493 DISCUSSION SECTION 1

WITH NOAH LUNTZLARA

If you are unsure of the precise mathematical definition of any of the following, please look it up. Thanks!

- cyclic group
- abelian group
- $C_n (= \mathbb{Z}/n\mathbb{Z})$
- S_n and cycle notation for elements of S_n
- GL_n
- order of a group (i.e. $|G|$)
- order of an elements of a group
- homomorphism
- isomorphism
- automorphism

In discussion, we attempted each part of each of the following problems in five minutes or less.

(Ex. 1) Let G be a group where every element $g \in G$ satisfies $g^2 = e$. Prove that G is abelian.

(Ex. 2) Let x, y, z be elements of a group such that $xyz = e$. Does it follow that $yzx = e$? That $yxz = e$?

(Ex. 3) Show that any finite group of even order has an element of order 2.

(Ex. 4) Let G be a group. Define a new operation $\circ$ on G, $a \circ b = ba$.

- (a) Prove $(G, \circ)$ is a group. This group is called G^{op}, the opposite group of G.
- (b) Show $G \cong G^{\text{op}}$.

(Ex. 5) Let G be a group. Define

$$\text{Aut}(G) = \{\text{isomorphisms} G \to G\}.$$

- (a) Prove $(\text{Aut}(G), \text{composition})$ is a group.
- (b) For $g \in G$, define $\phi_g : G \to G : h \mapsto ghg^{-1}$. Show that $\phi_g \in \text{Aut}(G)$.
- (c) Show that $\Phi : G \to \text{Aut}(G) : g \mapsto \phi_g$ is a homomorphism.
- (d) Show that if $G = S_3$, then Φ is an isomorphism.
- (e) (*Bonus*) Show that if $G = S_n$, then Φ is an isomorphism if and only if $n \neq 6$.

Date: September 7, 2018.