396 DISCUSSION SECTION 2

WITH NOAH LUNTZLARA

(Ex. 1) Hello! Today we're going to prove the Orbit-Stabilizer Theorem in a few steps. Let $G \circlearrowright S$, and fix $s \in S$.

(a) Show that if $g \cdot s = h \cdot s$, then $g^{-1}h \in \mathrm{Stab}_G(s)$.

(b) Show that for any fixed $t \in \mathrm{Orb}_G(s)$,

$$\{g \in G \mid g \cdot t = s\} \in G/\mathrm{Stab}_G(s).$$

(c) Prove the Orbit-Stabilizer Theorem.

Whenever we have a group action $G \circlearrowright S$, we get a *quotient set*, S/G, which simply denotes the set of G-orbits in S.

(Ex. 2) Prove Burnside's lemma:

$$|G| \cdot |S/G| = \sum_{g \in G} \mathrm{fix}(g),$$

where $\mathrm{fix}(g)$ denotes the set $\{s \in S \mid gs = s\}$ of points that are fixed by g.

[Hint: "It just defines an equivalence relation that partitions the set. It's that simple." –D. Winsor]

(Ex. 3) Use Burnside's Lemma to count the number of non-isomorphic graphs on four vertices. Check your answer by drawing all of them.

(Ex. 4) Use Burnside's Lemma to count the number of ways to 2-color the sides of an octahedron (my favorite solid).

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