

Sharable Motion Planning for Connected Automated Vehicles

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A sharable motion planning method for connected automated vehicles is proposed. The motion planning comprises of three consecutive parts: path planning, collision checking, and velocity planning. The path planning utilizes a three clothoid-based Hermite interpolation, and it generates a G2 smooth, feasible path that can be shared via V2X communication. The collision checking is based on the swept volume, whose continuous boundary can be computed efficiently. The velocity planning generates a smooth velocity profile using a two-step approach: a piecewise-constant acceleration profile is designed first and then smoothed by taking into account the jerk limit. Urban local planning scenarios are used to demonstrate the advantages of the proposed method compared to other planning methods.

Topics / Autonomous Driving Systems

1. INTRODUCTION

Motion planning is the core part of any autonomous driving systems since it determines the desired future motion of the vehicle while avoiding collisions with stationary objects and with other road participants. Sharing the planned path and velocity among vehicles via wireless vehicle-to-everything (V2X) communication can improve safety and mobility. To that end, efficient encoding of planned motion is essential. While existing motion planning methods for mobile robots are well suited for complex scenarios, it is hard to compactly encode the planned motion and to share it with other agents. In this paper, we target this challenging problem.

Considering autonomous driving applications for urban roads with structured environments, typically a set of waypoints can be defined along the route [1]. Thus, interpolating curve planning, that can generate smooth paths between the waypoints, is well suited for such scenarios. In this paper, we propose an encoding-friendly motion planning method that uses three-clothoid Hermite interpolation as the path primitive [2]. Thus, the planning problem is turned into a system of algebraic equations. Given the determined smooth path, a swept volume-based collision checking algorithm is used to compute the boundary of the continuous future trace of the vehicle's body. The velocity planning includes two steps: we generate a piecewise constant acceleration plan followed by constant jerk smoothing. The planned path and velocity profiles are smooth, continuous functions of the traveled distance and they can be represented with few parameters. Therefore, the plans can be shared via V2X communication in an efficient manner. Applying the proposed planning can potentially enhance safety of automated driving in urban environments by taking into account other agents' accurate intended motion.

2. MOTION PLANNING PROBLEM

The bicycle model of the form

$$\begin{aligned}\dot{x} &= v \cos \psi, \\ \dot{y} &= v \sin \psi, \\ \dot{\psi} &= \frac{v}{l} \tan \gamma, \\ \dot{v} &= a,\end{aligned}\tag{1}$$

is considered when formulating the motion planning problem. Here x, y denote the position of rear axle center in Earth-fixed frame, ψ denotes the orientation of the body (i.e., yaw angle), v denotes longitudinal velocity, a denotes longitudinal acceleration, γ denotes front wheel steering angle, and l is wheelbase. The vehicle model is augmented with the constraints

$$\begin{aligned}|\gamma| &\leq \gamma_{\max}, \\ |\dot{\gamma}| &\leq \Omega_{\max}, \\ a_{\min} &\leq a \leq a_{\max}, \\ |\dot{a}| &\leq j_{\max},\end{aligned}\tag{2}$$

where γ_{\max} is the maximum allowed front steering angle, Ω_{\max} is the maximum steering angle rate, a_{\min} and a_{\max} are the lower and upper bounds of the longitudinal acceleration, and j_{\max} denotes the longitudinal jerk limit.

The local motion planning problem requires to generate feasible desired trajectories of the rear axle center point. From the no-slip condition of the wheels used in the bicycle model (1), the instantaneous curvature κ can be expressed by the steering angle γ and the wheel base l as

$$\kappa = \frac{1}{l} \tan \gamma.\tag{3}$$

Thus, the constraint on the steering angle in (2) can be translated to constraint on the curvature of the path. The trajectory can be represented as a function of traveled distance s of the rear axle center. That is, the trajectory can be expressed by the vector $[x(s), y(s), \psi(s), v(s)]$, where $[x(s), y(s)]$ denotes position, $\psi(s)$ denotes orientation, and $v(s)$ denotes velocity.

The boundary conditions of the local motion planning problem are given by higher-level planner. These boundary conditions consist of initial and final configurations (global positions and orientations), velocities and the initial steering angle. Meeting the given initial steering angle is important in ensuring the continuity of curvature. The local motion planning problem, with change of basis, is equivalent to planning problem defined in the vehicle's local coordinate system with boundary conditions

$$\begin{aligned} x(0) &= 0, & x(T) &= \Delta x, \\ y(0) &= 0, & y(T) &= \Delta y, \\ \psi(0) &= 0, & \psi(T) &= \Delta \psi, \\ v(0) &= v_0, & v(T) &= v_f, \\ \gamma(0) &= \gamma_0, & \gamma(T) &= \gamma_f, \end{aligned} \quad (4)$$

where the final time T is not determined for now.

Through arclength parameterization, trajectory planning can be divided into separate path planning and velocity planning. Three-clothoid Hermite interpolation is used for path planning. Given the current and the desired configuration of the ego vehicle, the path planning problem is transformed into a system of nonlinear algebraic equations, which can be solved efficiently using numerical iterations [3]. The advantages of using clothoids for local path planning are the fast feasibility decision based on offline feasibility charts, and the fast continuous collision checking based on the swept volume of the local plan.

2. PATH PLANNING

The path is generated by solving a system of eight algebraic equations with ten unknowns as formulated in equation (9) in the Appendix. These can be further simplified to two equations which can be solved efficiently, for example, by using the Newton-Raphson method. The proposed path planning has the following four desired properties: (i) exactly meets the boundary conditions, (ii) G2 smoothness, (iii) fast feasibility validation, (iv) encodability using a few parameters.

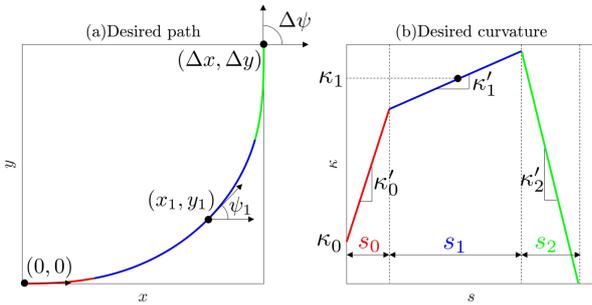


Fig. 1 Three-clothoid path for a 90-degree left turn with the key quantities indicated.

An example of a three-clothoid local path is shown in Fig. 1 where the physical meanings of the parameters are indicated. The path can be expressed by a continuous curvature function of the arclength s with eight parameters as

$$\kappa(s) = \begin{cases} \kappa_0 + \kappa'_0 s, & \text{if } 0 \leq s < s_0, \\ \kappa_1 + \kappa'_1 \left(s - s_0 - \frac{s_1}{2} \right), & \text{if } s_0 \leq s < s_0 + s_1, \\ -\kappa'_2 (s - s_0 - s_1), & \text{if } s_0 + s_1 \leq s, \end{cases} \quad (5)$$

Where the initial curvature κ_0 is given by the boundary condition, κ_1, κ_2 denote initial curvature of the first and the second clothoids, $\kappa'_0, \kappa'_1, \kappa'_2$ denote sharpness of the clothoids, and s_0, s_1, s_2 denote length of the clothoids. The existence of unconstrained solution of the local path is proven in [2], with systematic selection of free arclength parameters $s_0, s_2 > 0$. In this paper, for simplicity, we make the two free parameters equal, that is, we define $s_p = s_0 = s_2$. Thus, when encoding the path plan to the V2X messages, six parameters are used.

For efficient computation of feasibility of a given boundary condition, an offline feasibility chart is computed. In the space expressed by cartesian product of relative configuration $(\Delta x, \Delta y, \Delta \psi)$, initial steering angle γ_0 , and free arclength parameter s_p , we can express the boundary between feasible region and infeasible region. According to (3) the feasibility boundary is given by

$$|\kappa(s)| = \frac{1}{l} \tan \gamma_{\max}. \quad (6)$$

An example of the slice of feasibility chart is shown in Fig. 2 (a) with a feasible and a non-feasible points marked F and N. The feasible path (case F) is shown in Fig. 2 (b) with the corresponding curvature depicted in Fig. 2 (c). On the other hand, Fig. 2 (d) and (e) display a non-feasible path (case N) where the maximum curvature (indicated by horizontal dashed black lines) is exceeded.

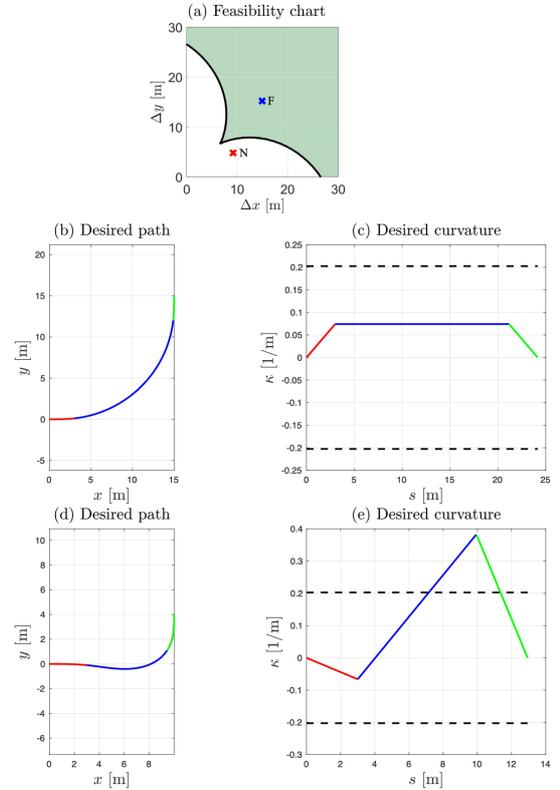


Fig. 2 (a) Feasibility chart for 90 deg turn. (b)-(c) feasible path and curvature for case F. (d)-(e) non-feasible path and curvature for case N.

3. SWEEP VOLUME-BASED COLLISION CHECKING

The swept volume expresses the union of the future traces of all points of the vehicle's body, which forms a closed set in the workspace. We can compute the boundary of the swept volume by computing the future traces of the left and right corners of the rear axle and the left and right corners of the front bumper as illustrated in Fig. 3. If the swept volume of the autonomous vehicle has no intersection with the regions occupied by static obstacles, the path can be identified as collision free. This can be extended to moving obstacles by checking intersections between predicted swept volumes of the objects and the swept volume of the autonomous vehicle.

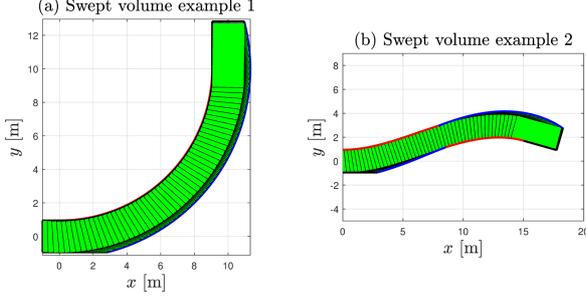


Fig. 3 Swept volume examples for three-clothoid paths. Panel (a) shows the swept volume boundary along the path for 90 deg turn similar to one shown in Fig. 2 (b).

4. VELOCITY PLANNING

Given the curvature $\kappa(s)$ derived in the path planning layer, the velocity planner uses a two-step approach to compute a velocity plan $v(s)$ along the path. When computing the velocity profile, the longitudinal acceleration constraint in (2) as well as constraints for comfort are considered. Comfort in lateral direction is achieved by limiting the lateral acceleration while longitudinal jerk is limited for longitudinal comfort.

First, constant acceleration values are computed for each of the three segments taking into account constraints representing longitudinal and lateral acceleration limits. Given initial curvature κ_j and sharpness κ_j' of the j^{th} segment ($j = 0,1,2$), the longitudinal acceleration of the segment is computed as

$$a_j = \min_{0 \leq s \leq s_j} \frac{1}{2s} \left(\frac{a_{\text{lat,max}}}{|\kappa_j + \kappa_j' s|} - v_j^2 \right). \quad (7)$$

Next, the acceleration profile is smoothed using constant jerk corrections with parameter j_c . The resulting velocity plan is a second order smooth function of the arc length s and can be expressed as in (8). Here S_1, S_2 denotes the smoothing intervals of the first and second segments, v_1, v_2 denotes the initial velocities of the second and third segment, which can be computed in the constant acceleration planning, while the initial velocity v_0 for the first segment is given by the boundary condition.

$$v(s) = \begin{cases} \sqrt{v_0^2 + 2a_0 s}, & \text{if } 0 \leq s < s_0 - S_1, \\ \sqrt{v_0^2 + 2a_0(s - s_0) - j_c(s^2 - s_0^2)}, & \text{if } s_0 \leq s < s_0 + S_1, \\ \sqrt{v_1^2 + 2a_1 s}, & \text{if } s_0 \leq s < s_0 + S_1 - S_2, \\ \sqrt{v_1^2 + 2a_1(s - s_0 - S_1) - j_c(s^2 - (s_0 + S_1)^2)}, & \text{if } s_0 + S_1 - S_2 \leq s < s_0 + S_1, \\ \sqrt{v_2^2 + 2a_2 s}, & \text{if } s_0 + S_1 \leq s < s_0 + S_1 + S_2. \end{cases} \quad (8)$$

5. IMPLEMENTATION RESULTS

In Fig. 4 the proposed path planning method is compared to other benchmark planners including a quintic spline [4], Bezier curve [5], and Dubins' path for a 90-degree left turn and for a lane change maneuver. Our proposed method gives a constrained continuous curvature profile while keeping reasonably short path length.

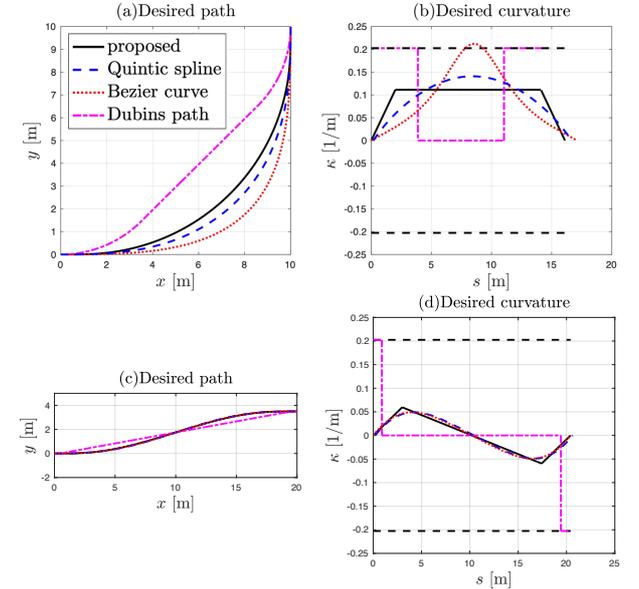


Fig. 4 (a)-(b) Path planning result for a 90-degree left turn compared to benchmark methods. (c)-(d) Path planning result for a lane change maneuver compared to benchmark methods.

In Fig. 5 the velocity plans are shown for the paths planned in Fig. 4. The constant acceleration plan (solid green curve) provides three different constant acceleration values for the three clothoid segments as given by (7). After constant jerk smoothing, the final velocity plan (solid black curve) is obtained via (8). The final velocity plan abides both acceleration and jerk constraint to ensure ride comfort. The plan also meets the initial conditions. When encoding the velocity plan to V2X messages, twelve parameters are used.

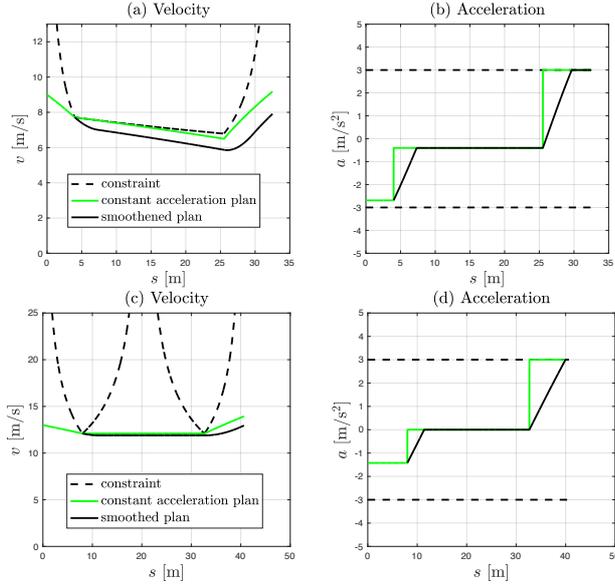


Fig. 5 (a)-(b) Velocity plan for a 90-degree left turn.
(c)-(d) Velocity plan for lane change.

6. CONCLUSION

A continuous local motion plan was proposed for automated vehicles that can be easily encoded and can be shared via V2X communication with limited bandwidth. A three clothoid-based G2 Hermite interpolation method is used for path planning that can generate a smooth local trajectory. The corresponding velocity plan is formed by constant acceleration and constant jerk-based method. The local plan is G2 continuous while it can be expressed with a few parameters. A novel collision checking method, which considers the swept volume of the vehicle, was also proposed. Compared to other interpolating spline-based local planning techniques, the proposed method is superior in terms of smoothness, exactness, and compactness, while giving comparable light computation load. Considering the versatility and easy tuning of the parameters of the proposed motion planner, it can be applied to wide variety of vehicle configurations (e.g., with different turning radius limitations) and complex road/intersection geometries. Furthermore, since the steering and acceleration limits (in rate and magnitude) have been incorporated in the path/velocity design, tracing the planned motion is achievable using simple path-following controllers.

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APPENDIX: Three-clothoid planning equation

With curvature continuity condition and boundary conditions, one can derive eight nonlinear algebraic equations with ten variables:

$$\begin{aligned}
 x_1 - \frac{s_1}{2} A\left(\frac{\kappa'_1 s_1^2}{4}, -\frac{\kappa_1 s_1}{2}, \psi_1\right) &= s_0 A(\kappa'_0 s_0^2, \kappa_0 s_0, 0), \\
 x_1 + \frac{s_1}{2} A\left(\frac{\kappa'_1 s_1^2}{4}, \frac{\kappa_1 s_1}{2}, \psi_1\right) &= \Delta x - s_2 A(\kappa'_2 s_2^2, 0, \Delta\psi), \\
 y_1 - \frac{s_1}{2} B\left(\frac{\kappa'_1 s_1^2}{4}, -\frac{\kappa_1 s_1}{2}, \psi_1\right) &= s_0 B(\kappa'_0 s_0^2, \kappa_0 s_0, 0), \\
 y_1 + \frac{s_1}{2} B\left(\frac{\kappa'_1 s_1^2}{4}, \frac{\kappa_1 s_1}{2}, \psi_1\right) &= \Delta y - s_2 B(\kappa'_2 s_2^2, 0, \Delta\psi), \\
 \frac{\kappa'_1 s_1^2}{8} - \frac{\kappa_1 s_1}{2} + \psi_1 &= \frac{\kappa'_0 s_0^2}{2} + \kappa_0 s_0, \\
 \frac{\kappa'_1 s_1^2}{8} + \frac{\kappa_1 s_1}{2} + \psi_1 &= \frac{\kappa'_2 s_2^2}{2} + \Delta\psi, \\
 -\frac{\kappa'_1 s_1}{2} + \kappa_1 &= \kappa_0 + \kappa'_0 s_0, \\
 \frac{\kappa'_1 s_1}{2} + \kappa_1 &= -\kappa'_2 s_2,
 \end{aligned} \tag{9}$$

where the Fresnel integrals are given by

$$\begin{aligned}
 A(a, b, c) &= \int_0^1 \cos\left(\frac{a}{2} t^2 + bt + c\right) dt, \\
 B(a, b, c) &= \int_0^1 \sin\left(\frac{a}{2} t^2 + bt + c\right) dt.
 \end{aligned} \tag{10}$$

This can be simplified to two nonlinear equations with four unknowns [3]. By selecting proper values of s_0, s_2 , the equations can be solved using the Newton-Raphson method.