396 DISCUSSION SECTION 1

WITH NOAH LUNTZLARA

(Ex. 1) A warm-up problem. Prove the following form of Lagrange's Theorem:

If H is a subgroup of G, and K is a subgroup of H, then $[G : K] = [G : H][H : K]$.

Recall the definition of the *orbit* and *stabilizer* of an element under a group action: if G acts on S and $s \in S$, we define

$$\mathrm{Orb}_G(s) = \{gs \mid g \in G\}; \ \mathrm{Stab}_G(s) = \{g \in G \mid gs = s\}.$$

Pop Quiz: Where do these folks live? $\mathrm{Orb}_G(s) \subseteq \qquad$, $\mathrm{Stab}_G(s) \subseteq \qquad$?

(Ex. 2) Prove the Orbit-Stabilizer theorem: if a finite group G acts on S, then for any $s \in S$ we have

$$|G| = |\mathrm{Orb}_G(s)| \cdot |\mathrm{Stab}_G(s)|.$$

Whenever we have a group action $G \circlearrowright S$, we get a *quotient set*, S/G, which simply denotes the set of G-orbits in S.

(Ex. 3) Prove Burnside's lemma:

$$|G| \cdot |S/G| = \sum_{g \in G} \mathrm{fix}(g),$$

where $\mathrm{fix}(g)$ denotes the set $\{s \in S \mid gs = s\}$ of points that are fixed by g.

(Ex. 4) Use Burnside's Lemma to count the number of non-isomorphic graphs on four vertices. Check your answer by drawing all of them.

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