Robustness of Connected Vehicle Systems

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Since a connected cruise controller utilizes motion information from multiple human-driven vehicles ahead, its performance significantly depends on human driving behavior. Therefore, robust controller designs required to maintain a desired level of performance against uncertainties in human parameters. In this paper, we apply structured singular value analysis to select robust control gains that guarantee string stability against uncertainties arising from the feedback gains and reaction time delays of the human drivers ahead. We demonstrate the results on a connected vehicle system where a connected automated vehicle follows three human-driven vehicles.

1 INTRODUCTION

In order to improve active safety, passenger comfort, and traffic efficiency, automation of passenger vehicles has become a widely studied area over the past few decades. While adaptive cruise control (ACC) was invented to alleviate human drivers from the constant burden of speed control [1], it only allows an equipped vehicle to respond to its immediate predecessor [2]. To overcome this limitation, cooperative adaptive cruise control (CACC) was proposed, which allows several automated vehicles to drive cooperatively through vehicle-to-vehicle (V2V) communication [3, 4, 5, 6]. However, to fully exploit the benefits of V2V communication, the V2V signals from nearby human-driven cars should also be utilized before automated vehicles become widespread. Therefore, connected cruise control (CCC) was proposed that allows an automated vehicle to respond to multiple human-driven vehicles ahead via ad-hoc V2V communication [7, 8, 9, 10].

Since both CACC and CCC designs rely on the dynamics of multiple preceding vehicles, parameter uncertainties among these vehicles need to be considered in order to guarantee robust performance. For CACC designs, [11, 12, 13] considered the uncertainties among an automated platoon and synthesized robust controllers using the $\mathcal{H}_\infty$ framework. Some other methods were also used in [14, 15, 16, 17] to discuss the effects of unmodeled dynamics, stochastic communication delay, and measurement noise. However, to obtain robust CCC designs, a systematic method is needed to not only consider the uncertain feedback parameters but also the reaction time delay of nearby human drivers. In this paper, we use structured singular value analysis [18, 19] to evaluate the effects of uncertain human parameters on CCC design, and demonstrate the results by ensuring robust head-to-tail string stability on a four-vehicle configuration.

2 CONNECTED VEHICLE SYSTEMS

In this section we describe the longitudinal dynamics of a connected vehicle system. We consider a heterogeneous chain of vehicles where all vehicles are equipped with V2V devices and some are capable of automated driving, as shown in Fig. 1. When an automated vehicle receives motion information broadcasted from several vehicles ahead, it may choose to use the information in its motion control (see the dashed arrows), and thus, it becomes a connected automated vehicle. Such a V2V-based controller then defines a connected vehicle system consisting of the connected automated vehicle and the preceding vehicles whose motion signals are used by the connected automated vehicle.

Inside this connected vehicle system, we denote the connected automated vehicle as vehicle 0, and the preceding vehicles as vehicles 1, $\ldots$, $n$. The longitudinal dynamics of human-driven vehicle $i$ can be described by

$$\dot{h}_i(t) = v_{i+1}(t) - v_i(t),$$

$$\dot{v}_i(t) = \alpha_i (v_i(h_i(t - \tau_i)) - v_i(t - \tau_i)) + \beta_i (v_{i+1}(t - \tau_i) - v_i(t - \tau_i)),$$ (1)

for $i = 1, \ldots, n$, where $h_i$ and $v_i$ are the headway and speed of vehicle $i$, $\alpha_i$ and $\beta_i$ are the control gains and the time delay $\tau_i$ accounts for the reaction time delay of a human driver plus the actuator delay of the vehicle. Moreover, $V_i(h_i)$ is the range policy function that describes the desired velocity based on headway
Since we are interested in how speed perturbations for the connected automated vehicle in (1) cannot be tuned for the connected automated vehicle of these vehicles is required. Thus, the feedback gains and delay for the preceding vehicles and \( h_0 \) the connected automated vehicle will tend to zero regardless of the initial conditions. Instead, we focus on how the connected automated vehicle responds to speed perturbations propagating through the system. When the speed fluctuation \( \varepsilon_0 \) of the connected automated vehicle has smaller amplitude than the input \( \varepsilon_n \), we call the connected automated vehicle design head-to-tail string stable. That is, head-to-tail string stability allows speed perturbations to be amplified among the uncontrollable vehicles 1, \ldots, n, and we focus on how the connected automated vehicle attenuates the perturbations. Being head-to-tail string stable not only enables a connected automated vehicle to enjoy better active safety, fuel economy, and passenger comfort, it can also improve the efficiency of the traffic flow [8].

We assume zero initial conditions for (5-6) and obtain

\[
\tilde{V}_0(s) = \sum_{i=1}^{n} T_{i,0}(s)\tilde{V}_i(s),
\]

\[
\tilde{V}_i(s) = T_{i+1,i}(s)\tilde{V}_{i+1}(s),
\]

where \( \tilde{V}_0(s) \) and \( \tilde{V}_i(s) \) denote the Laplace transform of \( \tilde{v}_0(t) \) and \( \tilde{v}_i(t) \), and the link transfer functions are

\[
T_{i,0}(s) = \frac{(a_i \kappa_0 + b_i \kappa_0 + 1) e^{-\sigma_1 s}}{s^2 + a_i \kappa_0 + b_i \kappa_0 + 1 + n \beta_i e^{-\sigma_1 s}},
\]

\[
T_{i+1,i}(s) = \frac{(a_i \kappa_1 + b_i \kappa_1 + 1) e^{-\sigma_1 s}}{s^2 + (a_i \kappa_1 + b_i \kappa_1 + 1) e^{-\sigma_1 s}}.
\]

for \( i = 1, \ldots, n \). Thus, the head-to-tail transfer function of the connected vehicle system is

\[
G_{n,0}(s) = \frac{\tilde{V}_0(s)}{\tilde{V}_n(s)} = \det(T(s)),
\]

where the transfer function matrix is

\[
T(s) =
\begin{bmatrix}
T_{1,0}(s) & -1 & 0 & \cdots & 0 \\
T_{2,0}(s) & T_{2,1}(s) & -1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
T_{n-1,0}(s) & T_{n-1,1}(s) & \cdots & T_{n-1,2}(s) & \cdots & -1 \\
T_{n,0}(s) & T_{n,1}(s) & \cdots & T_{n,2}(s) & \cdots & T_{n,n-1}(s)
\end{bmatrix}
\]

see [8] for the proof.

The criterion for head-to-tail string stability at the linear level is guaranteed if the perturbations are attenuated for any frequency, that is, if

\[
|\det(T(\omega))| < 1, \quad \forall \omega > 0,
\]

where we substituted \( s = \omega \). In order to facilitate robustness analysis we rewrite (12) as

\[
1 - |\det(T(\omega))\delta| < 0, \quad \forall \omega > 0,
\]

where \( \delta \) is an arbitrary complex number inside the unit circle in the complex plane, that is, \( \delta \in \mathbb{C} \), \( |\delta| < 1 \).
Figure 2: Example configuration: (a) Connected vehicle structure. (b) Block diagram.

Figure 3: Example transfer functions \([T_{3,2}(i\omega)], [T_{2,1}(i\omega)], [G_{3,0}(i\omega)]\) and corresponding simulations with \(\omega^* = 0.6 \text{ [rad/s]}\).

To illustrate the head-to-tail string stability, here we consider a connected automated vehicle using motion information from three vehicles ahead \((n = 3)\), as shown in Fig. 2(a). The transfer function matrix for this connected vehicle system is

\[
T(s) = \begin{bmatrix}
T_{1,0}(s) & -1 & 0 \\
T_{2,0}(s) & T_{2,1}(s) & -1 \\
T_{3,0}(s) & T_{3,1}(s) & T_{3,2}(s)
\end{bmatrix},
\]

(14)

where the elements \(T_{1,0}(s), T_{2,0}(s), T_{3,0}(s), T_{2,1}(s)\) and \(T_{3,2}(s)\) are given by (9), while (10) gives the head-to-tail transfer function

\[
G_{3,0}(s) = T_{3,2}(s)\left(T_{2,1}(s)T_{1,0}(s) + T_{2,0}(s)\right) + T_{3,0}(s).
\]

(15)

The flow of information is illustrated by a block diagram in Fig. 2(b). We consider the case when the preceding vehicles \(i = 1, 2\) have parameters \(\alpha_i = 0.2 \text{ [1/s]}, \beta_i = 0.4 \text{ [1/s]}, \kappa_i = 0.9 \text{ [1/s]}, \tau_i = 0.9 \text{ [s]}, \) and the design parameters are \(a_{1,0} = 0.4 \text{ [1/s]}, b_{1,0} = 0.2 \text{ [1/s]}, b_{2,0} = 0.4 \text{ [1/s]}, b_{3,0} = 0.4 \text{ [1/s]}, \kappa_0 = 0.9 \text{ [1/s]}, \) and \(\sigma_{1,0} = \sigma_{2,0} = \sigma_{3,0} = \sigma = 0.6 \text{ [s]}\) for the connected automated vehicle.

In Fig. 3(a) we plot the head-to-tail transfer function \([G_{3,0}(i\omega)]\) of the connected automated vehicle (solid gray curve) and the link transfer function \([T_{3,2}(i\omega)]\) that describes how vehicle 2 responds to the motion of vehicle 3 (dotted purple curve). Here this is equal to \([T_{2,1}(i\omega)]\) as vehicles 2 and 1 have the same parameters. While the magnitude of the head-to-tail transfer function stays below 1, the link transfer functions of vehicles 2 and 1 reach beyond 1 for low frequencies. This indicates that speed perturbations at low frequency are amplified by vehicles 2 and 1 but eventually are suppressed by the connected automated vehicle. This observation is supported by a simulation shown in Fig. 3(b), where the speed input \(v_3(t) = 15 + 5\sin(\omega^* t)\) is applied with \(\omega^* = 0.6 \text{ [rad/s]}\). The color code corresponds to the vehicle colors in Fig. 2(a).

Note that the results shown in Fig. 3 strongly depend on the parameters of the preceding vehicles. The same control parameters used in Fig. 3 may behave poorly with a different set of parameters \(\kappa_i, \alpha_i, \beta_i, \) and \(\tau_i\). In the forthcoming sections, we will assume additive perturbation in these parameters denoted by \(\tilde{\kappa}_i, \tilde{\alpha}_i, \tilde{\beta}_i, \) and \(\tilde{\tau}_i\), and apply robust control design to ensure head-to-tail string stability under these parameter changes.

### 3 ROBUST STRING STABILITY

Since a connected automated vehicle may not know the dynamics of the preceding vehicles \(1, \ldots, n\) accurately, the V2V-based controller should be robust against their parameter uncertainties beside from the model uncertainties of the connected automated vehicle itself. Based on the theory of robust control, we represent the system uncertainty in the \(m - \delta\) uncertain interconnection structure, as shown in Fig. 4 [18, 19]. We present the formulation of interconnected model for the two-vehicle configuration briefly, then extend the results for connected vehicle systems. Examples are given on a four-vehicle model, which can be easily generalized further.

#### 3.1 Two-vehicle configuration

To introduce the robust string stability, we start with the simplest case where vehicle \(i\) only uses information from the vehicle immediately ahead, see Fig. 5(a). In this case the input of the nominal system is \(\dot{e}_i(t)\) while the output is \(\dot{V}_i(t)\), and the link transfer function is given by

\[
T_{i+1,i}(s) = \frac{(\alpha \kappa + \beta \delta) s e^{\tau s}}{s^2 + (\alpha \kappa + (\alpha + \beta) \delta) s e^{\tau s}},
\]

(16)

where we dropped the subscripts of the parameters \(\kappa, \alpha, \beta, \) and \(\tau\); see (9).

While the additive uncertainties \(\tilde{\alpha}, \tilde{\beta}, \) and \(\tilde{\kappa}\) result in additive uncertainty terms, an additive delay uncertainty \(\tilde{\tau}\) results in a multiplicative exponential term \(e^{-\tilde{\tau} s}\) in (16), that is,

\[
T_{i+1,i}(s) + \tilde{T}_{i+1,i}(s) = \frac{((\kappa + \tilde{\kappa})(\alpha + \tilde{\alpha}) + (\beta + \tilde{\beta}) s e^{-(\tau + \tilde{\tau}) s}}{s^2 + ((\kappa + \tilde{\kappa})(\alpha + \tilde{\alpha}) + (\alpha + \tilde{\alpha} + \beta + \tilde{\beta}) s e^{-(\tau + \tilde{\tau})}},
\]

(17)

where \(\tilde{T}_{i+1,i}(s)\) represents the uncertainty. In order to formulate the uncertainties in a way that can be represented by the \(m - \delta\) interconnection structure, we use the approximation

\[
e^{-\tilde{\tau} s} \approx e^{-\tau s} \left(1 + \tilde{\vartheta}(s)\right),
\]

(18)

where

\[
\tilde{\vartheta}(s) = \frac{\tilde{\tau} s}{1 + \tilde{\tau} s / 3.456}, \quad \epsilon \in \mathbb{C}, |\epsilon| < 1
\]

(19)

estimates the uncertainty, see [20]. By taking into account the uncertain parameters, the block diagram in Fig. 5(b) can be drawn. This illustrates how uncertainties affect the system and allows one to construct the \(m - \delta\) structure by solving a number of algebraic equations.

Based on Fig. 5(b), one can derive the linear system of equations in the form

\[
\begin{bmatrix}
y \\
z
\end{bmatrix} = \begin{bmatrix} m(s) & u \end{bmatrix} w,
\]

(20)

\[
u = \delta(s)y,
\]

(21)
where \( \mathbf{m}(s) \) is partitioned as
\[
\mathbf{m}(s) = \begin{bmatrix}
\mathbf{m}_{1,1}(s) & \mathbf{m}_{1,2}(s) \\
\mathbf{m}_{2,1}(s) & \mathbf{m}_{2,2}(s)
\end{bmatrix}.
\]
(22)

More precisely, the generalized transfer function matrix \( \mathbf{m}(s) \) can be given by (23-24), and the matrix of uncertainties are collected in
\[
\delta(s) = \text{diag} \left[ \tilde{\kappa}, \tilde{\alpha}, \tilde{\beta}, \tilde{\tau} \right].
\]
(25)

The transfer function between the input \( w = \tilde{V}_{i+1} \) and output \( z = \tilde{V}_i \) with uncertainties can be written in terms of upper linear fractional transformations (LFT), such that
\[
\mathcal{F}_u(\mathbf{m}(s), \delta(s)) = \mathcal{T}_{i+1,i}(s) + \mathcal{T}_{i+1,i}(s)
\]
\[
\frac{m_{2,2}(s)}{m_{2,1}(s)} \mathbf{m}_{2,1}(s) \delta(s) \left( \mathbf{I} - \mathbf{m}_{1,1}(s) \delta(s) \right)^{-1} \mathbf{m}_{1,2}(s)
\]
\[
= \frac{((\kappa + \tilde{\kappa})(\alpha + \tilde{\alpha}) + (\beta + \tilde{\beta})s) e^{-st}(1 + \tilde{\vartheta}(s))}{s^2 + ((\kappa + \tilde{\kappa})(\alpha + \tilde{\alpha}) + (\alpha + \beta + \tilde{\beta})s) e^{-st}(1 + \tilde{\vartheta}(s))}
\]
(26)

under the condition \( \text{det}((\mathbf{I} - \mathbf{m}_{1,1}(s) \delta(s)) \neq 0 \).

Note that the elements of (25) are not normalized and some of them might be complex, while others are real parametric uncertainties. Therefore we introduce the weight matrix \( \mathbf{r}(s) \) in the form
\[
\mathbf{r}(s) = \text{diag} \left[ \rho_1, \rho_2, \rho_3, \rho_4(s) \right],
\]
(27)

where \( \tilde{\kappa} = \rho_1 \delta_1^1, \tilde{\alpha} = \rho_2 \delta_2^1, \tilde{\beta} = \rho_3 \delta_3^1, \tilde{\vartheta}(s) = \rho_4(s) \delta_4^1 \), for \( |\delta_1^1| < 1, \delta_2^1 \in \mathbb{R}, k = 1, 2, 3, \text{and} |\delta_3^1| < 1, \delta_4^1 \in \mathbb{C} \). Thus, the normalized perturbation matrix \( \delta \) reads
\[
\tilde{\delta} = \text{diag} \left[ \delta_1^1, \delta_2^1, \delta_3^1, \delta_4^1 \right],
\]
(28)

and
\[
\bar{\mathbf{m}}(s) = \begin{bmatrix}
\mathbf{m}_{1,1}(s) \mathbf{r}(s) & \mathbf{m}_{1,2}(s) \\
\mathbf{m}_{2,1}(s) \mathbf{r}(s) & \mathbf{m}_{2,2}(s)
\end{bmatrix},
\]
(29)

such that \( \mathcal{F}_u(\bar{\mathbf{m}}(s), \delta(s)) = \mathcal{F}_u(\bar{\mathbf{m}}(s), \tilde{\delta}(s)) \).

In this paper we are interested in the robust stability analysis of connected vehicle systems, therefore we do not go further with the robust analysis of the two-vehicle configuration. Instead, we apply these results when building larger systems. In particular, the \( \mathbf{m} - \delta \) model is used to construct the \( \mathbf{M} - \Delta \) model for larger systems and investigate the robust string stability of the head-to-tail transfer function afterwards.

### 3.2 Four-vehicle system

To demonstrate that the framework presented above can be extended for large connected vehicles networks, we consider the four-vehicle scenario in Fig 6(a), where a connected automated vehicle utilizes V2V signals from three human-driven vehicles ahead, cf. Fig. 2(a). The corresponding block diagram with uncertainties is presented in Fig. 6(b), which is the extension of Fig. 2(b). While the nominal transfer function matrix is given in (14), we assume each parameter for vehicles 2 and 1 have certain certainties is presented in Fig. 6(b), which is the extension of Fig. 2(b).

Similarly to the two-vehicle configuration, we formulate the \( \mathbf{M} - \Delta \) uncertain interconnection structure for the connected vehicle system as
\[
\begin{bmatrix}
\mathbf{y}_1 \\
\mathbf{y}_2 \\
\mathbf{z}\end{bmatrix} = \mathbf{M}(s) \begin{bmatrix}
\mathbf{u}_1 \\
\mathbf{u}_2 \\
\mathbf{w}_j\end{bmatrix}, \quad \begin{bmatrix}
\mathbf{u}_1 \\
\mathbf{u}_2\end{bmatrix} = \Delta(s) \begin{bmatrix}
\mathbf{y}_1 \\
\mathbf{y}_2\end{bmatrix},
\]
(30)

where \( \mathbf{M}(s) \) and \( \Delta(s) \) is built up from \( \mathbf{m}(s) \) and \( \delta(s) \). Figure 6(b) is a graphical representation of the uncertain interconnection structure and it results in
\[
\mathbf{M}(s) = \begin{bmatrix}
\mathbf{m}_{1,1} & \mathbf{m}_{1,2} & \mathbf{m}_{1,3} & \mathbf{m}_{1,4} \\
0 & \mathbf{m}_{2,1} & \mathbf{m}_{2,2} & \mathbf{m}_{2,3} \\
\mathbf{m}_{3,1} T_{1,0} & \mathbf{m}_{3,2} T_{2,0} + \mathbf{m}_{3,3} T_{1,0} T_{2,1} & \mathbf{m}_{3,4} T_{3,2} & \mathbf{m}_{3,5} T_{3,3}
\end{bmatrix},
\]
(31)

where \( \mathbf{m}(1) \) and \( \mathbf{m}(2) \) represent the uncertain interconnection matrices of vehicles 1 and 2, see (22) and Fig. 6(b). The dependence on \( s \) is not spelled out for conciseness in (31). The uncertainty matrix is written as
\[
\Delta(s) = \begin{bmatrix}
\delta^{(1)}(s) & 0 \\
0 & \delta^{(2)}(s)
\end{bmatrix},
\]
(32)

where \( \delta^{(1)}(s) \) and \( \delta^{(2)}(s) \) are uncertainties of vehicles 1 and 2, respectively, as introduced in (25). Similarly, the weight matrix reads
\[
\mathbf{R}(s) = \begin{bmatrix}
\mathbf{r}(1)(s) & 0 \\
0 & \mathbf{r}(2)(s)
\end{bmatrix},
\]
(33)

where \( \mathbf{r}(1)(s) \) and \( \mathbf{r}(2)(s) \) represent the weights of uncertainties of vehicles 1 and 2 according to (27).

The perturbed transfer function between \( w = \tilde{V}_0(s) \) and \( z = \tilde{V}_0(s) \) can be expressed by upper linear fractional transfor-
where $\sigma(\Delta)$ denotes the largest singular value of $\Delta$. As $\mu(\omega)$ increases, a smaller perturbation value in $\Delta$ may lead to a singular $(I - M(\omega)\Delta)$ and results in string instability. When singularity holds for arbitrarily small perturbations, then $\mu(\omega) \to \infty$ and robustness cannot be guaranteed. On the other hand, if $\det(I - M(\omega)\Delta) \neq 0$ for any perturbation $\Delta$, then $\mu(\omega) = 0$ and the system is robust. Therefore, the condition for robust string stability against bounded parameter variation is

$$\mu(\omega) < 1, \quad \forall \omega > 0,$$

otherwise there exists a perturbation matrix $\Delta$, $\sigma(\Delta) < 1$ such that $\det(I - M(\omega)\Delta) = 0$. If $\mu(\omega)$ goes above 1, the system can lose string stability around that frequency for certain parameters within that perturbation level. If $\mu(\omega)$ remains below 1, then robustness is guaranteed for that perturbation level. The definition of $\mu$ according to (41) does not yield directly any tractable way to compute it, since the optimization problem is not convex in general, therefore multiple local extrema might exist [22]. Instead, we are interested in computing upper and/or lower bounds, for which several alternative formulations have been developed, see [18, 22, 23, 24]. In this paper, we use the 	exttt{mussv} function in MATLAB $\mu$-Analysis and Synthesis Toolbox [25], which implements these algorithms, and we only focus on the results, not on the numerical issues.

$$\mu(\omega) = \left( \min_{\Delta} \{ \sigma(\Delta) : \det(I - M(\omega)\Delta) = 0 \} \right)^{-1}, \quad (41)$$

Figure 6: Four-vehicle configuration with uncertainties: (a) Connectivity topology. (b) Block diagram.

$$\mathcal{F}_u(M(s), \Delta(s)) = M_{2,2}(s) + M_{2,1}(s)\Delta(s)(I - M_{1,1}(s)\Delta(s))^{-1}M_{1,2}(s), \quad (34)$$

under the condition

$$\det(I - M_{1,1}(s)\Delta(s)) \neq 0,$$ \hspace{1cm} (35)

which is related to the plant stability boundaries under parameter uncertainty.

Recall the string stability criterion (13), similarly, the perturbed transfer function (34), needs to satisfy

$$1 - \mathcal{F}_u(M(\omega), \Delta(\omega))\delta_0 \neq 0, \quad \forall \omega > 0$$ \hspace{1cm} (36)

and for any complex number within the unit circle, that is, $\delta_0 \in \mathbb{C}, |\delta_0| < 1$. Using the Schur formula (see [21]), we rewrite (35) and (36) for $s = i\omega$ as

$$\det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} M_{1,1}(i\omega) & M_{1,2}(i\omega) \\ M_{2,1}(i\omega) & M_{2,2}(i\omega) \end{bmatrix} \Delta(i\omega) \begin{bmatrix} 0 \\ \delta_0 \end{bmatrix}\right) \neq 0.$$ \hspace{1cm} (37)

This can be rewritten in the compact form

$$\det(I - \tilde{M}(i\omega)\tilde{\Delta}) \neq 0,$$ \hspace{1cm} (38)

where

$$\tilde{M}(i\omega) = \begin{bmatrix} M_{1,1}(i\omega)R(i\omega) & M_{1,2}(i\omega) \\ M_{2,1}(i\omega)R(i\omega) & M_{2,2}(i\omega) \end{bmatrix}, \quad (39)$$

and $\delta_0$ is required to fulfill robust string stability.

In order to quantify the robustness of the system, we use the structured singular value analysis introduced by [18]. We define the $\mu$-value of $\tilde{M}(i\omega)$ as the inverse of the smallest $\sigma(\tilde{\Delta})$ when (38) fails at frequency $\omega$, i.e.,

$$\mu(\omega) = \left( \min_{\tilde{\Delta}} \{ \sigma(\tilde{\Delta}) : \det(I - \tilde{M}(i\omega)\tilde{\Delta}) = 0 \} \right)^{-1}, \quad (41)$$

and

$$\tilde{\Delta} = \text{diag}[\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7, \delta_8],$$ \hspace{1cm} (40)

with $\delta_i$ required to fulfill robust string stability.

Figure 7: Four-vehicle configuration: (a) Robust stability charts in the $(b_{2,0}, b_{1,0})$ plane for different values of $a_{1,0}$ and $b_{1,0}$. The black curves indicate the nominal string stability boundaries, red curves indicate the exact robust boundaries for 20% parameter uncertainty obtained from parameter sweeping, and blue curves indicate the robust boundaries obtained from $\mu$ analysis. (b) $\mu(\omega)$-curves (blue) and the nominal head-to-tail transfer function at parameter point A $(a_{1,0}, b_{1,0}, b_{2,0}) = (0.3, 0.2, 0.4, 0.4) [1/s]$. 

vehicle 1

vehicle 2
The results are presented in Fig. 7, where we assumed that each parameter of each uncertain vehicle is perturbed by the same percentage of their nominal value, i.e. \( \alpha_i, \beta_i, \kappa_i \) and \( \tau_i \) have identical relative uncertainties. The nominal human driver parameters are \( \kappa_i = 0.8 \) [1/s], \( \alpha_i = 0.25 \) [1/s], \( \beta_i = 0.5 \) [1/s] and \( \tau_i = 0.8 \) [s] (same for both vehicles for simplicity, \( i = 1, 2 \)), while the fixed parameters of the connected automated vehicle are \( \kappa_i = 0.6 \) [1/s] and \( \tau_i = 0.25 \) [s], respectively. Since \( \kappa_i \), \( \beta_i \), and \( \tau_i \) are the same percentage of their nominal value, i.e. \( \kappa_i / \kappa_0 = 0.8 \), \( \beta_i / \beta_0 = 0.5 \), and \( \tau_i / \tau_0 = 0.8 \), respectively. Since \( \kappa_i \), \( \beta_i \), and \( \tau_i \) are identical relative uncertainties. The nominal human driver parameters are \( \kappa_i = 0.8 \) [1/s], \( \alpha_i = 0.25 \) [1/s], \( \beta_i = 0.5 \) [1/s] and \( \tau_i = 0.8 \) [s] (same for both vehicles for simplicity, \( i = 1, 2 \)), while the fixed parameters of the connected automated vehicle are \( \kappa_i = 0.6 \) [1/s] and \( \tau_i = 0.25 \) [s], respectively. Since \( \kappa_i \), \( \beta_i \), and \( \tau_i \) are the same percentage of their nominal value, i.e. \( \kappa_i / \kappa_0 = 0.8 \), \( \beta_i / \beta_0 = 0.5 \), and \( \tau_i / \tau_0 = 0.8 \), respectively. Since \( \kappa_i \), \( \beta_i \), and \( \tau_i \) are identical relative uncertainties.

The \( \mu(\omega) \)-curves are shown in Fig. 7(a) and 7(b) corresponding to parameter point A \( (a_{1,0}, b_{1,0}, b_{2,0}, b_{3,0}) = (0.3, 0.2, 0.4, 0.4) \) [1/s], with uncertainty level 20%. The black curve indicates the absolute value of the nominal head-to-tail transfer function \( |G_{1,0}(j\omega)| \), while solid and dashed blue curves correspond to the upper and lower bounds of the exact \( \mu(\omega) \), respectively. Since point A in Fig. 7(a) is almost at the boundary (blue curve), the \( \mu(\omega) \)-curve almost reaches 1 at about \( \omega = 1.3 \) [rad/s].

4 CONCLUSION

We applied the structured singular value analysis to investigate the influences of uncertain human driver parameters on the head-to-tail string stability of connected cruise controllers. The method was applied in a four-vehicle configuration where the robustness results were used to design connected automated vehicles that could reject traffic perturbations well despite uncertain human gains and reaction time delay.

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