NUMERICAL PREDICTION OF HIGHER ORDER WAVE INDUCED LOADS ON TETHERED PLATFORMS

O.G. Nwogu and M.B. Irani
Hydraulics Laboratory, National Research Council
Ottawa, Canada

ABSTRACT

The high frequency vibration of the tendons of tension leg platforms (TLPs) with slender columns and pontoons is investigated in this paper. The equations for the two-dimensional rigid body motion of a TLP in heave, surge and pitch are solved in the time domain using a Runge-Kutta time-stepping procedure. The wave loads are computed using the relative velocity formulation of the Morison equation. High frequency tendon loading occurs at wave periods that are integer multiples of the platform heave and pitch periods. This is a result of different types of nonlinearities in the fluid loading. Here specifically, the contribution of two nonlinear effects are considered: the effect of integrating the kinematics up to the free water surface and the effect of the nonlinear drag term of the Morison equation. The numerical results confirm the occurrence of higher harmonic pitch motions predominantly due to free surface effects.

1. INTRODUCTION

The tension leg platform is one of the leading concepts for oil production in deep waters. The vertical tendons or tendons which connect the platform to the seabed are kept in tension by the excess buoyancy of the platform. The large axial stiffness of the tendons results in natural frequencies for the heave, pitch, and roll motions well above the exciting wave frequencies. However, recent studies have shown that the high frequency modes of motion can be excited, leading to high frequency vibrations of the tendons or tendon ringing. While the variance of the high frequency component of the tendon force is often much smaller than that of the wave frequency component, a proper understanding of this phenomenon is required for an accurate estimation of the probability distribution of the extreme tendon tensions, and tendon fatigue life predictions.

Most previous investigations of tendon ringing phenomenon have assumed it to be due to second order wave loading effects (e.g. De Boom et al. (1984), and Petrusas (1987)). This clearly explains the occurrence of tendon vibrations caused by waves with periods twice the natural heave and pitch periods. However, high frequency tendon loads were observed in model tests of a TLP in irregular waves with peak periods up to five times the platform natural periods. This must be due to effects higher than second order.

In the present paper, attention is focussed on platforms with column and pontoon diameters less than one-fifth of the shortest wavelength. The wave loads on the platform can thus be computed using the relative velocity formulation of the Morison equation. The nonlinear equations of motion are solved in the time domain. The contributions of two different nonlinear effects to the high frequency tendon force are investigated:

1. effect of integration of wave kinematics up to the free surface instead of the still water level;

2. effect of retaining the nonlinear drag damping term of the Morison equation.

The platform motions and tether forces are initially computed for regular waves with the kinematics obtained using the Fourier approximation of Reinecker and Fenton (1981). An efficient time domain procedure is then used to compute the motions and forces in irregular waves.

2. EQUATIONS OF MOTION

Consider the surge, heave, and pitch motions of a tethered platform due to long-crested waves propagating along the positive x direction (see Figure 1). The equations of motion for
the platform can be written as

$$\mathbf{M} \ddot{\mathbf{X}} + \mathbf{B} \dot{\mathbf{X}} + \mathbf{K} \mathbf{X} = \mathbf{F}(t)$$

(1)

where \( \mathbf{X} = (X_1, X_2, X_3) \) is the displacement vector with the subscripts 1, 2, 3 denoting surge, heave and pitch respectively, \( \mathbf{B} \) is the structural and wave radiation damping matrix, \( \mathbf{K} \) is the stiffness matrix due to hydrostatic and tendon restoring forces, and \( \mathbf{F} \) is the exciting force vector.

\[ 2.1 \] Hydrodynamic Force Vector

The wave loads on slender members of a structure can be computed using the relative velocity formulation of the Morison equation. The x and z components of the normal force acting on a segment of an arbitrarily inclined cylinder (see Figure 2) can be expressed as

$$\begin{align*}
\begin{bmatrix}
\Delta F_x \\
\Delta F_z
\end{bmatrix} &= \rho C_M A_c \Delta s \begin{bmatrix}
\dot{u}_n \\
u_n
\end{bmatrix} + \rho C_D D_c \Delta s \begin{bmatrix}
u_n' \\
u_n''
\end{bmatrix}
\end{align*}$$

(2)

where \( C_M \) and \( C_D \) are the inertia and drag coefficients respectively, \( D_c \) and \( A_c \) are the projected width and cross-sectional area of the cylinder, \( \rho \) is the density of water, and \( (u_n', u_n'') \) and \( (\dot{u}_n, n_n) \) are the x and z components of the relative velocity, \( v' \), and acceleration respectively, normal to the cylinder.

If the cylinder is located between coordinates \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) relative to the principal platform axes, then direction cosines can be defined as follows:

$$\cos \alpha = \frac{x_2 - x_1}{L}, \quad \cos \beta = \frac{y_2 - y_1}{L}, \quad \cos \gamma = \frac{z_2 - z_1}{L}$$

(3)

where \( L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \).

The components of the normal velocity and acceleration vectors are given by

$$\begin{align*}
\dot{u}_n &= (u - \dot{X}_1 - z \dot{X}_3) \sin^2 \alpha - (w - \dot{X}_2 + z \dot{X}_3) \cos \alpha \cos \gamma \\
u_n &= (w - \dot{X}_2 + z \dot{X}_3) \sin^2 \gamma - (u - \dot{X}_1 - z \dot{X}_3) \cos \alpha \cos \gamma \\
\dot{u}_n &= \dot{u} \sin^2 \alpha - \dot{w} \cos \alpha \cos \gamma \\
u_n &= \dot{w} \sin^2 \gamma - \dot{u} \cos \alpha \cos \gamma
\end{align*}$$

(5)

where \( u, w \) and \( \dot{u}, \dot{w} \) are the horizontal and vertical water particle velocities and accelerations respectively.

The total hydrodynamic force on the platform is composed of a Morison type force on the cylindrical members and a force due to the hydrodynamic pressure acting on the corner members which connect the columns to the pontoons. In order to compute the forces (or moment), the cylindrical members of the TLP are first divided into \( N_s \) segments. The kinematics are evaluated at the centroids of the segments, and then used to calculate the segment forces \( \Delta F_{zi} \) and \( \Delta F_{ai} \). The forces due to the hydrodynamic pressure, \( p_i \), acting on \( N_{cm} \) corner members are also evaluated. The total exciting force vector is thus given by

$$\begin{align*}
\begin{bmatrix}
F_1 \\
F_2 \\
F_3
\end{bmatrix} &= \begin{bmatrix}
N_s \sum_{i=1}^{N_t} \Delta F_{zi} + \sum_{i=1}^{N_{cm}} p_i A_{zi} n_z \\
N_s \sum_{i=1}^{N_t} \Delta F_{ai} + \sum_{i=1}^{N_{cm}} p_i A_{ai} \\
N_s \sum_{i=1}^{N_t} (\Delta F_{zai} - \Delta F_{aiz}) + C_p \sum_{i=1}^{N_{cm}} p_i (A_{zai} n_z - A_{azi} z_i)
\end{bmatrix}
\end{align*}$$

(6)
where \((x_i, z_i)\) are the coordinates of the centroid of the \(i\)th segment, \(A_x, A_z\) are the projected areas of the corner members in the \(x\) and \(z\) directions, and

\[
n_{z} = \begin{cases} 
+1 & \text{members facing } -x \text{ direction} \\
-1 & \text{members facing } +x \text{ direction}
\end{cases}
\]  

(7)

and \(C_p\) is an empirical pressure coefficient. The above computation of the wave forces ignores possible hydrodynamic interaction between the cylindrical members.

2.2 Mass Matrix

The mass matrix \(M\) consists of the structural mass matrix \(m\) and an added mass matrix \(a\). The added mass of the submerged portion of an arbitrarily inclined cylinder can be expressed in terms of the direction cosines (see Patel, 1989). The components of one-half of the symmetric added mass matrix of each cylindrical member are:

\[
\begin{align*}
a_{11} &= m_x \sin^2 \alpha \\
a_{12} &= -m_x \cos \alpha \cos \gamma \\
a_{13} &= a_{11} \bar{x} - a_{12} \bar{z} \\
a_{22} &= m_x \sin^2 \gamma \\
a_{23} &= a_{12} \bar{x} - a_{22} \bar{z} \\
a_{33} &= a_{22} \bar{z}^2 - 2a_{12}(x \bar{z})_m + a_{11} \bar{z}^2
\end{align*}
\]  

(8)

where

\[
\begin{align*}
m_x &= \rho(C_M - 1)A_xL \\
\bar{x} &= (x_1 + x_2)/2 \\
\bar{z} &= (z_1 + z_2)/2 \\
x_m^2 &= (x_1^2 + x_2^2 + x_3^2)/3 \\
z_m^2 &= (z_1^2 + z_2^2 + z_3^2)/3 \\
(x \bar{z})_m &= (2x_1x_2 + 2x_2x_3 + x_1x_3 + 2x_2x_1)/6
\end{align*}
\]

In computing the added mass matrix components, it should be noted that \(z_2 \leq 0\) for the submerged portion of the cylinder. The added mass of the corner members can be approximated as the mass of the displaced volume of water.

The non-zero components of the symmetric structural mass matrix for the total platform are:

\[
\begin{align*}
m_{11} &= m \\
m_{13} &= m x_p \\
m_{22} &= m \\
m_{33} &= m(r_p^2 + z_p^2)
\end{align*}
\]  

(9)

where \(m\) is the mass of the platform in air, \(z_p\) is the \(z\) coordinate of the center of gravity, and \(r_p\) is the radius of gyration for pitch.

2.3 Damping Matrix

The damping matrix includes contributions from structural damping as well as linear wave radiation damping. The matrix can be assumed to be diagonal with the components expressed as percentages of critical damping. The hydrodynamic damping, due to the drag effect, is proportional to the relative velocity between the structure and the fluid, is considered separately in the exciting force vector.

2.4 Stiffness Matrix

The stiffness matrix is calculated from the hydrostatic and tendon forces required to restore the platform to its equilibrium position for an arbitrary displacement. The hydrostatic stiffness contribution for the heave and pitch modes is determined for \(N_{cw}\) surface piercing cylindrical members. The \(i\)th member has a waterplane area \(A_{wi}\) and moment of inertia \(I_{zi}\) about its \(z\) axis. The stiffness contribution from the tendons can be derived (see for example Patel, 1989) by assuming the tendons to be taut cables with a known initial tension, \(T_0\), initial length, \(L_0\), and axial stiffness, \(k_i\). The tendons are assumed to be weightless, and fluid and dynamic effects on the tendons are neglected. Assuming symmetry in the horizontal plane, the non-zero components of the linearized stiffness matrix can be written as:

\[
\begin{align*}
K_{11} &= N_T T_0/L_0 \\
K_{13} &= \sum_{i=1}^{N_T} z_i T_0/L_0 \\
K_{22} &= N_T k_i' + \sum_{i=1}^{N_{cw}} pg A_{wi} \\
K_{33} &= \sum_{i=1}^{N_T} \left[ T_0 \left( z_i^2 + k_i x_i^2 \right) + \sum_{i=1}^{N_{cw}} pg A_{wi} (x_i^2 + I_{zi} + \rho V g (z_i - z_p)) \right]
\end{align*}
\]  

(10)

where \(N_T\) is the number of tendons, \(x_i\) and \(z_i\) are the \(x\) and \(z\) coordinates of the attachment point of the tendons respectively, and \(z_p\) is the \(z\) coordinate of the centroid of the waterplane area. While the complete time varying nonlinear stiffness matrix can easily be incorporated into a time domain model, linearized stiffnesses are used in order to investigate the effect of other nonlinearities.

2.5 Simulation of Wave Kinematics

The water particle velocities, accelerations, and hydrodynamic pressure are computed at the instantaneous, displaced position of the platform. For regular waves, the Fourier approximation method of Reinecker and Fenton (1981) is used to calculate the wave kinematics and pressure. The kinematics are calculated for all the submerged segments up to the instantaneous water surface elevation.

In irregular waves, linear theory is used to compute the wave kinematics. The water surface elevation time series \(\eta(x = 0, t)\)
is specified. The water surface elevation may be represented as a summation of $N$ regular waves with different amplitudes and frequencies, that is,

$$
\eta(0, t) = \sum_{n=1}^{N} (a_n \cos \omega_n t + b_n \sin \omega_n t) \tag{11}
$$

where the Fourier coefficients, $a_n$ and $b_n$ are obtained from a direct Fast Fourier Transform (FFT) of the input time series.

At any instant of time the kinematics and pressure are given by linear wave theory as

$$
u(x, z, t) = \sum_{n=1}^{N} \frac{\omega_n \cosh[k_n(z + d)]}{\sinh[k_n d]} \left[ a_n \cos(k_n x - \omega_n t) - b_n \sin(k_n x - \omega_n t) \right] \tag{12}
$$

$$
w(x, z, t) = \sum_{n=1}^{N} \frac{\omega_n \sinh[k_n(z + d)]}{\sinh[k_n d]} \left[ a_n \sin(k_n x - \omega_n t) + b_n \cos(k_n x - \omega_n t) \right] \tag{13}
$$

$$\ddot{u}(x, z, t) = \sum_{n=1}^{N} \frac{\omega_n^2 \cosh[k_n(z + d)]}{\sinh[k_n d]} \left[ a_n \sin(k_n x - \omega_n t) + b_n \cos(k_n x - \omega_n t) \right] \tag{14}
$$

$$\ddot{w}(x, z, t) = -\sum_{n=1}^{N} \frac{\omega_n^2 \sinh[k_n(z + d)]}{\sinh[k_n d]} \left[ a_n \cos(k_n x - \omega_n t) - b_n \sin(k_n x - \omega_n t) \right] \tag{15}
$$

$$p(x, z, t) = \sum_{n=1}^{N} \rho_g \frac{\omega_n^2 \cosh[k_n(z + d)]}{\cosh[k_n d]} \left[ a_n \cos(k_n x - \omega_n t) - b_n \sin(k_n x - \omega_n t) \right] \tag{16}
$$

The kinematics above the still water level are extrapolated from the still water values using the partial derivative of the kinematic variable at the still water level. For any kinematic variable $v$, this can be expressed as

$$v(x, z, t) = v(x, 0, t) + z \frac{\partial v}{\partial z}(x, 0, t) \quad \text{for } z > 0 \tag{17}
$$

In order to reduce the amount of computational effort involved in computing the kinematics at the displaced position of the platform at each instant of time, the direct FFT of $\eta(0, t)$ is performed only once and the summations in equations (12) to (16) are done directly at each time step.

The equations of motion are solved efficiently using a fourth order Runge-Kutta numerical integration scheme. The three coupled second order differential equations are first rewritten as six first order equations and then integrated using the algorithm given by White (1974).

3. RESULTS AND DISCUSSION

The numerical procedure described in the preceding section was validated with results obtained from model tests of a TLP comprising of relatively slender columns and pontoons. The complete model test results cannot be presented here due to the proprietary nature of the study. However, a few non-dimensional plots are presented to demonstrate the limitations of the present computation method.

The time series of the surge, heave, and pitch motions in an irregular sea state are compared with the computed time histories in Figure 3. The corresponding spectral densities of the measured and computed motions are shown in Figure 4. The surge and pitch motions at the exciting wave frequencies are
The amplitude of the high frequency pitch motions are overestimated by the numerical model. This is due to the added mass term \( a_{13} \), which couples the surge and pitch response. The model does not take into account the frequency dependent nature of the added mass coefficient. The analysis was repeated with \( a_{13} \) set equal to zero. The spectral densities of the measured and computed pitch motion are plotted in Figure 5. The computed pitch motion now compares favourably with the measured motion at the higher frequencies. However, the pitch motion at the exciting wave frequencies is no longer well estimated. These results indicate the importance of modelling the surge-pitch coupling as well as the frequency dependence of the hydrodynamic coefficients. The variation of the added mass coefficient with frequency can be incorporated into a time domain model using the convolution integral method described by Van Oortmerssen (1976).

Most previous authors (e.g. De Boom et al., 1984) assume the resonant heave and pitch responses are due to second order wave loads arising from the sums of the different frequency components in a sea state. In order to determine whether the observed resonant pitch response is due to such a quadratic effect, the coherency between the measured pitch response and the square of the water surface elevation at the higher frequencies was evaluated and is shown in Figure 6. Also shown on the figure are the spectral densities of the square of the water surface elevation and the pitch response. The figure shows a poor correlation between the resonant pitch motions and sum frequency wave excitation, indicating that the high frequency motions are due to effects higher than second order. This should be expected since there is very little energy in the sum frequency wave spectrum around the pitch natural frequency. A more formal analysis can be carried out to determine whether the measured high frequency TLP motions are due to second order loads by using cross-bispectral analysis methods (e.g. Kim et al., 1989) to estimate the quadratic coherence functions.
Figure 6. Coherence function for high frequency pitch response

Numerical computations were carried out to evaluate the relative contributions of two different nonlinear effects to the high frequency motions and forces. The main characteristics of the TLP model are given in Table 1. The platform consists of a square deck supported by four circular columns. The columns are connected at the bottom by four circular pontoons. The platform is moored by one tendon in each corner, attached at the center of each column. The platform properties are similar to those described by Vickery (1988). The natural periods are 79 s, 2.6 s, and 2.9 s for surge, heave and pitch respectively.

Table 1. Main Particulars of TLP Model

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between column centerlines</td>
<td>58.2 m</td>
</tr>
<tr>
<td>Column diameter</td>
<td>14 m</td>
</tr>
<tr>
<td>Pontoon diameter</td>
<td>8.9 m</td>
</tr>
<tr>
<td>Draft</td>
<td>26.6 m</td>
</tr>
<tr>
<td>Total platform height</td>
<td>56.3 m</td>
</tr>
<tr>
<td>Structural mass (t)</td>
<td>18200 tonnes</td>
</tr>
<tr>
<td>Displacement</td>
<td>28190 tonnes</td>
</tr>
<tr>
<td>Initial tension per tendon (T0)</td>
<td>2500 tonnes</td>
</tr>
<tr>
<td>Axial stiffness per tendon (k')</td>
<td>5100 tonnes/m</td>
</tr>
<tr>
<td>Pitch radius of gyration (r_y)</td>
<td>33 m</td>
</tr>
<tr>
<td>Vertical position of center of gravity (z_p)</td>
<td>1.3 m</td>
</tr>
<tr>
<td>Water depth</td>
<td>410 m</td>
</tr>
</tbody>
</table>

The two nonlinear effects considered are:

1. effect of integration of the water particle kinematics up to the instantaneous water surface elevation instead of the still water level (Type I);
2. effect of nonlinear drag loading (Type II).

Table 2. Amplitudes of the harmonics of tendon tensions

<table>
<thead>
<tr>
<th>Nonlinearity</th>
<th>Tension in Upstream Tendon (MN)</th>
<th>Tension in Downstream Tendon (MN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A(1)</td>
<td>A(2)</td>
</tr>
<tr>
<td>Full</td>
<td>7.9</td>
<td>2.4</td>
</tr>
<tr>
<td>Type I</td>
<td>8.4</td>
<td>2.4</td>
</tr>
<tr>
<td>Type II</td>
<td>8.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The platform motions and tendon forces were computed for a regular wave with a height of 12 m and period of 8.7 s, which is three times the natural pitch period. Inertia and drag coefficients, C_D = 1 and C_M = 2 are assumed. The structural and radiation damping ratio is assumed to be 1% of critical damping. The computations were repeated with the two nonlinear effects modelled separately. In the first case, the drag term was linearized using the equivalent linearization method while the kinematics were integrated up to the instantaneous water surface elevation. In the second case, the nonlinear drag term was retained while the kinematics were integrated up to the still water level. A Fourier analysis was used to determine the amplitudes of the first, second, and third harmonics (A(1), A(2), A(3)) of the tendon force. The results of this investigation are summarized in Table 2. It can be seen from Table 2 that the third harmonic tendon force is due primarily to wave loading in the free surface zone. The contribution of the nonlinear drag force is negligible.

In irregular waves, the intermittency of submergence of the columns in the crest to trough zone results in wave loading at frequencies greater than the wave frequencies. Figure 7 shows a comparison of the spectral density of the water particle ac-

Figure 7. Spectral densities of water particle acceleration
ceration for an irregular wave, described by a JONSWAP spectrum with $H_m = 15 \text{ m}$, $f_p = 0.07 \text{ Hz}$, and $\gamma = 3.3$. The accelerations were computed at elevations $z = -5 \text{ m}$, $z = 0 \text{ m}$ and $z = 5 \text{ m}$. The fluid acceleration in the splash zone is seen to have components over a broad range of frequencies. The high frequency component of the acceleration excites the pitch response which incuces high frequency vibrations in the tendons.

4. CONCLUDING REMARKS

The relative velocity formulation of the Morison equation has been used to compute the motions of a TLP and the tendon forces. High frequency, resonant pitch motions are observed due to effects higher than second order. For the TLP investigated in this paper, these motions are primarily due to the intermittency of submergence of the columns in the free surface zone. There are also other possible sources of high frequency loads in the tendons which are not addressed in this paper. These include wave slamming on the deck and columns and excitation of the bending modes of vibration of the tendons. The time domain numerical model can reasonably predict the observed low frequency, wave frequency and high frequency responses provided surge-pitch coupling as well as the frequency dependent nature of the hydrodynamic coefficients are taken into account. The results presented here are of a preliminary nature and additional work is being carried out to improve the numerical model.

REFERENCES


