

493 DISCUSSION SECTION 3

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If you are unsure of the precise mathematical definition of any of the following, please look it up. Thanks!

- cyclic group
- abelian group
- order of a group (i.e. $|G|$)
- order of an element of a group
- homomorphism, isomorphism
- subgroup
- **image of homomorphism**
- **kernel of homomorphism**
- **normal subgroup (\trianglelefteq)**
- **simple group**

In discussion, we attempted each part of each of the following problems in five minutes or less.

(Ex. 1) Show every subgroup of an abelian group is normal.

(Ex. 2) Let G be a group and $H \subseteq G$ be a subgroup.

(a) Show that there exists a group L and a homomorphism $\psi : L \rightarrow G$ such that $H = \text{im}(\psi)$.

(b) Suppose there is a group K and a homomorphism $\phi : G \rightarrow K$ such that $H = \ker(\phi)$. Show that $H \trianglelefteq G$.

(Ex. 3) Show that if G is simple and $\phi : G \rightarrow H$ is a group homomorphism, then either $\text{im}(\phi)$ is trivial, or it is isomorphic to G . (Or both, of course.)

(Ex. 4) Show $G \trianglelefteq G$ and $\{e\} \trianglelefteq G$.

(Ex. 5) (a) Show that all simple abelian groups are cyclic.

(b) Show that all simple abelian groups are finite.

(c) Classify simple abelian groups.