493 DISCUSSION SECTION 2

WITH NOAH LUNTZLARA

If you are unsure of the precise mathematical definition of any of the following, please look it up. Thanks!

- cyclic group
- abelian group
- $C_n (= \mathbb{Z}/n\mathbb{Z})$
- S_n and cycle notation for elements of S_n
- GL_n
- order of a group (i.e. $|G|$)
- order of an element of a group
- homomorphism
- isomorphism, automorphism
- **subgroup**

In discussion, we attempted each part of each of the following problems in five minutes or less.

(Ex. 1) Let G, H be groups, and let $\phi : G \to H$ be a surjective group homomorphism.
  (a) Show that if G is cyclic, then H is cyclic.
  (b) Show that if G is abelian, then H is abelian.

(Ex. 2) Let G be a group. Show that a nonempty subset $H \subseteq G$ is a subgroup if and only if $\forall a, b \in H$, $ab^{-1} \in H$. (Recall that by definition, a subgroup is a subset of a group satisfying the three criteria (a) $e_G \in H$, (b) $\forall h \in H$, $h_G^{-1} \in H$, (c) $\forall h, h' \in H$, $h \cdot_G h' \in H$.)

(Ex. 3) Define $Z(G) = \{g \in G \mid gh = hg \, \forall h \in G\}$. (This set is called the *center* of G.)
  (a) Show that $Z(G)$ is a subgroup of G.
  (b) When is $Z(G) = G$?
  (c) Can $Z(G) = \{e\}$ when G is not the trivial group?

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