alpha = \frac{\epsilon}{2(N - 1)} \). Consequently, inequality (7) in the proof of Theorem 1 is established.