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The Revenge of the Lumpy Potatoes

"There is no math without courage."

-Bar Roytman

The sky is a livid green. Four in the afternoon, the cloud cover makes it look like eight-thirty. It's just started to rain, but I'm walking quickly through the Diag, taking long strides, and I reach my dorm a few minutes before it starts really pouring. Taking off my backpack, I open the front pocket and slide out a book: Munkres' *Analysis on Manifolds.* It's the textbook for my class, Math 395.

395 is a second year analysis course taken mostly by sophomores who, like me, are majoring in mathematics. The professor is David Barrett, a short, burly, energetic complex analyst with a commanding lecture style and a knack for writing good problem sets. This week we're learning about manifolds, mathematical objects which generalize the notion of a curved surface to higher dimensions. Eventually we'll do calculus on them, so I need to make sure the definition is second-nature. I flop down on my bed and open the orange-and-white cover of *Munkres.*

I've just started to read when I notice something on the floor by the window: it's a potato–large, round, and covered in lumps. I sit up on the bed and stare at it. Rising slowly, I cross the room and bend down to pick it up. It's exactly what it seems–a lumpy brown potato, slightly heavy and cool in my open hand.

Feeling a little foolish, I glance around the room wondering where it could have come from. Rain drums against the closed window. Everything is much darker now–storm-clouds rumble in the inky sky outside. My eye catches the mirror, and I reel back: in the reflection, the object I'm holding is not a potato, but a miniature human head with a pudgy bearded face. Professor Barrett's eyes peer out at me from the mirror as he laughs, "The function isn't necessarily orientation-preserving!" I drop the potato and step back again, but trip on the rug, which has somehow become a huge rubber möbius band wrapping around the room. As I stumble backwards, its stretchy, non-orientable surface slides out from under me. There is a flash of lightning, and I fall against the mirror, which gives way under the weight of my shoulder. I topple, slip through the mirror, and then I'm falling through space…

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At some point, I realize that the space I'm in is no longer three-dimensional. I'm a single point zipping along a curve, my position given by a continuously differentiable function of time. I hear the whistling sound of compact supports going by, and all around me I see wavy iridescent forms filled with multicolored light.

The scene changes, and I'm walking through a convex open set. In the distance, a grad student paces up and down. As I approach, he turns to me, and I recognize his face.

“Bar!”

I call out.

“What are you doing here? I thought you were at UCLA.”

Without saying a word, he draws nearer, and I notice the dark circles under his perpetually bleary eyes. They look like the eyes of a much older man. As I look into them, I hear his voice, but it seems to come from all around me, or else from inside my own head. His lips don't move.

“Hello, Noah. Your essay's already over five hundred words, so it's probably time for a thesis statement by now.”

He pauses, then goes on,

“Anyway, visualization is very important in mathematics. That's one of the crucial things you learn as you get further into the subject. You can memorize definitions all you want, but to prove any sort of theorem, you're going to need some intuition about what's going on. Visual intuition is nice, and a picture almost always helps.

“Sometimes once you're able to picture something in your head, you see right away how to prove something about it. For example, in single-variable calculus, the statement of the Mean Value Theorem makes not a lot of sense the first time you read it, written out in symbols. But if you just draw the picture with the tangent line and the two endpoints, it becomes much clearer why it should be true. Then, when you go back to the symbols and work out a proof, you always refer to your picture, because it's the only way you get a complete idea of what's happening.

“But that doesn't mean math is easy. Visualization is the single hardest part of what we do. It can take years for the brightest mathematicians in the world to translate something into a picture that helps at all with a proof, and it often takes a huge amount of background work to make the translation rigorous enough to use. Visualization is as difficult as it is essential. Getting good at it will take everything you've got; but without it, you won't get anywhere.”

He pauses again, and I'm just about to ask him a question when he continues,

“I know you're going to ask me about manifolds. Well, you have the definition: a manifold is a subset of n-dimensional space which is locally diffeomorphic to an open subset of k-dimensional space. You can make that more precise, which is helpful, but it can also be helpful to make it much less precise. A manifold is basically a lumpy potato.”

With that, Bar finishes speaking and drowsily fades away. His eyes remain, hovering in space, and grow bigger and bigger until they fill half the sky. Then they fade too, and behind them I can see higher dimensional manifolds: 3-manifolds in 5-space, 4-manifolds in 7-space, on and on, and they're all just lumpy potatoes. Higher and higher dimensions, until, in the limit, I see an infinite-dimensional manifold burning bright with the light of a thousand suns.

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“Noah, Noah!”

Someone's shaking me awake. It's my roommate, Andrew Gitlin. The sunlight is streaming in through the window.

“It's morning, time to go to class. Did you fall asleep reading *Munkres*?”

“I guess so,” I answer,

“oh, and I think I figured out how to do problem five. You just have to draw the picture.” □