Proof Techniques

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1   Quantifiers

Remind yourselves of existential and universal quantifiers.

(1) Write the Intermediate Value Theorem in mathematical language.

(2) Write the Archimedean Property in mathematical language.

(3) Write the fact that $\mathbb{Q}$ is dense in $\mathbb{R}$ in mathematical language.

(4) Give the negation of the statement: If $a > 0$, then $ax^2 + bx + c = 0$ has no real-valued solutions whenever $c > 0$.

(5) Prove by contrapositive: If for all $\varepsilon > 0$, $c < d + \varepsilon$, then $c \leq d$. [Hint: Negate the following statements: (i) For all $\varepsilon > 0$, $c < d + \varepsilon$. (ii) $c < d + \varepsilon$.]

2   Logical Statements

(6) All shibes are doggos. No doggos are cades. Determine whether the following statements are **true**, **false**, or **there is not enough information**.

    (a) "I am shibe or I am cade."

    (b) "If I am shibe, then I am cade."

    (c) "I am shibe and I am cade."

(7) Cade says: "If it fits, I sits". Cade is wrong. Does it fit?

(8) The shepherd said "If the sheep are not restless then they'll be grazing normally, and if they are grazing normally then they'll be around the lake right now!" He looked at the lake but there were no sheep there. Are the sheep restless?

(9) Bob said, "If I study and pass, then passing implies that I haven't studied." Bob was wrong. Did he pass?

(10) Captain Morgan is plotting to win the race. He says "If I sabotage my opponents rigging and my opponents don't sabotage my rigging then I'll win the race, if and only if my crew stays awake." If the Captain was correct in his thinking and his crew did not stay awake, find out if the Captain won the race.

(11) Jane will win Trivial Pursuit or Andy doesn't cheat, but not both, and either Andy cheats or Cathy wins. If Andy cheats, then Jane will sulk. Jane is not sulking. Who won?

3 Casework

(12) Find all $x \in \mathbb{R}$ satisfying $|2x + 3| = 3 - x$.

(13) Prove that for all $a \in \mathbb{Z}$, $2 \cdot \left| a - \frac{3}{2} \right| \in \mathbb{N}$

(14) Prove that for all $x \in \mathbb{R}$, we have $|x + 5| + |x - 2| \geq 7$.

(15) Prove *Nesbitt's Inequality*. It states that in an ordered field (F, P), if $a, b, c \in P$,

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$

[Hint: Let $x = \frac{a}{b+c}, y = \frac{b}{c+a}, z = \frac{c}{a+b}$, and show that $xy + yz + zx + 2xyz = 1$.]

Also, have you considered other ways customization can make your clients' lives easier? We've configured rollout trash and recycling containers, drops for sharps, electrical connection holes and grommets and under-cabinet lighting. None are difficult or especially costly, but all add that high-end feel to a space that can make all the difference for day-to-day users.

Lastly, think before you drawer! (We're making this a catchphrase, we swear.) Everyone puts in more drawers than they really need. Not only are they pricey, they're also often the most unused of spaces.

In our experience, deep and wide drawers are more useful to people than lots of small stacks – and they cost about the same. Ergonomically, base cabinets cause people to have to reach inside, and therefore are sometimes less useful, but they can also save you money if you're trying to meet a budget. In small spaces, they often work better, too. And remember, you can never place a drawer under a sink – it simply doesn't work!

Where are your connections? Leave space for electrical, plumbing and HVAC. Outlets need to be high enough that they clear the splash (or are completely within a tall splash). Relocating wall fixtures is an expensive and time consuming casework planning mistake to deal with later.

Shorten utility cabinets so that they don't interfere with sprinklers. Most states require 18 inches of clearance from the bottom of the sprinkler head to the top of shelving or furniture. Forgetting this detail can result in costly construction changes later.

Do your cabinets align vertically? Leading lines are not just for horizontal use – making sure the wall and base cabinets are the same width adds consistency to your design. Done well, no one notices this detail. If it's off, though, the comments seemingly never end.

Did you field measure? It's important in both new construction and renovations. The last thing you want is to have designed a great bank of 24-inch cabinets and when you go to install them, see that the wall is actually 21 inches deep. Now your cabinets stick out three inches past the wall. (Probably not the best look; and yes, we have seen it happen.) As you know, no room is perfect, no corner is 100-percent square. It's best to always measure and verify.

Make sure you plan for door clearances, too. In one installation we visited, several cabinets that swung towards the wall, had no filler srip or a very small filler, so the door could not open to 90 degrees before the handle hit the wall. Another of the 5 common casework planning mistakes is that the space isn't measured beforehand, so selected products and hardware don't fit properly.

Did you forget about the sink?

People always go to the biggest sink first, but we're here to say, resist the temptation. Why? Well, just because you have a 30-inch cabinet doesn't mean you have room for a 30-inch sink. The measurement is the outside measurement of the cabinet.

Taking Shield's cabinets as an example, we have half-inch thick verticals on each side of the cabinet, which leaves an inside dimension of 29 inches available for a sink. This logic applies to countertops, too – you can't have a 25-inch deep countertop with a cove finish and 24-inch sink set within. There's simply not enough room.

And then we get to faucets: even if you can fit a large sink, it doesn't mean you can fit the hardware you want with it. We have some horror stories of too-large sinks that didn't leave enough room for faucets. Some can be turned sideways (see the photo), but some simply have to be returned or exchanged for a spout model, or even a smaller sink.