395 DISCUSSION SECTION 8-ISH

WITH NOAH LUNTZLARA

(Ex. 1) Recall what you learned in the two guest discussion sections.

We start with the problems we didn't get to last time.

Unless stated otherwise (and it's never stated otherwise) G is a group, S is a set, and G acts on S.

The *orbit* of $s \in S$ is the set

$$\mathrm{Orb}_G(s) = \{gs \mid g \in G\}.$$

Notice that this is a **subset of S**.

(Ex. 4) Suppose G and S are both finite. Of the following statements, which ones are true? Justify either way.
- (a) The sizes of the orbits sum to $|S|$.
- (b) The product of the orders of the orbits equals $|G|$.
- (c) The number of orbits divides $|S|$
- (d) The number of orbits divides $|G|$.
- (e) The orbits have order dividing $|S|$.
- (f) The orbits have order dividing $|G|$.

(Ex. 5) Let G act on itself by *conjugation*: $g \cdot h = ghg^{-1}$. The orbits under this action are called *conjugacy classes*.
- (a) The union of all conjugacy classes with order 1 is called the *center* of G, denoted $Z(G)$. Show that $g \in Z(G)$ if and only if it commutes with every element of G.
- (b) Since the orbits partition G, we can write the order of G as the sum of the orders of its conjugacy classes:

$$|G| = \sum_{\mathcal{O} \in \{\mathrm{Orb}_G(s) \mid s \in S\}} |\mathcal{O}|.$$

  This is called the *class equation* for G. Write down the class equation for
  - (i) $G = \mathbb{Z}/3\mathbb{Z}$
  - (ii) $G = S_3$
  - (iii) $G = D_4$

The *stabilizer* of $s \in S$ is the set

$$\mathrm{Stab}_G(s) = \{g \in G \mid gs = s\}.$$

Note that this is a **subset of G**.

(Ex. 6) [D. Copeland] Show that the stabilizer of n in the action of S_n on $\{1, \dots, n\}$ is isomorphic to S_{n-1}.

(Ex. 7) Show that $\mathrm{Stab}_G(s) \trianglelefteq G$.

(Ex. 8) [D. Copeland] Find the orbits and stabilizers for the following parts of a cube. Notice anything?
- (a) a single vertex
- (b) an edge
- (c) a face

(Ex. 9) State and prove the *Orbit-Stabilizer Theorem*.

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