P2P RIDESHARING WITH RIDE-BACK ON HOV LANES: TOWARDS A PRACTICAL
ALTERNATIVE MODE FOR DAILY COMMUTING

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ABSTRACT

We present a matching and pricing mechanism for a P2P ridesharing system which ensures a ride-back for the matched riders. This service is thus presented as an alternative to driving solo for daily commuting. The matching algorithm is formulated as a min-cost max-flow problem which is exact and fast solvable on polynomial time. The mechanism modelling is based on the Vickrey-Clarke-Groves (VCG) mechanism that is known to be efficient, incentive compatible and individually rational. However, VCG runs on a budget deficit in a ridesharing setting. To address this issue, participants are classified into drivers and riders according to a novel multiparameter reserve price that fixes the revenue shortage problem, making the system financially self-sustainable, but in detriment of not being efficient anymore. Agents’ utility functions include cost-sharing savings and HOV travel time savings. The parametric study uses origin-destination demand data from Southern California Association of Governments (SCAG) and travel times are extracted from a professional webmap mapping service. Results show the method has revenue surplus over most of the reserve price parameter space and offers high matching rates due to the inclusion of HOV travel time savings and reserve price structure. The reserve prices are drawn from empirical distributions of value of time and unit distance cost.

Keywords: ridesharing, mechanism design, ride-back, commuting, multiparameter reserve price, HOV lanes.
1 INTRODUCTION

Traffic congestion in urban areas is a major concern that leads to losses of about $260 billion nationwide or $960 per commuter every year according to (1). Several alternatives have been put in place during the last decades to ameliorate congestion. Transportation demand management, congestion pricing, public transportation, and additional infrastructure investments such as High Occupancy Vehicle lanes (HOV) have served to reduce this externality. However, during the last decade, a historical alternative, ridesharing, has seen its revival due to the advent of smartphones and mobile internet.

Peer-to-peer (P2P) ridesharing systems suffer from the “chicken-and-egg” problem that hampers their implementation, namely not reaching a potential critical mass that makes them a reliable alternative. One of the reasons why riders might be skeptical of leaving their vehicles at home and participating in a ridesharing system is the uncertainty of finding a ride back home. To address this concern, we model the operations of a single-hop ridesharing system (one in which each matched driver makes a detour to carry a single passenger) with a guaranteed ride-back in the evening, targeted for commuter trips. This study is the first to model this customer attraction policy on a P2P ridesharing matching and pricing scheme.

Ridesharing reduces travel cost per traveler due to sharing trip monetary costs among multiple people. On the other hand, under certain traffic conditions, HOV lanes can offer travel time savings to any vehicle with an occupancy higher than one or two passengers. Current studies on pricing for ridesharing systems have overlooked this possibility, despite being known that carpoolers positively value travel time savings (2). We thus understand P2P ridesharing as a differentiated mode that not only offers distance cost savings but also travel time savings in the presence of HOV lanes, focusing on peak hour commuting. However, potential system users must be knowledgeable of the resulting travel time savings so that they can take this factor into consideration when it comes to making mode choice decisions.

The objective of this paper is to present a matching and pricing mechanism for a P2P ridesharing system which ensures a ride-back for the matched riders and accounts for potential HOV travel time savings. A parametric study investigates whether such a system can serve as an effective and financially self-sustainable alternative mode to commuting solo. Since there is value heterogeneity across the commuter population, the system operator must ensure that these values are elicited truthfully from users and prevent selfish manipulation of the system. We address these issues by using mechanism design solutions.

The mechanism is based on a modified version of Vickrey-Clarke-Groves (VCG) mechanism, which maximizes social welfare and ensures truthfulness and voluntary participation from users. The article models the operation of a single-hop ridesharing system with guaranteed ride back, which can be formulated as a min-cost max-flow formulation that is of polynomial nature and fast to solve using standard linear programming methods.

Unfortunately, there is a tradeoff between economic efficiency, truthfulness, budget balancedness and individual rationality. It is known (3) that generally it does not exist a mechanism that is guaranteed to satisfy the four properties at the same time. Actually, in practice, ridesharing systems usually depend on external subsidies to cover operational costs and make the system attractive for users. In this study, we use reserve prices to address this issue.

To assess realistic quantitative benefits of the proposed system, we present a real-world study of the Los Angeles metropolitan area. Origin-Destination demand volume data for this study comes from the Southern California Association of Governments (SCAG) model, OD travel times are queried through Google Maps API, and individual valuations of time savings and distance savings are drawn from empirical distributions.

This paper brings multiple new contributions. We introduce the ride-back, enabling ridesharing to be an alternative mode to commuting. We are the first to consider HOV lane travel time savings on a P2P ridesharing (truthful) pricing scheme, which at the same time helps to control budget deficit problem. Finally, we provide a novel min-cost max-flow modeling of the problem which allows it to be solved rapidly in polynomial time which will be critical for an eventual implementation of dynamic ridesharing.

The paper is divided as follows. Section 2 presents the review on existing literature, section 3 provides the method preliminaries, section 4 describes the mechanism, section 5 describes the data used and its treatment, section 6 contains the numerical results and finally, we state conclusions and further research.
2 LITERATURE REVIEW

There has been a recent interest in using mechanism design for pricing in ridesharing operations (4), (5), (6), (7) and (8). Most of the literature is concerned with using pricing mechanisms not only to address incentive issues but also in how to increase the probability of matching and therefore the overall efficiency of the system. These problems are common in either day-to-day ridesharing or real-time dynamic ridesharing. Alternatively, (9), (10), (11), (12), and (13) are some examples of studies that focus on system efficiency and performance, and leave out the pricing component.

(4) proposes a parallel auction scheme based on the sealed-bid second price auction. They examine how the use of auctions to allocate rides can lead to more efficient outcome than allocating rides based on Vehicle Kilometer Travelled. (5) proposes also an online posted-price mechanism for a centrally managed autonomous mobile on demand transportations systems. Their solution is incentive compatible, individually rational and budget balanced and performs better than their online auction benchmark.

(6) studies the viability of ridesharing plans using GPS data. The method used is a VCG-based mechanism and they address the budget deficit problem imposing a positive budget constraint in line with (14). Agents’ utility function is the sum of travel time and delayed waiting time, but they do not include distance savings. Their approach is not incentive-compatible.

(7) examines several theoretical solutions on how to control deficit when using the VCG mechanism for ridesharing problems. They show that VCG’s deficit is unbounded if there are no reserve prices and propose a double reserve price scheme to control for this deficit. Alternative policies to reduce deficit are to limit the maximum detour of drivers. Agents’ utilities are based on time proportional gas savings.

(8) explores a new method to increase the probability of matching in dynamic ridesharing systems. A new bilateral trading scheme is proposed between a user already present in the system who is assigned a ride and a just arrived user could not be assigned any matched due to lack of available drivers. The system proposes the two drivers a posted price based on historical data to try to exchange the ride between. If they accept the price, the ride is being transferred and the payment executed. System efficiency and matching probability is increased.

In other applications, VCG mechanisms have been used in procurement auctions for distributed transportation procurement (15), e-commerce logistics (16, 17). Specific dynamic primal-dual algorithms have been developed to reach the VCG outcome. These dynamic auctions reveal less information from bidders and are simpler to understand for bidder than static VCG.

Our article fills the literature gap in pricing on P2P ridesharing systems by considering for the first time HOV lane time savings. Our work also uses more complete utility functions that include time valuation, distance valuation and time savings due to HOV lanes. This is also the first case of the integration of a ride-back in the ride-sharing problem.

3 METHOD PRELIMINARIES

The economic benchmark presented here is based on a mechanism design solution. Mechanism design is a field of microeconomics that studies the design of economic mechanisms. Mechanisms are methods that help a system operator to reach a particular goal, such as efficiency or revenue, while addressing potential selfish manipulation from rational economic agents who hold some information private. In our setting, we have agents who need to be assigned a role of rider or driver and a trip plan, according to their private preferences. These agents hold some information private, which is how much they value their time and cost per distance. The system operator is interested in assigning rides that maximize social welfare, while taking care of not running into budget deficit. That is, requiring an external subsidy to implement the allocation.

We provide next a short introduction to the necessary mechanism design concepts. Excellent introductions on this discipline can be found in (18, 19).

We define a mechanism $M$ as a pair of functions $k: \Theta \to K, p: \Theta \to \mathbb{R}$. We call the first function the phase allocation rule that maps the type space $\Theta$ with the allocation space $K$. Type is the private information of the agent. The second function is the payment rule, which maps $\Theta$ to the real line, the payment space. In our notation, $k$ can represent either the allocation function or an element in the allocation space $K$. We assume private information, therefore agents are allowed to misreport their type $\theta_i$. Agents are
assumed to have a quasilinear utility function $u_i = v_i - p_i$, in which $v_i$ is the valuation of individual $i \in I$ and $p_i$ is the price charged to $i$, which can be negative or positive.

**Definition:** A mechanism $M$ is (allocative) efficient if for any type vector $\theta$:

$$\sum_{i \in I} v_i(k(\theta), \theta_i) \geq \sum_{i \in I} v_i(k', \theta_i) \quad \forall k' \in K$$

(1)

**Definition:** A mechanism $M$ is weakly budget balanced iff for any type vector $\theta$:

$$\sum_{i \in I} p_i(\theta) \geq 0$$

(2)

The question is now whether the sum of payments, the budget balance, is weakly positive or not. If it is not the case, the mechanism requires an external subsidy to enforce the optimal allocation. Otherwise, we say that the mechanism is financially self-sustainable.

A fundamental concept in mechanism design is truthfulness, or incentive compatibility (IC). Under IC, an agent is better off eliciting its type $\theta_i$ truthfully rather than reporting a type $\theta'_i \neq \theta_i$, conditional on the rest of agents reporting truthfully.

**Definition:** (Dominant-Strategy) Incentive Compatibility (DSIC)

$$u_i(\theta_i', \theta_{-i}) \geq u_i(\theta_i', \theta_{-i}), \quad \forall \theta_i' \in \Theta_i, \forall i \in I$$

(3)

Where $u_i(\theta_i', \theta_{-i})$ is agents’ $i$ utility when he reports $\theta_i'$ and the rest of agents report truthfully $\theta_{-i}$. The subindex $-i$ indicates all agents except $i$. Finally, we are interested in the voluntary participation of agents, that is, if users are better off participating into the mechanism than staying outside. We call this individually rational.

**Definition:** A mechanism $M$ is ex-post individually rational for any type vector $\theta$:

$$u_i(\theta) \geq 0$$

(4)

We model the actual benefits of the ridesharing system with the well-known VCG mechanism. VCG is a direct revelation mechanism that is known to be efficient and (dominant-strategy) incentive compatible. VCG actually belongs to the family of Groves mechanisms, which are known to be the only ones that are efficient and (dominant-strategy) incentive compatible. Under certain conditions met in our situation and which will be elaborated later on in the text, VCG is also ex-post individually rational.

VCG is calculated in two sequential steps. First, the allocation rule which is the sum of all valuations, is maximized, and second, the prices are computed by a closed formula:

$$k^* = \arg \max_{k \in K} \sum_{i \in I} v_i(k)$$

(5)

$$p_i = \sum_{j \neq i \in I} v_j(k^*_i) - \sum_{j \neq i \in I} v_j(k^*)$$

(6)

The price $p_i$ of agent $i$ is the social cost or externality that an agent exerts into the system. It is actually the difference in the sum of other agents’ valuations when $i$ is not present and when $i$ is present. If the difference is positive, it means that agent $i$ creates a negative externality for its presence, thus, pays a positive price. Equivalently, if the difference is negative, the price paid is negative and agent $i$ receives a payment.

It is known, that under certain conditions, the outcome of the VCG mechanism is individually rational. These conditions are choice set monotonicity and no-negative externalities. The first condition states that the choice set $K$ increases as additional agents are introduced into the system. This is true for the
ridesharing problem: adding a driver or rider, only increases the number of potential rider-driver matching options. The second property is no-negative externalities. A mechanism is said to have no-negative externalities property when \( \forall i \in I, \forall \theta \in \Theta, \forall k_{\cdot i}^*(\theta_{\cdot i}) \in K_{\cdot i}: \)

\[
v_i(k_{\cdot i}^*(\theta_{\cdot i}), \theta_{\cdot i}) \geq 0 \quad \tag{7}
\]

According to our definition of valuations, not considering agent \( i \) in \( k_{\cdot i} \) leads to a valuation of user \( i \) of zero, since that agent will travel by himself along his shortest path, taken as valuation reference. Thus, we conclude that VCG is individually rational in our ridesharing setting.

It could be argued though, that an increasing presence of matched riders on HOV lanes could create a congestion externality and the former result would not hold. However, for the purpose of this article, the pool of participants considered is small, corresponding to the case of a ridesharing company that is starting to provide this service. In case the ridesharing service would become successful enough to create substantial congestion, the mechanism should be embedded into an equilibration framework that includes such congestion effect. Eventually, more normal lanes should be converted to HOV lanes and provide enough supply for this beneficial service.

Finally, an interesting result of VCG following (23) is that, in situations where VCG is ex-post individually rational, VCG is as budget balanced as any efficient mechanism can be. In other words, no other mechanism in that situation can have a more positive budget balance than VCG. Even when compared to weaker Bayesian Incentive Compatible mechanisms. Therefore, VCG is a useful benchmark for the budget deficit-efficiency tradeoff we examine in this article and we can overlook weaker implementation concepts. However, as it is shown in (7), VCG’s budget deficit on the ridesharing setting is unbounded. VCG will be modified by reserve prices to control for such deficit.

4 MATCHING AND PRICING MECHANISM

We present now the matching mechanism. We define two sets of agents: the drivers set \( D \), and the riders set \( R \). Some drivers will provide service during the morning \( D_M \), others during the evening \( D_E \) and the rest during both peak times \( D_{ME} = D_M \cap D_E \). Similarly, we can define sets \( R_M \) and \( R_E \) to include riders who wish to participate in the system in the morning and evening respectively, where \( R = R_M \cup R_E \). Agents follow a quasilinear utility as mentioned earlier: \( u_i = v_i - p_i \). Users’ valuations are the difference between the cost of solo driving and the cost of participating in the system, that is, the cost of being matched.

Each agent \( i \) has two private parameters to be elicited: \( \theta^l_i \), the valuation of every mile driven and \( \theta^t_i \), the valuation of every minute of the agent’s travel time. The first one can be estimated from users’ car model, mileage and insurance information or alternatively, by inquired by user. The second parameter is user-specific and can be directly elicited or obtained by means of a short SP survey. In addition, agents submit an origin, a destination, an earliest departure time and latest arrival time. On the other hand, the ridesharing system operator has to provide accurate information of travel time savings, based on either the real-time state of the network, or historical data.

Drivers’ valuation \( v_d \) is the negative of the detour cost in distance \( \delta^d_{d,r} \) and time \( \delta^t_{d,r} \), of driving a rider \( r \) plus the travel time cost savings \( \epsilon^t_{HOV} \) in case the pair of agents can use an HOV lane.

\( \epsilon^t_{HOV} \) is the difference between rider’s shortest path time on regular lane \( S_{P,REG}^r \) minus his shortest path on HOV lane, \( S_{P,HOV}^r \). This travel time savings depend only on the rider since the rider and driver travel through the rider’s shortest path. The rider’s valuation \( v_r \) is the distance cost savings of being driven thorough his/her shortest path \( S_{P}^r \) plus the travel time savings of using an existing HOV lane. Note that the utility of not being matched is zero for both riders and drivers. Agents’ valuation equations are shown next:

\[
\epsilon^t_{HOV} = S_{P,HOV}^r - S_{P,REG}^r \quad \tag{8}
\]

\[
v_r = \theta^t_r S_{P}^r + \theta^t_r \epsilon^t_{HOV} > 0 \quad \tag{9}
\]

\[
v_d = -\theta^d_d \delta^d_{d,r} - \theta^t_d \delta^t_{d,r} + \theta^t_d \epsilon^t_{HOV} \quad \tag{10}
\]
Setting reserve prices have two functions: to reduce the budgetary deficit of the mechanism, and to classify the agents as drivers, riders or unmatched. This is particularly practical since in day-to-day ridesharing services participants are often ready to play both roles: driver and rider. Reserve prices for the VCG mechanism are normally set in single-parameter environments, that is, the valuation of the agent is the actual type. In our case, our environment is multi-parameter since each agent has two private parameters. Furthermore, we have linear valuations in which each parameter type is multiplied by a variable, time or distance.

Our reserve prices are not simple scalars, but a function that maps the type and valuation to the reserve price space. If prices were just scalars for driving or being driven, there would be inefficiency due to agents travelling through routes of different lengths in both time and distance being discarded. Also, individual rationality and incentive compatibility could not be guaranteed. We define two constants, $\rho^t$ and $\rho^l$, which are reserve prices for value of time (VOT) and unit distance cost VOD (value of distance), respectively. These constants are calibrated by the system operator in function of agents’ population characteristics.

We present next the VCG mechanism with multiparameter reserve prices ($\rho^t, \rho^l$):

1. Agents report their type $\theta_i = (\theta_i^t, \theta_i^l)$.
2. Agents are classified as drivers or riders according to:
   a. An agent becomes a potential rider if $\theta_i^t \geq \rho^t$ and $\theta_i^l \geq \rho^l$.
   b. An agent becomes a potential driver if $\theta_i^t < \rho^t$ and $\theta_i^l < \rho^l$.
   c. An agent is unmatched otherwise.
3. The efficient allocation is solved.
4. VCG prices $p_i^{VCG}$ are calculated.
5. Agents are charged the following prices:
   \[
   p_r = \max\{\rho^l S_p^l + \rho^t e^{HOV}, p_r^{VCG}\} \\
   p_d = \max\{-\rho^l \delta_d^l - \rho^t \delta_d^t + \rho^t \min\{\delta_d^l, e^{HOV}\}, p_d^{VCG}\}
   \]

Essentially, the purpose of the above reserve price is, for the riders, to get charged at least the maximum payment that the limit type rider $(\theta_i^t, \theta_i^l) = (\rho^t, \rho^l)$ can pay such that this agent’s individual rationality is respected. This guarantees a larger positive revenue from the drivers. On the other hand, the reserve price for drivers ensures that no driver is compensated more than the minimum compensation for the binding type driver. Thus, the payments that drivers receive are minimized while respecting still their individual rationality constraints.

The main reason why this mechanism has a lower deficit than the VCG mechanism is because the reserve prices filter agents such that higher valuation agents are the riders, who have a positive payment, and the lower valuation agents are the drivers, who receive payments. Thus, positive payments are increased and negative payments are minimized. Furthermore, the travel time savings due to HOV lanes helps have a positive effect on the budget. HOV-related payments from riders is always larger than those received by the drivers since the former are larger than the latter, while $e^{HOV}$ is the same for both agents.

**Proposition:** the mechanism is incentive compatible and individually rational.

Proof: let $i$ be a rider. If $p_i^{VCG} > \rho^l S_p^l + \rho^t e^{HOV}$, his individually rationality is guaranteed by the argument made earlier on the standard VCG mechanism. Else, since $\theta_i^t < \rho^t$ and $\theta_i^l < \rho^l$ the reserve price charged is always smaller than the valuation. Analogous is the case in which $i$ is selected as a driver. Suppose that agent $i$ is classified as a driver. If the driver misreports a higher type, he/she will be classified as a rider and charged either a reserve price that will violate his/her individual rationality, or will be charged a VCG payment that will not maximize his/her utility, since VCG is incentive compatible. Analogously, if the agent is classified as a rider, by misreporting a lower type his/her utility will not be maximized. ■
To ensure truthfulness, it is important that the optimization of the allocation rule is exact, not approximate. Otherwise, truthfulness cannot be guaranteed (24). Our problem formulation is a min-cost max-flow, which has exact integer solution since the constraint matrix is totally unimodular. We guarantee that the solution is optimal by using standard linear programming techniques.

An aspect left aside so far are the users who do not own a vehicle or do not have access to one temporarily. From the mechanism definition, users who do not own a car would mostly fall into the category of drivers, since they would have to specify a VOD of either zero or very low one in case their default mode of transportation is public transit. A second similar case arises, which is that of users who own a car but they do not have it temporarily available and would still wish to share a ride. For instance, due to mechanical failure or because a family member had to use that car.

An idea to accommodate these two types of users is to set up a “leftover pool” that takes advantage of the budget surplus resulting from the main optimization and provide these agents when a ride from the unmatched drivers or through an external service when possible or desirable, as it is often done in carpooling companies. Obviously, measures need to be established to prevent users from manipulating and gaming the system. Some examples are (1) providing a justification for not having the car available, (2) setting up a maximum number of leftover pool rides for each user and (3) setting up a penalty for lying about having a car available. For users who do not own a car, a separate, more expensive fare could be set, since they do not contribute to the driver supply. Naturally, priority in the leftover pool will be given to the users with a history participating into the system with a vehicle.

We present the allocation rule on a min-cost max-flow formulation, see Figure 1. Once agents are classified as drivers and riders, they are grouped in columns. Assuming a daytime problem, with ride-backs being in the evening (as opposed to night-shift travel), the first column consists of the morning drivers, and the second column, of the morning riders. The riders are duplicated for the evening matching, and finally there is the evening driver column. Each matching arc is assigned a cost $c_{ij} \in \{0, c_{dr} = -(v_r(r,d) + v_d(d,r))\}$. Rider morning to rider evening arcs have cost equal to zero.

At the initial extreme, there is a source vertex, which generates $b_s = \min\{|D_M|, |R|, |D_E|\}$ units of flow. At the other end, there is a sink vertex that attracts $b_t = -b_s$ units of flow. Source-to-driver and driver-to-sink arcs have cost equal to zero. In order to make the allocation individually rational and as efficient as possible, we add a source-to-sink arc with zero cost and capacity equal to $b_s$. This link prevents the existence of negative welfare driver-rider matchings. The rest of arcs have capacity equal to one.

![FIGURE 1 Min-cost max-flow graph for the ridesharing with ride back problem](image-url)
The above graph is built after executing a preprocessing step in which agents that do not intersect on space and time do not have an arc connecting them. This considerably reduces the size of the graph and therefore the optimization time. The VCG prices for agent $i$ are calculated by subtracting the edges that connect to agent $i$ vertices and solving the optimization again. By using the former solution as initial solution, the optimization speed is increased considerably. Please note that there always a solution, diverting all the flow through the source-to-sink auxiliary arc.

The min-cost max flow formulation can be equivalently represented as the following linear program:

$$\min \sum_{(i,j)\in A} -c_{ij}x_{ij}$$

s.t.

$$\sum_{(i,j)\in \delta_i^+} x_{ij} - \sum_{(j,i)\in \delta_i^-} x_{ji} = b_i$$

$$b_i = \begin{cases} 
\min\{|D_M|, |R|, |D_E|\}, & i = s \\
-\min\{|D_M|, |R|, |D_E|\}, & i = t \\
0, & \text{else}
\end{cases}$$

$$u_{ij} = 1, \forall (i,j) \in A - \{(s,t)\}$$

$$u_{ij} = b_s, \forall (i,j) = \{(s,t)\}$$

$$0 \leq x_{ij} \leq u_{ij}, \forall (i,j) \in A$$

Equation (11) is the objective function, the sum of the negative of all edge costs times the edge flows $x_{ij}$. Constraints (12) represent the flow conservation at every node, where (13) defines nodes’ supply terms $b_i$, $\delta_i^+$ and $\delta_i^-$ are the set of outgoing and incoming edges from node $i$. Constraints (14, 15) define edge capacities $u_{ij}$, according to the discussion made earlier for the graph.

5 DATA SOURCES AND TREATMENT

This article relies on five data sources. Origin and destination nodes and their AM and PM peak demands for a 3 hour period are extracted from the Southern California Association of Goverment (SCAG) model 2012. In total, we consider 3051 nodes corresponding to 9,308,601 OD pairs. Our study area, shown in Figure 2, covers Los Angeles County and parts of San Bernardino County and Orange County which encapsulate the totality of Southern California HOV network. Since the SCAG model is a planning model, the trip tables from this model are only available at an aggregate level. For the purpose of our study, we distribute the peak-hour demand throughout the duration of morning and evening peak hours based on a uniform random distribution.

OD travel times and distances are obtained by using Google Maps API. The following information was requested at 8AM and 5PM for Wednesday July 20th (a typical work day): total travel time given traffic, total trip distance, total freeway distance if an HOV is present. The presence of HOV lanes was determined by parsing the current HOV-equipped freeways from the retrieved trip instructions. Querying travel times from Google Maps is a costly task in time and money, since there are quotas for their usage. For this reason, the nodes were aggregated geographically to reduce the number of OD queries. This was done by means of a k-means clustering procedure. The number of clusters was set to 136 which led to small circular areas of 2.5 km of radius in average, with an average max radius of 6km. Figure 2 shows the study area with the nodes, cluster centroids and (average and maximum) cluster areas.

Final ODs distances and times are computed with some approximations. If the distance between the origin and the destination is shorter than 3 km or both nodes belong to the same cluster, the OD trajectory is Euclidean and the speed is set to 25mph (40kmh). Otherwise, the ODs are composed by three segments: origin node to cluster centroid of origin node, cluster centroid origin node to cluster centroid destination node and a final segment from cluster centroid destination to the destination node. This extra distances are run at 25mph (40kmh) as well.
Google Maps does not provide HOV travel times. To include the HOV travel time savings, we obtained the relationship between mainlane speed and HOV speed from NGSIM data collected at I-80 (25). This data covers the period from 5:00PM to 5:30PM. We extracted the average speed for the regular lanes (11 mph), the average speed for the HOV lane (27 mph) and established an affine relationship between the HOV lane speed $s_{HOV}$, and the regular lane speed $s_{ML}$, given the free flow speed $s_{FF}$ of 65 mph. This relationship provides the HOV speed given the regular lane speed obtained from google maps, after having subtracted the above mentioned non-freeway segments of the path.

$$s_{HOV} = s_{FF} - s_{FF} - \frac{27}{s_{FF} - s_{ML}}$$

(16)

With more extensive data on HOV lane speeds, the system could propose optimal departure and pickup times for the rider and driver and reach further time savings gains. The same relationship is used for both peak periods.

We acknowledge that some degree of positive correlation could exist between VOT and VOD. Higher income people are observed to value time more and to purchase vehicles with higher operating costs (26). But given the complexity of vehicle purchase decision and that the VOT increases with income less than proportionally (27), we assume independence between VOT and VOD. Thus, their values will be drawn from two independent distributions. Agents’ VOT are drawn from the VOT distribution estimated in (28). Their distribution is estimated from a Swedish SP survey based on car alternatives, accounting for income levels and latent variables.

Driving costs per distance are obtained from AAA’s Your Driving Costs study for year 2014. This study gives the costs per mile for the following vehicle segments: small, medium and large sedan, SUV 4WD and minivan. This costs include fuel, maintenance, tire, insurance and depreciation costs. Since we assume that agents keep their own vehicle, we discount insurance and depreciation costs for the driving costs. These latter items account for around 60% of of the costs. The VOD distribution is obtained by weighting the former costs by a distribution of vehicle segments. The latter is obtained from the car sales in 2016 from January to June 2016 (29, 30).
6 SIMULATION RESULTS

This section presents the simulation results of our control policy on the study area described above. The total number of agents is always set to 2500 and six different seeds are used for pseudo-random population characteristics generation.

6.1 Percent of drivers, riders and supply to demand ratio

We first observe the effect that the reserve prices exert on the induced supply and demand of the ridesharing services. Please note that the percent of drivers and riders does not add to a hundred since those vehicles that do not satisfy the reserve prices are left unmatched and drive by themselves. Figure 3 shows how increasing $\rho^c$ and $\rho^l$ has a positive effect on the percent of drivers and a negative effect on the quantity of riders. This leads to the ratio riders to drivers increasing as we decrease the prices. We expect thus, higher social welfare on the lower reserve price section. From the graph on the right, we can observe that the interesting regions for our sensitivity analysis are on the 10 to 30 $$/h range for the VOT and on the 0.35 to 0.6 $$/mile VOD (value of distance) range.

![Figure 3 Percent of drivers, riders and ratio of riders to drivers in function of the reserve prices.](image)

6.2 Sensitivity analysis over both reserve price components.

We next analyze the sensitivity of the matching rate in our model to the reserve prices, setting the maximum detour for the drivers to 20 min. Figure 4 shows the revenue per vehicle, total social welfare SW (the sum of all agents’ utilities) and percent of matched participants.

![Figure 4 Revenue per vehicle, total social welfare and percent of matched participants.](image)

The percent of matched participants decreases with $\rho^c$ and increases with $\rho^l$. While the former effect is expected, since the ratio of riders to drivers increases from below one to more than, the latter requires more attention. The main explanation is that by raising the reserve price on distance, the agents who become riders have higher cost savings and higher probability of being matched. This eventually
compensates the decline in matching due to the decreasing ratio of riders to drivers. An observation worth making is that a high percent of vehicles matched could increase congestion in the HOV lane and reduce its travel time benefits. However, which such a low number of participants simulated, this effect is considered to have little impact. This could be the case of the initial states of the implementation of the ridesharing policy, in which the system is still gaining critical mass.

From the center plot, we observe that total participant social welfare is correlated with the percent of matched participants. From this sensitivity analysis, we conclude that the most favorable reserve prices are low \( \rho^t \) and high \( \rho^l \). That is, around 10$/h and 0.55$/mi respectively. The leftmost figure shows how the average revenue per vehicle stays positive over all the reserve price parametric domain, except for the corner with high \( \rho^t \) and low \( \rho^l \), which is of little operational interest. Over the low \( \rho^t \), high \( \rho^l \) region, the revenue oscillates between 18 and 38 cents per vehicle. The instability of the revenue surface is explained by the non-convexity of the reserve price expressions.

### 6.3 Sensitivity analysis on maximum driver detour and VOT reserve price.

We analyze now the consequences of limiting the maximum detour length on drivers. Maximum driver detour is an operational parameter that the system manager can use to control for revenue and efficiency. Maximum detour has two effects on the system: while increasing it may increase the total number of matches and therefore, social welfare, it also increases deficit since drivers need to be compensated for longer detours, while riders keep paying similar fares.

To be able to analyze the influence of maximum detour, we fix the VOD reserve price to 0.54 cents per mile, which is the IRS mileage rate for 2016. This mileage rate accounts for fixed and variable costs for operating a vehicle. Moreover, this mileage rate turned out to be the maximum social welfare point in the previous section. We enrich the detour analysis with sensitivity on the VOT reserve price.

Leftmost plot in Figure 5 shows how the revenue per vehicle decreases as the maximum detour increases, leading to negative revenues above the 25 min of maximum detour. The reserve price on the VOT seems to slightly increase revenue, due to higher savings from HOV lane per vehicle and also for reducing the actual number of agents matched, as seen in Figure 4: fewer matches are made, but more revenue per match is collected. Similarly, increasing the detour increases the average utility per vehicle and total social welfare since more vehicles are matched. Finally, increasing the reserve price on VOT has a positive effect on the social welfare per vehicle but a negative one on total social welfare. An explanation of the latter may be due to the increase in revenue mentioned earlier.

Finally, we analyze the effect the maximum detour has on the percent of riders matched with the same driver for both morning and evening tour. The results are shown in Figure 6. We observe how an increase on the maximum detour time for drivers translates into more drivers being matched with the same driver for both trips. As drivers’ travel time budgets become longer, the set of matched potential matches per rider increases and the most efficient one is selected. If, for a rider, his most efficient driver is selected
during the morning, this same driver should be selected as well during the evening.

FIGURE 6 Percent of matched riders with the same driver for both trips

This trend is present over all the reserve price range, but it is clearer for lower VOT reserve prices since the ratio riders to drivers increases. From these results, we conclude that modelling the ride-back separately is generally beneficial for a system operator interested in social welfare, while for a revenue oriented operator, bundling the morning and evening ride into a round trip may help increasing revenue.

7 CONCLUSION AND FURTHER RESEARCH
We presented an economic benchmark of the interaction of P2P ridesharing with HOV lanes. Ridesharing was introduced as an alternative mode for commuting that can provide not only cost savings by sharing the trip, but also time savings due to the possibility of HOV lane use. The ridesharing service, modelled as an economic mechanism, is based on the VCG mechanism which is known to be efficient, incentive compatible and individually rational in the ridesharing environment. However, VCG is known to run on budget deficit, making the system financially unsustainable without external subsidy.

A multiparameter reserve price was introduced to control for this budget deficit. Moreover, it efficiently classifies agents between drivers and riders before the matching, thus, and so eliminates the need to specify exogenously who is driver and who is rider. The reserve price successfully manages to collect enough revenue for most of the parameter space, making the system financially sustainable. This is the case of low reserve price time cost and high reserve price for distance cost, the combination in which the sum of agents’ utility is maximized. Finally, we conclude that modelling the ride-back as a separate complementary trip increases social welfare and the percent of participants matched.

We suggest next several lines of further research to extend this paper. First, extending the reserve price method to more complicated ridesharing settings such as multi-hop day-to-day ridesharing. This would require careful examination on satisfying incentive compatibility, since it is not guaranteed when the allocation rule is not exact. Second, using richer utility functions that could include extra time savings, such as parking savings for the rider. Third, modelling the congestion resulting from the interaction between matching percentage and HOV travel time benefits. This could be addressed by embedding the mechanism into an equilibration framework. Moreover, planning and simulation models that have sharing economy inherent in them should be developed. Fourth, adding a day-to-day choice-related behavior component in agents would provide a better understanding of the actual ridership volume of this system.

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