

More Linear Algebra Problems

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1. Let S and K be $n \times n$ matrices. If S is symmetric and K is skew-symmetric, then SK is _____.
(Prove your answer.)
2. If the sum of two unit vectors is a unit vector, then what is the angle between them?
3. (a) Find all 1×1 orthogonal matrices.
(b) Show that all 2×2 orthogonal matrices are either a rotation (of the form $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$)
or a reflection (of the form $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$).
4. Consider the 3×3 matrix $A = \begin{bmatrix} 3 & 0 & 2 \\ 4 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.
 - (a) Find 3×3 matrices Q and R such that Q is orthogonal, A is upper-triangular, and $A = QR$.
 - (b) Find 3×3 matrices \mathcal{O} and P such that \mathcal{O} is orthogonal, P is lower-triangular, and $A = P\mathcal{O}$.
 - (c) What is $Q\mathcal{O}$?
 - (d) What is PR ?