Evaluating the strength of identification in DSGE models. An a priori approach*

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Abstract

This paper presents a new approach to parameter identification analysis in DSGE models wherein the strength of identification is treated as property of the underlying model and studied prior to estimation. The strength of identification reflects the empirical importance of the economic features represented by the parameters. Identification problems arise when some parameters are either nearly irrelevant or nearly redundant with respect to the aspects of reality the model is designed to explain. The strength of identification therefore is not only crucial for the estimation of models, but also has important implications for model development. The proposed measure of identification strength is based on the Fisher information matrix of DSGE models and depends on three factors: the parameter values, the set of observed variables and the sample size. By applying the proposed methodology, researchers can determine the effect of each factor on the strength of identification of individual parameters, and study how it is related to structural and statistical characteristics of the economic model. The methodology is illustrated using the medium-scale DSGE model estimated in Smets and Wouters (2007).

Keywords: DSGE models, Identification, Information matrix, Cramér-Rao lower bound

JEL classification: C32, C51, C52, E32

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1 Introduction

There is a considerable consensus among academic economists and economic policy makers that modern macroeconomic models are rich enough to be useful as tools for policy analysis. It is also well understood that when structural models are used for quantitative analysis, it is crucial to use parameter values that are empirically relevant. The best way of obtaining such values is to estimate and evaluate the models in a formal and internally consistent manner. This is what the new empirical dynamic stochastic general equilibrium (DSGE) literature attempts to do.

The estimation of DSGE models exploits the restrictions they impose on the joint probability distribution of observed macroeconomic variables. A fundamental question that arises is whether these restrictions are sufficient to allow reliable estimation of the model parameters. This is known as the identification problem in econometrics, and to answer it econometricians study the relationship between the true probability distribution of the data and the parameters of the underlying economic model (Koopmans (1949)). Such identification analysis should precede the statistical estimation of economic models (Manski (1995)).

Although the importance of parameter identification has been recognized, the issue is rarely discussed when DSGE are estimated. Examples of models with unidentifiable parameters can be found in Kim (2003), Beyer and Farmer (2004) and Cochrane (2007). That DSGE models may be poorly identified has been pointed out by Sargent (1976) and Pesaran (1989). More recently, Canova and Sala (2009) summarize their study of identification issues in DSGE models with the conclusion: "it appears that a large class of popular DSGE structures can only be weakly identified".

Most of the existing research on identification in DSGE models follows the econometric literature where weak identification is treated as a sampling problem, i.e. as an issue within the realm of statistical inference (see e.g. Stock and Yogo (2005) and the survey in Andrews and Stock (2005)). For this reason the effort has been devoted to either devising tests for detecting weak identification (Inoue and Rossi (2008)), or to developing methods for inference that are robust to identification problems (Guerron-Quintana et al. (2009)). This paper proposes an alternative approach, based on the premise that identification in DSGE models can be treated as a property of the underlying economic model, and, as such, may be studied without reference to a particular sample of data. This approach is in the spirit of the classical literature on identification, and is based on the fact that DSGE models provide a complete characterization of the data generating process. Thus, any identification problem in the data must have its origin in the underlying model. This is different from the typical situation in structural econometrics, where the mapping from the economic model to the data is known only partially. For instance, the degree of correlation between instruments and endogenous variables in the simple linear instrumental variables model depends on nuisance parameters, which, in the absence of a fully-articulated economic model, have no structural interpretation. In a general equilibrium setting all reduced-form parameters become functions of structural parameters and one can investigate how the instruments' strength is determined by the properties of the underlying model.

In the context of DSGE models important identification-related question include: (1) which model parameters are identified and which are not; (2) how well identified are the identifiable parameters; (3) if the identification of some parameters fails or is weak, is this due to data limitations, or is it intrinsic to the structure of the model; (4) what structural or statistical properties of the model are most important determinants of the strength of identification of the parameters; (5) how the answers to (1)-(4) vary across different regions in the parameter space and for different sets of observables. The purpose of this paper is to show how answers to questions like these can be obtained for any linearized DSGE model.

A central tool in the proposed approach is the expected Fisher information matrix, the use of which for identification analysis was first suggested by Rothenberg (1971). As Rothenberg points out, the information matrix "is a measure of the amount of information about the unknown parameters available in the sample". Information deficiency results in identification problems and is associated with singular or nearly-singular information matrix. In addition to the purely statistical dimension of these problems there is also an economic modelling aspect, which is often far more important. Parameters are unidentifiable or weakly identified if the economic features they represent have no empirical relevance at all, or very little of it. This may occur either because those features are unimportant on their own, or because they are redundant given the other features represented in the model. These issues are particularly relevant to DSGE models, which are sometimes criticized of being too rich in features, and possibly overparameterized (Chari et al. (2009)). This paper shows how one can distinguish between the statistical and economic modelling aspects of identification problems, and provides tools for determining the causes leading to them.

Papers related to this one are Iskrev (2010) and Komunjer and Ng (2009), which consider the parameter identifiability question, and Canova and Sala (2009), which is focused on the weak identification problem. Iskrev (2010) presents an identifiability condition that is easier to use and more general than the one developed here. The condition is based on the Jacobian matrix of the mapping from theoretical first and second order moments of the observable variables to the deep parameters of the model. The condition is necessary and sufficient for identification with likelihood-based methods under normality, or with limited information methods that utilize only first and second order moments of the data. However, that paper does not deal with the weak identification issue, which is the main theme of this paper. Komunjer and Ng (2009) derive a similar rank condition for identification using the spectral density matrix. The paper of Canova and Sala (2009) was the first one to draw attention to the problem of weak identification in DSGE models, and to discuss different strategies for detecting it. Those include: one and two dimensional plots of the estimation objective function, estimation with simulated data, and checking numerically the conditioning of matrices characterizing the mapping from parameters to the objective function. The paper of Canova and Sala (2009) differs form the present paper in several ways. First, they approach parameter identification from the perspective of a particular limited information estimation method, namely, equally weighted impulse response matching. In addition to the model and data deficiencies discussed above, weak identification in that setting may be caused by the failure to use some model-implied restrictions on the distribution of the data, and by the inefficient weighing of the utilized restrictions. Consequently, it may be very difficult to disentangle the causes and quantify their separate contribution to the identification problems. Second, it is very common in DSGE models to have identification problems that stem from a near observational equivalence involving a large number of parameters. This means that the objective function is flat with respect to all of the parameters as a group. The plots used in Canova and Sala (2009) are limited to only two parameters at a time, and it is far from straightforward to select the appropriate pairs from a large number of free parameters. Third, Canova and Sala (2009) do not discuss the role of the set of observables for identification. The effect of using different observables for the estimation of a DSGE model is investigated in Guerron-Quintana (2010), who finds that the parameter estimates and the economic and forecasting implications of the model vary substantially with the choice of included variables. The last and perhaps most important difference is in the approach itself. While it is possible in principle to address all identification questions discussed here by conducting Monte Carlo simulations, this is hardly a viable strategy for an a priori identification analysis of most DSGE models. Estimating a multidimensional and highly non-linear model even once is a numerically challenging and time consuming exercise. Doing that many times and for a large number of parameter values is completely impractical. In contrast, the tools used in this paper are simple, easy to apply, and general.

The remainder of the paper is organized as follows. Section 2 introduces the class of linearized DSGE models, and outlines the derivation of the log-likelihood function and the Fisher information matrix for Gaussian models. Section 3 explains the role of the Fisher information matrix in the analysis of identification, and discusses different aspects of the a priori approach to identification. The methodology is illustrated, in Section 4, with the help of the medium-scale DSGE model estimated in Smets and Wouters (2007). Section 5 discusses an a priori analysis of identification strength in a Bayesian setting. Concluding comments are given in Section 6.

2 Preliminaries

This section provides a brief discussion of the class of linearized DSGE models and the derivation of the log-likelihood function and the Fisher information matrix for Gaussian models

2.1 Setup

A DSGE model is summarized by a system of non-linear equations. Currently, most studies involving either simulation or estimation of DSGE models use linear approximations of the original models. That is, the model is first expressed in terms of stationary variables, and then linearized around the steady-state values of these variables. Let $\hat{\boldsymbol{z}}_t$ be a $m-$dimensional vector of the stationary variables, and $\hat{\boldsymbol{z}}^*$ be the steady state value of $\hat{\boldsymbol{z}}_t$. Once linearized, most DSGE models can be written in the following form

$$\boldsymbol{\Gamma}_0(\boldsymbol{\theta})\boldsymbol{z}_t = \boldsymbol{\Gamma}_1(\boldsymbol{\theta})\,\mathrm{E}_t\,\boldsymbol{z}_{t+1} + \boldsymbol{\Gamma}_2(\boldsymbol{\theta})\boldsymbol{z}_{t-1} + \boldsymbol{\Gamma}_3(\boldsymbol{\theta})\boldsymbol{\epsilon}_t \tag{2.1}$$

where $\boldsymbol{z}_t = \hat{\boldsymbol{z}}_t - \hat{\boldsymbol{z}}^*$, and the structural shocks $\boldsymbol{\epsilon}_t$ are independent and identically distributed n-dimensional random vectors with $\mathrm{E}\,\boldsymbol{\epsilon}_t = \mathbf{0}$, $\mathrm{E}\,\boldsymbol{\epsilon}_t\boldsymbol{\epsilon}_t' = I_n$. The elements of the matrices $\boldsymbol{\Gamma}_0$, $\boldsymbol{\Gamma}_1$, $\boldsymbol{\Gamma}_2$ and $\boldsymbol{\Gamma}_3$ are functions of a $k-$dimensional vector of deep parameters

$\boldsymbol{\theta}$, where $\boldsymbol{\theta}$ is a point in $\boldsymbol{\Theta} \subset \mathbb{R}^k$. The parameter space $\boldsymbol{\Theta}$ is defined as the set of all theoretically admissible values of $\boldsymbol{\theta}$.

There are several algorithms for solving linear rational expectations models (see for instance Blanchard and Kahn (1980), Anderson and Moore (1985), King and Watson (1998), Klein (2000), Christiano (2002), Sims (2002)). Depending on the value of $\boldsymbol{\theta}$, there may exist zero, one, or many stable solutions. Assuming that a unique solution exists, it can be cast in the following form

$$\boldsymbol{z}_t = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{z}_{t-1} + \boldsymbol{B}(\boldsymbol{\theta})\boldsymbol{\epsilon}_t \tag{2.2}$$

where the $m \times m$ matrix $\boldsymbol{A}$ and the $m \times n$ matrix $\boldsymbol{B}$ are unique for each value of $\boldsymbol{\theta}$.

In most applications the model in (2.2) cannot be taken to the data directly since some of the variables in $\boldsymbol{z}_t$ are not observed. Instead, the solution of the DSGE model is expressed in a state space form, with transition equation given by (2.2), and a measurement equation

$$\boldsymbol{x}_t = \boldsymbol{C}(\boldsymbol{\theta})\boldsymbol{z}_t + \boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{u}_t + \boldsymbol{\nu}_t \tag{2.3}$$

where $\boldsymbol{x}_t$ is a l-dimensional vector of observed variables, $\boldsymbol{u}_t$ is a q-dimensional vector of exogenous variables, and $\boldsymbol{\nu}_t$ is a p-dimensional random vectors with $\mathrm{E}\,\boldsymbol{\nu}_t = \boldsymbol{0}$, $\mathrm{E}\,\boldsymbol{\nu}_t\boldsymbol{\nu}_t' = \boldsymbol{Q}$, where $\boldsymbol{Q}(\boldsymbol{\theta})$ is $p \times p$ symmetric semi-positive definite matrix.

For a given value of $\boldsymbol{\theta}$, the matrices $\boldsymbol{A}$, $\boldsymbol{\Omega} := \boldsymbol{B}\boldsymbol{B}'$, and $\hat{\boldsymbol{z}}^*$ completely characterize the equilibrium dynamics and steady state properties of all endogenous variables in the linearized model. Typically, some elements of these matrices are constant, i.e. independent of $\boldsymbol{\theta}$. For instance, if the steady state of some variables is zero, the corresponding elements of $\hat{\boldsymbol{z}}^*$ will be zero as well. Furthermore, if there are exogenous autoregressive (AR) shocks in the model, the matrix $\boldsymbol{A}$ will have rows composed of zeros and the AR coefficients. As a practical matter, it is useful to separate the solution parameters that depend on $\boldsymbol{\theta}$ from those that do not. I will use $\boldsymbol{\tau}$ to denote the vector collecting the non-constant elements of $\hat{\boldsymbol{z}}^*$, $\boldsymbol{A}$, and $\boldsymbol{\Omega}$, i.e. $\boldsymbol{\tau} := [\boldsymbol{\tau}_z', \ \boldsymbol{\tau}_A', \ \boldsymbol{\tau}_\Omega']'$, where $\boldsymbol{\tau}_z$, $\boldsymbol{\tau}_A$, and $\boldsymbol{\tau}_\Omega$ denote the elements of $\hat{\boldsymbol{z}}^*$, $\mathrm{vec}(\boldsymbol{A})$ and $\mathrm{vech}(\boldsymbol{\Omega})$ that depend on $\boldsymbol{\theta}$.

2.2 Log-likelihood function and the information matrix

The log-likelihood function of the data $\boldsymbol{X} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{\mathrm{T}}]$ can be derived using the prediction error method whereby a sequence of one-step ahead prediction errors $\boldsymbol{e}_{t|t-1} = \boldsymbol{x}_t - \boldsymbol{C}\hat{\boldsymbol{z}}_{t|t-1} - \boldsymbol{D}\boldsymbol{u}_t$ is constructed by applying the Kalman filter to the obtain one-step ahead forecasts of the state vector $\hat{\boldsymbol{z}}_{t|t-1}$. Assuming that the structural shocks $\boldsymbol{\epsilon}_t$ are jointly Gaussian, it follows that the conditional distribution of $\boldsymbol{e}_{t|t-1}$ is also Gaussian with zero mean and covariance matrix given by $\boldsymbol{S}_{t|t-1} = \boldsymbol{C}\boldsymbol{P}_{t|t-1}\boldsymbol{C}'$, where $\boldsymbol{P}_{t|t-1} = \mathrm{E}\left(\boldsymbol{z}_t - \hat{\boldsymbol{z}}_{t|t-1}\right)\left(\boldsymbol{z}_t - \hat{\boldsymbol{z}}_{t|t-1}\right)'$ is the conditional covariance matrix of the one-step ahead forecast, and is also obtained from the Kalman filter recursion. This implies that the log-likelihood function of the sample is given by

$$\ell_T(\boldsymbol{\theta}) = \text{const.} - \frac{1}{2}\sum_{t=1}^{\mathrm{T}} \log\det(\boldsymbol{S}_{t|t-1}) - \frac{1}{2}\sum_{t=1}^{\mathrm{T}} \boldsymbol{e}_{t|t-1}^{'}\boldsymbol{S}_{t|t-1}^{-1}\boldsymbol{e}_{t|t-1} \tag{2.4}$$

The ML estimator $\hat{\boldsymbol{\theta}}_T$ is the value of $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ which maximizes (2.4). As I will discuss in Section 3.1, the precision of $\hat{\boldsymbol{\theta}}_T$ is determined by the inverse of the Fisher information matrix, defined as

$$\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta}) := \mathrm{E}\left[\left\{\frac{\partial \ell_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right\}'\left\{\frac{\partial \ell_T(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right\}\right] \tag{2.5}$$

The next result, due to Klein and Neudecker (2000), provides an explicit expression for the Fisher information matrix for Gaussian models.

Theorem 1. *The expected Fisher information matrix is given by*

$$\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta}) = \sum_{t=1}^{T} \mathrm{E}\left[\left(\frac{\partial \boldsymbol{e}_{t|t-1}}{\partial \boldsymbol{\theta}'}\right)'\boldsymbol{S}_t^{-1}\left(\frac{\partial \boldsymbol{e}_{t|t-1}}{\partial \boldsymbol{\theta}'}\right)\right] + \frac{1}{2}\sum_{t=1}^{T}\left(\frac{\partial \mathrm{vec}(\boldsymbol{S}_t)}{\partial \boldsymbol{\theta}'}\right)'(\boldsymbol{S}_t \otimes \boldsymbol{S}_t)^{-1}\left(\frac{\partial \mathrm{vec}(\boldsymbol{S}_t)}{\partial \boldsymbol{\theta}'}\right) \tag{2.6}$$

The asymptotic information matrix, defined as the limit of (2.6), can be computed using the following result (see Ljung (1999))

Theorem 2. *Let* $\boldsymbol{S}_\infty = \boldsymbol{C}\boldsymbol{P}_\infty\boldsymbol{C}'$*, where* $\boldsymbol{P}_\infty = \lim_{\mathrm{T}\to\infty} \boldsymbol{P}_{t|t-1}$ *is the steady state covariance* *matrix of the one-step ahead forecast vector* $\hat{\boldsymbol{z}}_{t|t-1}$. *Then*

$$\boldsymbol{\mathcal{I}}(\boldsymbol{\theta}) = \mathrm{E}\left[\left(\frac{\partial \boldsymbol{e}_{t|t-1}}{\partial \boldsymbol{\theta}'}\right)' \boldsymbol{S}_{\infty}^{-1}\left(\frac{\partial \boldsymbol{e}_{t|t-1}}{\partial \boldsymbol{\theta}'}\right)\right] + \frac{1}{2}\left(\frac{\partial \mathrm{vec}(\boldsymbol{S}_{\infty})}{\partial \boldsymbol{\theta}'}\right)' (\boldsymbol{S}_{\infty} \otimes \boldsymbol{S}_{\infty})^{-1} \left(\frac{\partial \mathrm{vec}(\boldsymbol{S}_{\infty})}{\partial \boldsymbol{\theta}'}\right) \tag{2.7}$$

To evaluate either (2.6) or (2.7), one needs the derivatives of the reduced-form matrices $\boldsymbol{A}$, $\boldsymbol{\Omega}$ and $\boldsymbol{C}$ with respect to $\boldsymbol{\theta}$. Explicit formulas for computing these derivatives can be found in Iskrev (2010). Therefore, the full information matrix and all measures of identification strength discussed below can be evaluated analytically.

Since the Gaussian assumption is sometimes difficult to justify, it is important to understand the role it plays here. It has two important consequences. First, the likelihood function involves only first and second-order moments of the observables. Therefore, for an efficient estimation of the parameters it is sufficient to use the model-implied restrictions on these moments only. Second, the Gaussian assumption facilitates the computation of the optimal weights one should place on the restrictions to achieve efficiency. To see this, note that the ML estimator can be interpreted as a generalized method of moments (GMM) estimator, where the moment function is given my the difference between the vector of theoretical first and second order moments and the vector of their sample counterparts. The optimal weighting matrix, given by the inverse of the covariance matrix of the moment function, is not available in closed-form unless Gaussianity is assumed. It can be shown that the inverse of the information matrix (2.7) is smaller than the asymptotic covariance matrix of an efficient GMM estimator for a general distribution. Thus, the measures of information strength computed using the information matrix provide an upper bound on the strength of identification for general estimation methods that utilize only first and second moments.

3 Identification Analysis

3.1 General principles

Let a model be parameterized in terms of a vector $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbb{R}^k$, and suppose that inference about $\boldsymbol{\theta}$ is made on the basis of T observations of a random vector $\boldsymbol{x}$ with a known joint probability density function $p(\boldsymbol{X};\boldsymbol{\theta})$, where $\boldsymbol{X} = [\boldsymbol{x}'_1, \dots, \boldsymbol{x}'_{\mathrm{T}}]'$. When considered as a function of $\boldsymbol{\theta}$, $p(\boldsymbol{X};\boldsymbol{\theta})$ contains all available sample information about the value of $\boldsymbol{\theta}$ associated with the observed data. Thus, a basic prerequisite for making inference about $\boldsymbol{\theta}$ is that distinct values of $\boldsymbol{\theta}$ imply distinct values of the density function. Formally, we say that a point $\boldsymbol{\theta}^o \in \boldsymbol{\Theta}$ is identified if

$$p(\boldsymbol{X};\boldsymbol{\theta}) = p(\boldsymbol{X};\boldsymbol{\theta}^o) \text{ with probability } 1 \Rightarrow \boldsymbol{\theta} = \boldsymbol{\theta}^o \tag{3.1}$$

This definition is made operational by using the following property of the log-likelihood function $\ell_T(\boldsymbol{\theta}) := \log p(\boldsymbol{X};\boldsymbol{\theta})$

$$\mathrm{E}_0\, \ell_T(\boldsymbol{\theta}^o) \geq \mathrm{E}_0\, \ell_T(\boldsymbol{\theta}), \text{ for any } \boldsymbol{\theta} \tag{3.2}$$

This follows from the Jensen's inequality (see Rao (1973)) and the fact that the logarithm is a concave function. It further implies that the function $H(\boldsymbol{\theta}^o,\boldsymbol{\theta}) := \mathrm{E}_0 \left(\ell_{\mathrm{T}}(\boldsymbol{\theta}) - \ell_T(\boldsymbol{\theta}^o)\right)$ achieves a maximum at $\boldsymbol{\theta} = \boldsymbol{\theta}^o$, and $\boldsymbol{\theta}^o$ is identified if and only if that maximum is unique. While conditions for global uniqueness are difficult to find in general, local uniqueness of the maximum at $\boldsymbol{\theta}^o$ may be established by verifying the usual first and second order conditions, namely: (a) $\frac{\partial H(\boldsymbol{\theta}^o,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}|_{\boldsymbol{\theta}=\boldsymbol{\theta}^o} = \mathbf{0}$, (b) $\frac{\partial^2 H(\boldsymbol{\theta}^o,\boldsymbol{\theta})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}'}|_{\boldsymbol{\theta}=\boldsymbol{\theta}^o}$ is negative definite. If the maximum at $\boldsymbol{\theta}^o$ is locally unique we say that $\boldsymbol{\theta}^o$ is locally identified. This means that there exists an open neighborhood of $\boldsymbol{\theta}^o$ where (3.1) holds for all $\boldsymbol{\theta}$. Global identification, on the other hand, extends the uniqueness of $p(\boldsymbol{X};\boldsymbol{\theta}^o)$ to the whole parameter space. One can show that (see Bowden (1973)) the condition in (a) is always true, and the Hessian matrix in (b) is equal to the negative of the Fisher information matrix. Thus, we have the following result of Rothenberg (1971),

Theorem 3. *Let $\boldsymbol{\theta}^o$ be a regular point of the information matrix $\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta})$ Then $\boldsymbol{\theta}^o$ is locally identifiable if and only if $\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta}^o)$ is non-singular.*

A point is called regular if it belongs to an open neighborhood where the rank of the matrix does not change. Without this assumption the condition is only sufficient for local identification. Although it is possible to construct examples where regularity does not hold (see Shapiro and Browne (1983)), typically the set of irregular points is of measure zero (see Bekker and Pollock (1986)). Thus, for most models the non-singularity of the information matrix is both necessary and sufficient for local identification. By definition, a model is (locally) identified if all points in the parameter space are (locally) identified. This can be checked by examining the rank of the information matrix at all points in

$\boldsymbol{\Theta}$.

Verifying that the model is identified, at least locally, is important since identifiability is a prerequisite for the consistent estimation of the parameters. Singularity of the information matrix means that likelihood function is flat at $\boldsymbol{\theta}^o$ and one has no hope of finding the true values of some of the parameters even with an infinite number of observations. Intuitively, this may occur for one of two reasons: either some parameters do not affect the likelihood at all, or different parameters have the same effect on the likelihood. This reasoning may be formalized by using the fact that the information matrix is equal to the covariance matrix of the scores, and therefore can be expressed as

$$\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta}^o) = \boldsymbol{\Delta}^{\frac{1}{2}} \tilde{\boldsymbol{\mathcal{I}}}_T(\boldsymbol{\theta}^o) \boldsymbol{\Delta}^{\frac{1}{2}} \tag{3.3}$$

where $\boldsymbol{\Delta} = \mathrm{diag}(\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta}^o))$ is a diagonal matrix containing the variances of the elements of the score vector, and $\tilde{\boldsymbol{\mathcal{I}}}_T(\boldsymbol{\theta}^o)$ is the correlation matrix of the score vector.

Hence, a parameter θ_i is locally unidentifiable if:

(a) Changing θ_i does not change the likelihood, i.e.

$$\Delta_i := \mathrm{E}\left(\frac{\partial \ell_T(\boldsymbol{\theta}^o)}{\partial \theta_i}\right)^2 = -\,\mathrm{E}\left(\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \theta_i^2}\right) = 0 \tag{3.4}$$

(b) The effect on the likelihood of changing θ_i can be offset by changing other parameters in $\boldsymbol{\theta}$, i.e.

$$\boldsymbol{\varrho}_i := \mathrm{corr}\left(\frac{\partial \ell_T(\boldsymbol{\theta}^o)}{\partial \theta_i}, \frac{\partial \ell_T(\boldsymbol{\theta}^o)}{\partial \boldsymbol{\theta}_{-i}}\right) = 1 \tag{3.5}$$

where $\frac{\partial \ell_T(\boldsymbol{\theta}^o)}{\partial \boldsymbol{\theta}_{-i}}$ is the partial derivative of the log-likelihood with respect to $\boldsymbol{\theta}_{-i} := [\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_k]$.

Both cases result in a flat likelihood function and lack of identification for one or more parameters. Weak identification, on the other hand, arises when the likelihood is not completely flat but exhibits very low curvature with respect to some parameters. The issue of detecting and measuring weak identification problems is discussed next.

3.2 Identification strength in the population

The rank condition ensures that the expected log-likelihood function is not flat and achieves a locally unique maximum at the true value of $\boldsymbol{\theta}$. In general this suffices for the consistent estimation of $\boldsymbol{\theta}$. However, the precision with which $\boldsymbol{\theta}$ may be estimated in finite samples depends on the degree of curvature of log-likelihood surface in the neighborhood of $\boldsymbol{\theta}^o$, of which the rank condition provides no information. Nearly flat likelihood means that small changes in the value of $\ell_T(\boldsymbol{\theta})$, due to random variations in the sample, result in relatively large changes in the value of $\boldsymbol{\theta}$ that maximizes the observed likelihood function. In this situations parameter identification is said to be weak in the sense that the estimates are prone to be very inaccurate even when the number of observations is large.

Intuitively, a weakly identified parameter is one which is either nearly irrelevant, because it has only a negligible effect on the likelihood, or nearly redundant, because its effect on the likelihood may be closely approximated by other parameters. Consequently, the value of that parameter is difficult to pin down on the basis of the information contained in the likelihood function. Using the notation introduced earlier, the two causes for weak identification may be expressed as $\Delta_i \approx 0$ and $\boldsymbol{\varrho}_i \approx 1$. The overall effect is determined by the interaction of the two factors. The particular measure of identification strength adopted here is

$$\boldsymbol{s}_i(\boldsymbol{\theta}) := \sqrt{\Delta_i\,(1 - \boldsymbol{\varrho}_i^2)} \tag{3.6}$$

The motivation behind using this measure comes from the following result which shows how $\boldsymbol{s}_i(\boldsymbol{\theta})$ is related to the Fisher information matrix.

Theorem 4. *Suppose that the Fisher information matrix $\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta})$ is not singular when evaluated at $\boldsymbol{\theta}$. Then $1/\boldsymbol{s}_i(\boldsymbol{\theta})$ is equal to the square root of the i-th diagonal element of $\boldsymbol{\mathcal{I}}_T^{-1}(\boldsymbol{\theta})$.*

The proof of Theorem 4 uses the decomposition of $\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta})$ shown in (3.3) and the properties of the correlation matrix. The result implies that $\boldsymbol{s}_i(\boldsymbol{\theta})$ possesses a number of useful properties discussed next.

Corollary 1. *Let $\tilde{\boldsymbol{\theta}}_{\theta_i}$ be the value of $\boldsymbol{\theta}$ that maximizes $\ell(\boldsymbol{\theta})$ given the value of θ_i. Also,* *let* $\ell_p(\theta_i) = \ell(\tilde{\boldsymbol{\theta}}_{\theta_i})$ *be the profile log-likelihood of* θ_i*. Then*

$$\boldsymbol{s}_i^2(\boldsymbol{\theta}) = \mathrm{E}\left(\frac{\partial \ell_p(\theta_i)}{\partial \theta_i}\right)^2 = -\,\mathrm{E}\left(\frac{\partial^2 \ell_p(\theta_i)}{\partial \theta_i^2}\right)$$

Corollary 1 shows that $\boldsymbol{s}_i(\boldsymbol{\theta})$ is the expected value of the curvature of the profile log-likelihood of θ_i. Thus $\boldsymbol{s}_i(\boldsymbol{\theta})$ is analogous, in the multiparameter setting, to the Fisher information $(-\,\mathrm{E}(\partial^2\ell(\theta)/\partial\theta^2))$ in the single parameter case. Note that, unlike $\Delta_i = -\,\mathrm{E}\left(\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \theta_i^2}\right)$, which measures the sensitivity of $\ell(\boldsymbol{\theta})$ to θ_i keeping the other parameters fixed, $\boldsymbol{s}_i(\boldsymbol{\theta})$ gives the sensitivity of the log-likelihood to θ_i when the other parameters are free to adjust optimally. Clearly, the latter sensitivity cannot be larger than the former, but may be much smaller when $\boldsymbol{\theta}_{-i}$ could be varied so as to compensate most of the effect of changing θ_i. This occurs when $\boldsymbol{\varrho}_i$ is large and indicates that there is little independent information about θ_i in the likelihood.

Corollary 2. *If* $\hat{\boldsymbol{\theta}}$ *is an unbiased estimator of* $\boldsymbol{\theta}$*, and* $\mathrm{std}(\hat{\theta}_i) := \sqrt{\mathrm{E}(\hat{\theta}_i - \theta_i)^2}$*, then*

$$\mathrm{std}(\hat{\theta}_i) \geq \frac{1}{\boldsymbol{s}_i(\boldsymbol{\theta})} \tag{3.7}$$

This corollary is a direct consequence of the Cramér-Rao lower bound inequality, which states that the covariance matrix of any unbiased estimator of $\boldsymbol{\theta}$ is bounded from below by the inverse of the Fisher information matrix. It shows that the identification strength of a parameter can be expressed in terms of bounds on a one-standard-deviation intervals for unbiased estimators of the parameter. Such intervals may be easier to interpret and more informative than the value of $\boldsymbol{s}_i(\boldsymbol{\theta})$ alone.

It is worth emphasizing that $\boldsymbol{s}_i(\boldsymbol{\theta})$ is derived from the population objective function - the expected log-likelihood. It is solely based on the model-implied restrictions on the statistical properties of the observables, and measures how informative they are for the value of θ_i. Since no data is required, $\boldsymbol{s}_i(\boldsymbol{\theta})$ can be used to evaluate the strength of identification of the parameters prior to observing any data.

The a priori analysis of identification strength is different, though related to, the problem of quantifying the sampling uncertainty arising in estimation. Typically, in order to accurately characterize the uncertainty about an estimate, one has to take into account the full shape of the actual log-likelihood function. In contrast, as an a priori measure of identification strength, $\boldsymbol{s}_i(\boldsymbol{\theta})$ only utilizes the curvature of the expected log-likelihood. To see the relation between the two approaches, consider the construction of likelihood-based confidence interval for individual parameters. Define the signed root log-likelihood ratio $r(\theta_i) := \text{sign}(\theta_i - \hat{\theta}_i)\sqrt{2\left(\ell(\hat{\boldsymbol{\theta}}) - \ell_p(\theta_i)\right)}$, where $\ell_p(\theta_i)$ is the profile log-likelihood of θ_i. Then an approximate $1 - \alpha$ confidence interval for θ_i is the set of all values such that

$$- q_{\alpha/2} \leq r(\theta_i) \leq q_{\alpha/2} \tag{3.8}$$

where $q_{\alpha/2}$ is the $\alpha/2$ upper quantile of the standard normal distribution.
Figure 1 shows plots of $r(\theta_i)$ and the corresponding $1 - \alpha$ confidence intervals for cases where the identification of θ_i is stronger (panel (a)) or weaker (panel (b)). In addition to the likelihood-based confidence intervals, which are indicated with dashed lines, the figure shows confidence intervals constructed using a linear approximation of $r(\theta_i)$; these are marked with dotted lines. Note that the difference between the two plots is the slope of the curves, which is larger when identification is stronger. Not surprisingly, the slope is related to the curvature of the log-likelihood function as the next corollary shows

Corollary 3. *If maximum likelihood estimate $\hat{\boldsymbol{\theta}}$ is an interior point of $\boldsymbol{\Theta}$, then*

$$\boldsymbol{s}_i(\hat{\boldsymbol{\theta}}) = \mathrm{E}\left(\frac{\partial r(\hat{\theta}_i)}{\partial \theta_i}\right) \tag{3.9}$$

Thus, $\boldsymbol{s}_i(\hat{\boldsymbol{\theta}})$ gives the expected value of the slope of the tangent to $r(\theta_i)$ at the maximum likelihood estimate of θ_i, indicated with thin solid line in Figure 1. The difference between using only the curvature and using the full shape of the log-likelihood function is seen by comparing the two types of confidence intervals. Clearly, the bounds of the intervals may be quite different when $r(\theta_i)$ is far from linear, or, equivalently, when the likelihood function is far from quadratic. On the other hand, for reasonably smooth functions, the length of the two types of intervals would provide similar answers regarding the strength of identification. Thus, while it may be deficient as a basis for statistical inference, the curvature of the likelihood function should provide an accurate assessment of the identification strength of the parameters.

One thing that should be noted about Figure 1 is that the x-axes in the two plots are the same. In fact, this is what allows us to say that θ_1 is better identified than θ_2 in the figure; the opposite could be true if the scales of the two plots were different. In general we cannot compare the the values of $\boldsymbol{s}_i(\boldsymbol{\theta})$ because the parameters may be measured in very different units. Another way to see this is to note that $\boldsymbol{s}_i(\boldsymbol{\theta})$ may be interpreted in terms of the percentage change in the value of the profile likelihood for one unit change in θ_i. To obtain a scale-independent measure in terms of the elasticity of the profile likelihood, we have to multiply $\boldsymbol{s}_i(\boldsymbol{\theta})$ by the absolute value of θ_i. I will refer to this measure as the relative strength of identification of θ_i and denote it by $\boldsymbol{s}_i^r(\boldsymbol{\theta})$. Using the definition of $\boldsymbol{s}_i(\boldsymbol{\theta})$, we have

$$\boldsymbol{s}_i^r(\boldsymbol{\theta}) = \sqrt{\theta_i \Delta_i} \times \sqrt{1 - \boldsymbol{\varrho}_i^2} \tag{3.10}$$

Note that the first term is simply the elasticity of the likelihood function with respect to θ_i, holding the other parameters constant. I will refer to it as the *sensitivity component* of the measure. The second term captures the effect of allowing the other parameter to adjust and is determined by the degree of collinearity among the elements of the score vector. I will refer to it as the *collinearity component* of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$.

3.3 Discussion

Theorem 4 shows how to compute the measure of identification strength once we have the Fisher information matrix. The latter is straightforward to obtain as discussed in Section 2.2. Consider what is involved in the computation of $\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta})$ in (2.6) or $\boldsymbol{\mathcal{I}}(\boldsymbol{\theta})$ in (2.7). Taking the linearized structural model in (2.1) together with the assumption about the distribution of $\boldsymbol{u}$ as given, the Fisher information matrix depends on: (1) the true value of $\boldsymbol{\theta}$, (2) the set of observed variables in $\boldsymbol{x}$, and, in the case of (2.6), on (3) the number of observations T.

That identification is parameter-dependent is a property of all non-linear models, and implies that $\boldsymbol{\theta}$ may be identifiable in some regions of the parameter space, and unidentified in others. Similarly, identification may be strong in some regions and weak in others. Thus, in order to understand the identification of the parameters as a property of the model, one has to study the behavior of the information matrix at all theoretically plausible values, i.e. everywhere in $\boldsymbol{\Theta}$.

The set of observed variables may be considered as a part of the econometric model, and in that sense as given. The practice in the empirical DSGE literature, however, shows that it is to some extent a matter of choice how many and which macroeconomic variables to include in the estimation. The relevance of this for identification is that some parameters may be well identified if certain endogenous variables are included in $\boldsymbol{x}$, and poorly identified or unidentified if these variables are (treated as) unobserved.

Finally, the value of T enters directly in the computation of $\boldsymbol{\mathcal{I}}_T(\boldsymbol{\theta})$, and therefore may affect the rank of that matrix. Having more observations may help identify parameters which are otherwise unidentifiable. Naturally, the sample size also matters for the strength of identification of $\boldsymbol{\theta}$.

The effect on identification of having different sets of observables or samples of different sizes can be investigated by making the appropriate changes in $\boldsymbol{C}$, the matrix which selects the observed among all model variables (see equation (2.3)), or by varying the value of T. Fixing these two dimensions of the statistical model, one can study how identification varies with the value of $\boldsymbol{\theta}$ by evaluating the information matrix at all points in the parameter space. There are two problems with implementing this in practice. First, it is usually impossible to know, before solving the model, for which values of $\boldsymbol{\theta}$ there are either zero or many solutions. Such points are typically deemed as inadmissible, and have to be excluded from $\boldsymbol{\Theta}$. A second problem arises from the fact that there are infinitely many points in $\boldsymbol{\Theta}$, and it is not feasible to evaluate the information matrix at all of them. In view of these difficulties, one approach is to start by specifying a larger set $\boldsymbol{\Theta}'$, such that the parameter space $\boldsymbol{\Theta}$ is a subset of $\boldsymbol{\Theta}'$, and evaluate the information matrix at a large number of randomly drawn points from $\boldsymbol{\Theta}'$, discarding values of $\boldsymbol{\theta}$ that do not imply a unique solution. The set $\boldsymbol{\Theta}'$ may be constructed by specifying a lower and an upper bound for each element of $\boldsymbol{\theta}$. Such bounds are usually easy to come by from the economic meaning of the deep parameters. An alternative approach is to define $\boldsymbol{\Theta}'$ by specifying some univariate probability distribution for each parameter θ_i. The benefit of this approach is that, by choosing the shape and parameters of the distribution, one can achieve a better coverage of the parts of the space that are believed to be more plausible. In practice the choice of distributions may follow the logic of specifying a prior distribution for a Bayesian estimation of DSGE models (see e.g. Del Negro and Schorfheide (2008)).

It should be stressed that the information matrix approach for identification analysis applies only to full information methods. Identification with full information is necessary but not sufficient for identification with limited information. The same applies to the strength of identification - a well identified model may still suffer from weak identification problems if the estimation approach is a limited information one. Thus, if a DSGE model is to be estimated with methods, such as impulse response matching, that do not utilize all model-implied restrictions on the distribution of the data, identification should be studied differently. A general rank condition for local identification in DSGE models, which applies to any estimation approach that utilizes only first and second moments of the data, is developed in Iskrev (2010). Applying that result, one can determine if $\boldsymbol{\theta}$ is identifiable from, for instance, the covariance and first-order autocovariance matrix of some observable endogenous variables. This is useful to know even in a full information setting since identification with limited information is sufficient, though not necessary, for identification with full information methods. Thus, finding that the rank condition in Iskrev (2010) is satisfied for some small number of second moments obviates the need to compute the information matrix, which is generally more computationally expensive. A second necessary condition for identification from Iskrev (2010), that does not depend on statistical model and the distributional assumptions in particular, concerns the invertibility of the mapping from $\boldsymbol{\tau}$ - the reduced-form parameters, to $\boldsymbol{\theta}$. Note that by the chain rule we have:

$$\frac{\partial \ell_T}{\partial \boldsymbol{\theta}'} = \frac{\partial \ell_T}{\partial \boldsymbol{\tau}'} \frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{\theta}'} \tag{3.11}$$

and therefore the information matrix may be written as

$$\boldsymbol{\mathcal{I}}_T = \left(\frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{\theta}'}\right)' \mathrm{E}\left\{\left(\frac{\partial \ell_T}{\partial \boldsymbol{\tau}'}\right)' \left(\frac{\partial \ell_T}{\partial \boldsymbol{\tau}'}\right)\right\} \left(\frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{\theta}'}\right) \tag{3.12}$$

Thus, the Jacobian matrix $\partial \boldsymbol{\tau} / \partial \boldsymbol{\theta}'$ must have full column rank in order for $\boldsymbol{\mathcal{I}}_T$ and its limit $\boldsymbol{\mathcal{I}}$ to be non-singular. If this condition does not hold some deep parameters are unidentifiable for purely model-related reasons, not because of deficiencies of the statistical model or lack of observations for some model variables. Furthermore, the properties of the Jacobian matrix, when it has full column rank, has implications for the strength of identification of $\boldsymbol{\theta}$. From (3.11) it is clear that the two types of weak identification problems discussed in Section 3.1 may be due to either one of the following two transformations - from $\boldsymbol{\theta}$ to $\boldsymbol{\tau}$, or from $\boldsymbol{\tau}$ to ℓ_T, or to both. The second transformation is partially determined by data limitations - how many and which of the model variables are included, and the number of observations. The first one depends only on the model, and the Jacobian matrix measures how sensitive are the elements of $\boldsymbol{\tau}$ to those of $\boldsymbol{\theta}$. A very low sensitivity means that relatively large changes in some deep parameters have a very small impact on the value of $\boldsymbol{\tau}$. Consequently, these parameters would be difficult to estimate even if one had data for all endogenous variables in the model. In that sense we may say that such deep parameters are poorly identified in the model. To find out what parameters are poorly identified, as well as what model features are causing the problem, one may proceed as in Section 3.3. Specifically, θ_i is weakly identified in the model if either one of the following two conditions holds:

(a) $\boldsymbol{\tau}$ is insensitive to changes in θ_i, i.e.

$$\frac{\partial \boldsymbol{\tau}}{\partial \theta_i} \approx \mathbf{0} \tag{3.13}$$

(b) The effect on $\boldsymbol{\tau}$ of changing θ_i can be well approximated by changing other parameters in $\boldsymbol{\theta}$, i.e.

$$\cos\left(\frac{\partial \boldsymbol{\tau}}{\partial \theta_i}, \frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{\theta}_{-i}}\right) \approx 1 \tag{3.14}$$

If (a) is true, changing θ_i while keeping the other deep parameters fixed has almost no effect on $\boldsymbol{\tau}$. This occurs when a parameter is almost irrelevant with respect to the steady state and the dynamics of the model. If (b) is true, the angle between $\partial\boldsymbol{\tau}/\partial\theta_i$, and the space spanned by the other columns of $\partial\boldsymbol{\tau}/\partial\boldsymbol{\theta}'$ is nearly zero. This means that, locally, the effect of changing θ_i is almost the same as changing one or more of the other deep parameters.

4 Application: Identification in the Smets and Wouters (2007) model

In this section I illustrate the identification analysis tools discussed above using a medium-scale DSGE model estimated in Smets and Wouters (2007) (SW07 henceforth). I start with an outline of the main components of the model, and then turn to the identification of the parameters.

4.1 The model

The model, based on the work of Smets and Wouters (2003) and Christiano et al. (2005), is an extension of the standard RBC model featuring a number of nominal frictions and real rigidities. These include: monopolistic competition in goods and labor markets, sticky prices and wages, partial indexation of prices and wages, investments adjustment costs, habit persistence and variable capacity utilization. The endogenous variables in the model, expressed as log-deviations from steady state, are: output (y_t), consumption

(c_t), investment (i_t), utilized and installed capital ($k_t^s,\ k_t$), capacity utilization (z_t), rental rate of capital (r_t^k), Tobin's q (q_t), price and wage markup ($\mu_t^p,\ \mu_t^w$), inflation rate(π_t), real wage (w_t), total hours worked (l_t), and nominal interest rate (r_t). The log-linearized equilibrium conditions for these variables are presented in Table 1. The last equation in the table gives the policy rule followed by the central bank, which sets the nominal interest rate in response to inflation and the deviation of output from its potential level. To determine potential output, defined as the level of output that would prevail in the absence of the price and wage mark-up shocks, the set of equations in Table 1 is extended with their flexible price and wage version (see Table 2). The model has seven exogenous shocks. Five of them - to total factor productivity, investment-specific technology, government purchases, risk premium, and monetary policy - follow AR(1) processes; the remaining two shocks - to wage and price markup, follow ARMA$(1, 1)$ processes.

The model is estimated using data of seven variables: output growth, consumption growth, investment growth, real wage growth, inflation, hours worked and the nominal interest rate. Thus, the vector of observables is given by

$$\boldsymbol{x}_t = [y_t - y_{t-1}, c_t - c_{t-1}, i_t - i_{t-1}, w_t - w_{t-1}, \pi_t, l_t, r_t,]' \tag{4.1}$$

and the exogenous term in the measurement equation (2.3) is given by $\boldsymbol{u}_t = 1$ for all t, and

$$\boldsymbol{D} = \left[\bar{\gamma}, \bar{\gamma}, \bar{\gamma}, \bar{\gamma}, \bar{\pi}, \bar{l}, \bar{r}\right]' \tag{4.2}$$

where $\bar{\gamma}$ is the growth rate of output, consumption, investment and wages, $\bar{\pi}$ is the steady state rate of inflation, $\bar{l}$ is the steady state level of hours worked and $\bar{r}$ is the steady state nominal interest rate. Since there is no measurement error, the last term in (2.3) is omitted.

The deep parameters of the model are collected in a 41-dimensional vector $\boldsymbol{\theta}$ given by[1]

$$\begin{aligned}\boldsymbol{\theta} = \quad & [\delta, \lambda_w, g_y, \varepsilon_p, \varepsilon_w, \rho_{ga}, \beta, \mu_w, \mu_p, \alpha, \psi, \varphi, \sigma_c, \lambda, \Phi, \iota_w, \xi_w, \iota_p, \xi_p, \sigma_l, \\ & r_\pi, r_{\triangle y}, r_y, \rho, \rho_a, \rho_b, \rho_g, \rho_I, \rho_r, \rho_p, \rho_w, \gamma, \sigma_a, \sigma_b, \sigma_g, \sigma_I, \sigma_r, \sigma_p, \sigma_w, \bar{\pi}, \bar{l}]'\end{aligned} \tag{4.3}$$

[1]Note that by definition $\bar{\gamma} = 100(\gamma - 1)$, and $\bar{r}$ is determined from the values of $\beta,\ \sigma_c,\ \gamma$ and $\bar{\pi}$ from $\bar{r} = 100(\frac{\bar{\pi}\gamma^{\sigma_c}}{\beta} - 1)$.

4.2 Identification Analysis

The identifiability of the parameters in the SW07 model was studied in Iskrev (2010). It was found that 37 out of the 41 parameters in (4.3) are locally identified; the remaining four parameters - ξ_w, ξ_p, ϵ_w andϵ_p, are not separately identifiable in the sense that in the linearized model ξ_w cannot be distinguished from ϵ_w, and ξ_p cannot be distinguished from ϵ_p. As in SW07, I will assume that ϵ_w and ϵ_p are known and are both equal to 10. The purpose of this section is to study the strength of identification of the remaining 39 parameters.

4.2.1 Identification strength at the posterior mean

As a preliminary step I evaluate the strength of identification of $\boldsymbol{\theta}$ at the posterior mean reported in SW07. The purpose of doing this is twofold: first, the posterior mean is of interest on its own as a value of $\boldsymbol{\theta}$ where the empirical properties of the DSGE model are in line with the post-war US data; second, the strength of identification at the posterior mean will be used as a reference point to which the results for other points in the parameter space are compared.

I evaluate the expected information matrix for $T = 156$ (the sample size in SW07), and compute the measures of identification strength when $\boldsymbol{\theta}$ is equal to the posterior mean. Table 4 reports intervals of the form $\hat{\theta}_i \pm \mathrm{std}(\hat{\theta}_i)$, where $\hat{\theta}_i$ denotes the posterior mean and $\mathrm{std}(\hat{\theta}_i) = 1/\boldsymbol{s}_i(\hat{\boldsymbol{\theta}})$ is the the Cramér-Rao lower bound on the standard deviation for θ_i. The meaning of the intervals depends on the exact sampling distribution of the estimator of $\boldsymbol{\theta}$. In the case of normally distributed and unbiased estimators, the reported intervals may be interpreted as liberal 68% confidence intervals.[2] With this interpretation in mind, it is interesting to compare the Cramér-Rao intervals with the corresponding one-standard-deviation Bayesian intervals (see Table 3). Although conceptually very different, by comparing the two types of intervals we can assess the contribution of the prior information in the estimation of the parameters.

Starting with the behavioral and technology parameters, listed in the upper panel of the table, the Cramér-Rao intervals are substantially larger, on average by 84%, than the Bayesian intervals. Most pronounced are the differences for the discount factor β, the wage stickiness parameter ξ_w, and policy response to inflation parameter r_π, for which the intervals shown in Table 4 are 165%, 162% and 130% larger than the respective

[2] That is, the actual coverage probability of the intervals is smaller than 68%.

Bayesian intervals. The differences are smallest for: the price stickiness parameter ξ_p (3%), the capital share α (27%) and the price indexation parameter ι_p (44%). Regarding the structural shock parameters, the Cramér-Rao intervals are on average 28% larger than the Bayesian intervals. Here the largest differences occur with respect to the MA coefficient of the price markup shock μ_p (74%), the autoregressive coefficient of the TFP shock ρ_a (56%), and the autoregressive coefficient of the monetary policy shock ρ_r (46%). Among all model parameters, only for the trend growth rate γ is the Bayesian credibility interval larger, with 24%, than the Cramér-Rao interval. Thus, the conclusion we may draw from this comparison is that the prior information used in the estimation of the model has a substantial effect in reducing the posterior uncertainty. The effect is stronger for the behavioral and technology parameters because of the relatively tighter priors, including the fact that δ, λ_w and g_y are assumed known in Smets and Wouters (2007).

In some cases it is easy to see, by inspecting the bounds in Table 4, whether the identification of a parameter is strong or weak. For instance, the intervals are clearly very large for parameters like ξ_w, σ_l or β, and very small for parameters like γ, ρ and λ. However, due to the very different parameter values, it is difficult to assess the relative strength of identification of different parameters. For instance it is not immediately clear from the standard deviation bounds whether the capacity utilization cost parameter (ψ) or the investment adjustment cost parameter (φ) is better identified. A scale-independent measure of the strength of identification is $\boldsymbol{s}_i^r(\boldsymbol{\theta})$, defined earlier as the absolute value of θ_i divided by the Cramér-Rao lower bound on the standard deviation of the parameter. This measure are also reported in Table 4, and we can see that in fact ψ is better identified than φ. Overall, the best identified are ρ_g, ρ_w, ρ_a, γ, ρ and ρ_p, with values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ between 87.5 and 15; the worst identified parameters are $\bar{l}$, β, ρ_r, σ_l, ι_p, λ_w, and r_y with values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ between .4 and 2.2.

The different degrees of identification strength reflect differences in the importance of the parameters in determining the empirical properties of the model. Identification is weak when a parameter is either nearly irrelevant, because it does not affect much the likelihood, or nearly redundant, because the effect of the parameter on the likelihood can be replicated by other parameters. The importance of these two factors can be determined using the factorization of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ in (3.10). The two components are shown in Table 4 under the labels **sens.** and **col.** Furthermore, the last column shows the values of the multiple correlation coefficients associated with the collinearity components, which are bounded between 0 and 1 and therefore easier to interpret. From there we can see that strong collinearity is an important factor for many parameters, and in particular for λ_w, ξ_w, σ_c, μ_p, μ_w ρ_p, r_π and λ. However, the overall strength of identification could still be high if the sensitivity components are sufficiently large, as is the case for λ, ρ_p and μ_w. On the other hand, there are parameters such as $\bar{l}$, β and ρ_p, which are poorly identified mainly because of their very low sensitivity components. Note that even if we set the collinearity terms to 1, which is the upper bound achieved when there is either zero collinearity or when all other parameters are assumed known, the relative strength of identification of these three parameters would still be lower than that of most other parameters.

The correlation coefficients $\boldsymbol{\rho}_i$ in Table 4 are computed with respect to all 38 parameters other than θ_i. It is reasonable to expect that there are small subsets of parameters, representing closely related features in the model, which are most important, while the others having only a marginal contribution. To find out what these subsets are, I compute the multiple correlation coefficients between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}(n)$, where $\boldsymbol{\theta}_{-i}(n)$ is a subset of n elements of $\boldsymbol{\theta}_{-i}$, for all possible n-tuples, and pick the largest one denoting it by $\boldsymbol{\varrho}_{i(n)}$. Table 5 shows the results for n between 1 and 4. For most parameters, and in particular for those with the highest collinearity values, the results with 4 parameters come very close to what we have for the coefficients of multiple correlation (i.e. for $\boldsymbol{\varrho}_i(n)$ with n=38) in Table 4. Note also that in a few cases, namely for the price markup shock coefficients (μ_p and ρ_p), and for the the wage stickiness and the steady state wage markup parameters (ξ_w and λ_w), we have very large coefficients of pairwise correlation (i.e. for $n = 1$). This suggest that the parameters in these pairs are difficult to distinguish on the basis of their effects on the likelihood. In general, however, it is not sufficient to examine the pairwise correlations in order to appreciate the full extent of the collinearity problems. For instance, the largest pairwise correlations for the monetary policy rule parameters r_π, ρ, and r_y are around .3, while the multiple correlation coefficients increase to .9 with 4 parameters, and are even larger when all 38 parameters are included.

One interpretation of large correlation coefficients in Table 5 is that some parameters play very similar roles in the structural model, and virtually the same model features are represented by several different parameters. However, we should keep in mind the fact that the likelihood functions represents only seven of the variables in the model (see eq. (4.1)). Thus, the collinearity patterns in Table 5, as well as the sensitivity values in

Tables 4, are partially determined by the choice of observables, and may change with a different or a larger set of variables. As suggested in Section 3.3, a simple way to find out whether the identification problems are independent of the observables is to study the Jacobian matrix of the reduced-form parameters $\boldsymbol{\tau}$ with respect to $\boldsymbol{\theta}$. Since $\boldsymbol{\tau}$ fully characterizes the steady state properties and the equilibrium dynamics of all model variables, parameters with very weak or highly collinear effects on $\boldsymbol{\tau}$ are likely to have similar problems with respect to any subset of observables. Table 6 reports measures of sensitivity and collinearity in the model. Specifically, the sensitivity to parameter θ_i is computed as the norm of the vector of elasticities of $\boldsymbol{\tau}$ with respect to θ_i; the collinearity of θ_i is measured by the cosine of the angle between $\partial\boldsymbol{\tau}/\partial\theta_i$ and the hyperplane spanned by the columns of $\partial\boldsymbol{\tau}/\partial\boldsymbol{\theta}_{-i}$. The table also shows measures of collinearity with respect to smaller sets of parameters, selected to maximize the value of the cosine among all possible sets of given size. A comparison with Tables 4 and 5 shows that, in many cases, the identification properties of the parameters can be traced back to the structural model. For example, parameters with very low sensitivity in terms of $\boldsymbol{\tau}$ also have low sensitivity components of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$. A notable exception is the trend parameter γ, which has one of the largest sensitivity components in Table 4 in spite of the relatively low sensitivity in terms of $\boldsymbol{\tau}$. This is due to the disproportionately large influence on the likelihood of the steady state component of the solution vector compared to most other elements of $\boldsymbol{\tau}$. Comparing the last four columns of Tables 5 and 6 reveals that the collinearity relationships with respect to the likelihood can be explained largely by the very similar effects of parameters on the solution of the model. There appears to be larger discrepancy between the two tables in terms of the overall measures of correlation (column 2 in Table 5) and collinearity (column 3 in Table 6). Particularly striking is the difference for the steady state parameter for hours ($\bar{l}$), for which the numbers are .815 and 0, respectively. The explanation for this difference is that, while $\bar{l}$ only affects the mean of hours worked and is the only parameter that does that (hence the zero), the value of $\boldsymbol{\varrho}_i$ also depends on the covariance matrix of the sample means of all observables; since the latter is not diagonal, this results in a non-zero multiple correlation for $\bar{l}$.

Some insights on the role of the observed variables in the identification of the parameters may be gained by comparing the strength of identification with and without each variable. This would tell us, for instance, which observable is most informative for a given parameter, and how much would be lost in terms of estimation precision if any one of the seven variables is not used in the estimation of the model. Table 7 reports the ratios of the values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ with six and with seven variables. All values are smaller then 1, which means that all variables are informative, albeit to different degrees, for all parameters. The most informative variables are the the ones whose exclusion leads to the largest decrease in the identification strength. Thus, $\triangle y_t$ is the most informative variable for 3 parameters (ρ_{ga}, σ_a and σ_g), $\triangle c_t$ is the most informative variable for 8 parameters ($\sigma_c, \lambda, \xi_w, \rho_b, \rho_g, \sigma_b, \lambda_w$ and g_y), $\triangle i_t$ is the most informative variable for 7 parameters ($\alpha, \psi, \varphi, \rho_I, \gamma, \sigma_I$ and δ), l_t is the most informative variable for 5 parameters ($\bar{l}, \Phi, \xi_p, \sigma_l$ and ρ_a), π_t is the most informative variable for 5 parameters ($\bar{\pi}, \iota_p, r_y, \rho$, and σ_p), $\triangle w_t$ is the most informative variable for 7 parameters ($\mu_w, \mu_p, \iota_w, \rho_p, \rho_w$, and σ_w) and r_t is the most informative variable for 5 parameters ($\beta, r_\pi, r_{\triangle y}, \rho_r$, and σ_r).

It is important to remember that the results discussed so far are conditional on the particular parameter values used to evaluate the information matrix. There is no guarantee that what was found regarding the (relative) strength of identification, the sensitivity and collinearity patterns among the parameters and the role of observables remains true in other regions of the parameter space. To establish that, it is necessary to apply the analysis to other points in $\mathbf{\Theta}$. This is the subject of the next section.

4.2.2 Identification strength in $\mathbf{\Theta}$

Here I proceed along the lines discussed in Section 3.3. Specifically, I draw randomly 100,000 points from $\boldsymbol{\Theta}$, which is defined using the prior distribution in SW07 (see Table 3), and evaluate the identification strength of the parameters at each point. As before, the expected information matrix is evaluated for $T = 156$. The results are summarized in Table 8 showing the means, the coefficient of variation,[3] and the deciles of the distributions of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$.

A close examination of the table sheds some light on the the generality of the findings regarding the strength of identification at the posterior mean. For more than half of the parameters the values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ at the posterior mean of $\boldsymbol{\theta}$ are smaller than the 2-th deciles in Table 8. For 7 of the 39 parameters identification at the posterior mean is weaker than the 1-th deciles, and only 9 of them are better identified at the posterior mean than the median values in Table 8. On average, the median values in Table 8 are 2.4 times larger than respective values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ at the posterior mean; however, the difference is much larger in the case of $\bar{l}$ (17.7 times), γ (11.8 times), ρ_r (5.9 times), ι_p (5 times), and goes in the opposite direction for parameters like ξ_w (.7 times), μ_p (.5 times), μ_w (.15 times)

[3]The coefficient of variation is defined as the standard deviation divided by the mean.

and ρ_w (.04 times). All of this suggests that the posterior mean value of $\boldsymbol{\theta}$ is located in a region of the parameters space where, with a few exceptions, the parameters in the model are much worse identified than in the rest of $\boldsymbol{\Theta}$. Furthermore, there are some striking differences in the ranking of the parameters in terms of identification strength when $\boldsymbol{\theta}$ is equal to the posterior mean, and the ranking in Table 8 using either the mean or the median values of the bounds. In particular, ρ_p, ρ_w, μ_w are better identified than most parameters at the posterior mean, but are among the worst identified in general, according to the results in Table 8. Conversely, ρ_r and ψ are relatively well identified in general, but are worse identified than most parameters at the posterior mean. Apart from this, the results at the posterior mean are consistent with those in Table 8 in ranking λ_w, ξ_w, $\bar{l}$, β, σ_l, and r_y among the worst identified, and γ, ρ, λ, ξ_p, Φ, σ_g, σ_a and σ_r among the best identified parameters in the model.

It is worth noting that the strength of identification of many parameters changes quite substantially as $\boldsymbol{\theta}$ varies in the parameter space. Comparing the first and ninth deciles in Table 8 shows that the highest values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ for the relatively weakly identified parameters exceed the lowest values of the relatively well identified ones. For instance, while only 5 parameters have values of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ greater than 10 in the first deciles, only 6 parameters have values of less than 10 in their ninth deciles. This implies that, with some exceptions, the strength of identification cannot be regarded as a constant feature of the model, but is something which depends on where in $\boldsymbol{\Theta}$ we evaluate the model.

As before, the results in Table 8 can be explained with the relative importance of each parameter in determining the statistical implications of the DSGE model. Tables 9 and 10 summarize the distributions of the sensitivity and collinearity (in terms of $\boldsymbol{\varrho}_i$) factors in the decomposition of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$. From there we see that γ is generally very well identified because both components are large and relatively stable across $\boldsymbol{\Theta}$. This means that γ is a parameter with a strong and unique effect on the probability distribution of the observables, irrespectively of the value of $\boldsymbol{\theta}$. From the other well identified parameters, ρ, λ, ξ_p and Φ tend to have much smaller collinearity terms which, however, is compensated by sufficiently large sensitivity components. The reverse is true for the shock parameters σ_g, σ_a and σ_r. Among the worst identified parameters, λ_w, ξ_w, σ_l and φ have large sensitivity and small collinearity components. Thus, in the SW07 model each one of these parameters represents a structural feature that is well identified empirically, but is associated with more than one deep parameters. The weak identification of $\bar{l}$ is mostly due to the very small sensitivity component, while that of β - to the relatively small values of both the sensitivity and the collinearity components.

The extend to which these properties of the parameters are inherent in the structure of the model, and not caused by the limited set of observables, is investigated by studying the behavior of the derivative of $\boldsymbol{\tau}$ as $\boldsymbol{\theta}$ varies in $\boldsymbol{\Theta}$. Table 11 provides information about the distribution of the elasticity of $\boldsymbol{\tau}$ with respect to $\boldsymbol{\theta}$, and Table 12 does the same for the collinearity among the columns of the matrix. The results are consistent with what was found for the posterior mean: the likelihood tends to be more sensitive to parameters to which $\boldsymbol{\tau}$ is very elastic, and strong collinearity with respect to the likelihood is typically associated with strong collinearity among the derivatives of $\boldsymbol{\tau}$. A few notable exceptions to these patterns are: (1) γ, which, as in Section 4.2.1, has a very strong effect on the likelihood, in spite of the very low elasticity of the reduced-form parameters to it; and (2) the markup shock parameters ρ_w, μ_p, μ_w tend to have relatively large multiple correlation coefficients and relatively low collinearity with respect to $\boldsymbol{\tau}$. These, and other less pronounced discrepancies may be explained with the process of constructing the likelihood function (see Section 2.2), which results in assigning very different weights to the elements of $\boldsymbol{\tau}$ as they enter in the information matrix for a particular set of observables. On the other hand, each parameter in $\boldsymbol{\tau}$ is weighted equally in the measures of sensitivity and collinearity with respect to $\boldsymbol{\tau}$.

The large values of $\boldsymbol{\varrho}_i$ in Table 10 suggest that the collinearity problems are pervasive in the SW07 model. This means that many model features can be well approximated by more than one deep parameter. From Section 4.2.1 we know that it is often possible to find small subsets of parameters that explain most of the collinearity captured by $\boldsymbol{\varrho}_i$. There is no guarantee, however, that the optimal selections of parameters remain the same for values of $\boldsymbol{\theta}$ different from the posterior mean. To explore this further, I proceed as in Section 4.2.1 and compute, for each θ_i, the coefficients of multiple correlation between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}(n)$ for all possible subsets of $n = 1, \ldots, 4$ elements of $\boldsymbol{\theta}_{-i}$. This is done for the parameter values corresponding to the deciles of $\boldsymbol{\varrho}_i$ in Table 10, and the results are reported in Tables 13 to 15 for the first, fifth and ninth deciles. As in Section 4.2.1, it is often sufficient to use four or fewer parameters to reach to a degree of collinearity close to the one with all 38 parameters. However, although there is some degree of consistency in the selected parameters, the optimal sets change quite substantially depending on where in the parameter space we evaluate the model. For instance, the effect of the labor supply elasticity parameter (σ_l) may be closest to that of either the wage stickiness parameter (ξ_w), the wage markup parameter (λ_w), the policy rule coefficient ($r_{\triangle y}$) or the intertemporal substitution parameter (σ_c); when two or more parameters are considered, the optimal selections include also other labor market-related parameters - μ_w, ι_w and σ_w, or preference parameters - λ and σ_c. The tables also indicate that there is a strong similarity among the monetary policy parameters ($r_\pi, r_y, r_{\triangle y}, \rho, \rho_r, \sigma_r$), the price markup ($\rho_p, \mu_p, \sigma_p$) and wage markup ($\rho_w, \mu_w, \sigma_w$) shock parameters.

4.3 Identification strength and model features

One of the more striking findings of the previous section is the large variability in the strength of identification across different regions in the parameter space. This suggests that most parameter in the model may be either very well or very poorly identified depending on where in $\boldsymbol{\Theta}$ we evaluate the model. A natural question to ask is: what determines these differences. The purpose of this section is to try to answer this question by establishing links between different features of the model and the strength of identification of the parameters.

An obvious place to start are the values of individual deep parameters. Naturally, there is a substantial variability in the values of $\boldsymbol{\theta}$ associated with the deciles of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ in Table 8. It is conceivable that these variations are to some extend systematic, i.e that there is a systematic relationship between the values of one or more parameters and the strength of identification of a given θ_i. Such relationship exists when the values of a parameter, say θ_j, in the region of $\boldsymbol{\Theta}$ where θ_i is well identified, are systematically different from the values in the region of $\boldsymbol{\Theta}$ where θ_i is poorly identified. This idea underlies a procedure known as Monte Carlo filtering (see Ratto (2008) for a detailed discussion and a different application in the context of DSGE models). The decision of whether the values θ_j in the two regions of $\boldsymbol{\Theta}$ are different is made on the basis of the outcome from a two-sample Kolmogorov-Smirnov test for equality of two distribution functions. The two-sided version of the test rejects if the two distributions are different, which indicates that there is a systematic relationship between the value of θ_j and the strength of identification of θ_i. A one-sided version may also be used to determine the sign of the relationship, i.e. whether larger values of θ_j are associated with stronger or weaker identification of θ_i.

Four types of outcomes are possible: (1) there is a systematic positive relationship; (2) there is a systematic negative relationship; (3) there is a systematic relationship which is positive for some values of θ_j and negative for other values; (4) there is no systematic relationship, i.e. the distributions of θ_j in the two subsets are the same. Figure 2 illustrates each of these cases, showing the cumulative distribution functions (CDFs) of the interest rate smoothing parameter (ρ), the Taylor rule coefficient on inflation (r_π), the autoregressive coefficient of the risk premium shock (ρ_b), and the Taylor rule coefficient on output growth ($r_{\triangle y}$) in regions of $\boldsymbol{\Theta}$ where the elasticity of intertemporal substitution (σ_c) is more strongly (solid line) and more weakly (dashed line) identified. The figure is constructed using the previously obtained sample from $\boldsymbol{\Theta}$ from which the two regions of the parameter space are defined using the 2-th and 8-th decile of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ for σ_c. In panel (a) the CDF of ρ in the region of stronger identification of σ_c is larger than in the region of weaker identification. This means that larger values of ρ tend to be associated with stronger identification of σ_c. The reverse is true for r_π - larger values tend to be associated with weaker identification of σ_c, as may be seen from panel (b) of Figure 2. In panel (c) the CDFs of ρ_b in the two regions of $\boldsymbol{\Theta}$ intersect at approximately $\rho_b = .5$. This means that the region of $\boldsymbol{\Theta}$ where σ_c is better identified is associated with smaller values of ρ_b when ρ_b is smaller than .5, and with larger values of ρ_b when it is greater than .5. In panel (d) we see that the CDFs of $r_{\triangle y}$ in the two regions of $\boldsymbol{\Theta}$ lie on the top of each other, implying that there is no systematic relationship between the values of that parameter and the strength of identification of σ_c. In all cases the conclusions from the visual inspection of the CDFs are supported by the outcomes from the two-sample Kolmogorov-Smirnov test.

Proceeding with visual inspection and formal testing for equality of the distributions for all possible pairs of parameters produces the results shown in Table 16 with the following notation: in cell (i, j) the sign "+" ("−") indicates that the cumulative distribution function of θ_j in the region of $\boldsymbol{\Theta}$ where θ_i is better identified is everywhere greater (smaller) than the cumulative distribution function of θ_j where θ_i is worse identified. With "±" are indicated the cases where the two distributions are different, but the cumulative distribution functions intersect. Lastly, an empty cell means that that the two distributions are the same.

From Table 16 it is clear that the strength of identification of most parameters is influenced by many of the structural features of the model. Furthermore, the plots of the distribution functions show that in most cases the parameter values on which the distribution functions assign positive mass are the same in the region of stronger as in the region of weaker identification. This means that there are no values that are exclusively associated with either stronger or weaker identification of the parameter in question. Some exceptions to this observation are presented in Figures 3 and 4. In panels (a) to (i) of Figure 3 are shown cases where very large values of a parameter are located only in the region of stronger identification and/or very small values are located only in the region of weaker identification; in panels (j) to (l) the opposite is true: very large values are located only in the region of weaker identification and/or very small values are located only in the region of stronger identification. Note that in most cases the relationship is between the value of a parameter and the strength of identification of the same parameter. Figure 4 is similar, except that it focuses on the effect of structural shock parameters. According to the plots, very persistent or very volatile shocks are associated with stronger identification for all except the last four of the depicted deep parameters.

In addition to quantifying the structural characteristics of the model, the value of $\boldsymbol{\theta}$ determines the statistical properties of the model variables. We can therefore ask whether there is a systematic relationship between properties such as persistence, volatility and correlation structure of the observables, and the strength of identification of the deep parameters. Some evidence for such relationships may be found in Table 16 since the shock parameters affect directly the degree of persistence and volatility of the variables, while many of the other deep parameters represent frictions which have similar effect through the transmission mechanism in the model. To study the question formally I use the same Monte Carlo filtering procedure as before. The variables' volatility and correlation are obtained from the theoretical covariance matrix, while the persistence is measured as the population value of the sum of the first five autoregressive coefficients.[4] As can be seen from Table 17, both persistence and volatility tend to be larger in the better identified regions for most parameters. Notable exceptions are $\bar{l}$, $\bar{\pi}$, γ and β, for which the relationship is reversed, and μ_w, μ_p, ρ_a and ρ_g, for which there is no or little evidence for systematic relationships. The results for the correlation structure are presented separately, in Tables 18 and 19, for the positive and negative values of the correlation coefficients. This accounts for the possibility that the sign of the correlation coefficients may affect the relationship between their magnitude and the strength of identification. This is indeed the case for some parameters; for instance, the correlation between consumption and the interest rate tends to be stronger when

[4]There is no unique universally accepted measure of persistence. The result I report are not sensitive to the number of autoregressive coefficients included in the sum.

positive but weaker when negative in the region where the Taylor rule parameters r_π and r_y are better identified. However, in most cases what matters is the absolute value of the correlation, and larger correlations of the variables are associated with stronger identification of the deep parameters.

Before concluding this section, a comment on the number of parameter draws is in order. It is difficult to say how many draws are required to cover the parameter space of model. The identification analysis in this section is only intended as an illustration of the general approach and not as a complete study of identification issues on the SW07 model. One possible test on the generality of the results is to check whether summary statistics of the distributions of the identification strength measures (e.g. the means, and the deciles) have converged. I have found that the results in in Table 8 change very little if instead of the full sample of $100,000$ I use only half of the draws.

5 Identification strength when θ is random

Until now the parameters were treated as non-random. This is consistent with the frequentist tradition of parametric statistical inference where $\boldsymbol{\theta}$ is regarded as unknown but fixed. It is also the natural approach if one wants to understand how the identification properties of a model depend on the particular value of $\boldsymbol{\theta}$. In this section I consider the case where $\boldsymbol{\theta}$ is regarded as a random variable, which is an essential characteristics of the Bayesian approach to inference.

According to the Bayesian point of view, there is no true unknown value of $\boldsymbol{\theta}$. Instead, one has prior beliefs about the probability of different values, and updates these beliefs on the basis of the evidence provided by the data. Specifically, if $p(\boldsymbol{\theta})$ is the prior distribution of $\boldsymbol{\theta}$, and $p(\boldsymbol{X};\boldsymbol{\theta})$ is the likelihood function, the Bayesian inference for $\boldsymbol{\theta}$ is based on the joint density:

$$p(\boldsymbol{X},\boldsymbol{\theta}) = p(\boldsymbol{X};\boldsymbol{\theta})p(\boldsymbol{\theta}) \tag{5.1}$$

The Bayesian information matrix may be defined similarly to (2.5)

$$\boldsymbol{\mathcal{J}} := \mathrm{E}_{\boldsymbol{X},\boldsymbol{\theta}}\left[\left\{\frac{\partial \log p(\boldsymbol{X},\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right\}'\left\{\frac{\partial \log p(\boldsymbol{X},\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right\}\right], \tag{5.2}$$

where the subscript indicates that the expectation is with respect to the joint probability density. Van Trees (1968) shows that the inverse of $\boldsymbol{\mathcal{J}}$ plays the role of a posterior bound analogous to the Cramér-Rao lower bound in classical statistics. In particular, if $\hat{\boldsymbol{\theta}}$ is a Bayesian estimator of $\boldsymbol{\theta}$, then the mean squared error of $\hat{\boldsymbol{\theta}}$ is bounded as follows

$$\mathrm{E}_{\boldsymbol{X},\boldsymbol{\theta}}\left[\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right)\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right)'\right] \geq \boldsymbol{\mathcal{J}}^{-1} \tag{5.3}$$

Unlike the Cramér-Rao lower bound, the inequality in (5.3) holds for biased estimators. It is assumed, however, that some regularity conditions are satisfied, one of which is that the prior $p(\boldsymbol{\theta})$ is zero at the boundary of its support (see Van Trees (1968) for details). Since $p(\boldsymbol{X},\boldsymbol{\theta}) = p(\boldsymbol{X};\boldsymbol{\theta})p(\boldsymbol{\theta})$, the Bayesian information matrix can be decomposed as the sum of two matrices

$$\boldsymbol{\mathcal{J}} = \boldsymbol{\mathcal{J}}_D + \boldsymbol{\mathcal{J}}_P, \tag{5.4}$$

where $\boldsymbol{\mathcal{J}}_D := \mathrm{E}_{\boldsymbol{\theta}}\left[\boldsymbol{\mathcal{I}}(\boldsymbol{\theta})\right]$ is the data component of $\boldsymbol{\mathcal{J}}$ and $\boldsymbol{\mathcal{J}}_P := \mathrm{E}_{\boldsymbol{\theta}}\left[\left\{\frac{\partial \log p(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right\}'\left\{\frac{\partial \log p(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}'}\right\}\right]$ is the prior component of $\boldsymbol{\mathcal{J}}$. Note that, like the Fisher information matrix, $\boldsymbol{\mathcal{J}}$ does not depend on a particular $\boldsymbol{X}$. Thus, it can be used to assess the strength of identification of $\boldsymbol{\theta}$ in a Bayesian setting prior to the estimation of the model. For instance, (5.3) implies that the root mean squared error (RMSE), or the standard deviation for unbiased estimators, of $\hat{\theta}_i$ is bounded from below by the i-th diagonal element of $\boldsymbol{\mathcal{J}}^{-1/2}$. The required expectations with respect to the prior distribution of $\boldsymbol{\theta}$ are straightforward to compute numerically. This is illustrated in Table 20, where I show the Bayesian bounds for the posterior mean in SW07, computed using the previously obtained sample of 100,000 draws from $\boldsymbol{\Theta}$. To make the comparison with the earlier results easier, the bounds are shown in terms of $\hat{\theta}_i\pm$ (the value of the bound for the RMSE of $\hat{\theta}_i$). Also, in the table I have replicated the posterior one-standard-deviation intervals, as well as the frequentist Cramér-Rao bounds. There is a remarkable similarity between the Bayesian a priori and posterior bounds. On average, the a priori bounds are about 5% wider, which may be explained with the fact that three parameters (δ, λ_w and g_y) were fixed in the estimation of $\boldsymbol{\theta}$. The frequentist bounds, on the other hand, are on average 50% wider than the Bayesian a priori bounds. As in Section 4.2.1, the differences are much smaller for the structural shock parameters and the growth rate γ, while for parameters like λ_w and ξ_w, the frequentist bounds are about three times as large as the Bayesian ones.

It is clear from (5.4) that the size of the Bayesian bounds is determined by the interaction of the likelihood and the prior distribution. Furthermore, unless both $\boldsymbol{\mathcal{J}}_D$ and $\boldsymbol{\mathcal{J}}_P$ are diagonal, the bound for each parameter in general depends on the prior distribution of every other parameter. A common approach to assess the sensitivity to priors is to re-estimate the models using increasingly diffuse priors. A simple alternative is to compute the derivative of the bound in (5.3) with respect to the prior precision parameters. In the case of independent priors, one could compute the derivatives with respect to the prior standard deviations. This would provide some indication on the relation between prior and posterior uncertainty. Table 21 reports measures of sensitivity to the prior uncertainty in the SW07 model, expressed in terms of the rate at which an increase in the prior standard deviation is translated into an increase in the bound on the RMSE. For instance, 1 in the (i, j) cell of the table means that 100% increase in the prior standard deviation of parameter θ_j leads to a 1% increase in the bound for parameter θ_i. Empty cells in the table indicate that the effect is less than 1%. As may be expected, the strongest effect of increasing the prior uncertainty of a parameter is on the posterior uncertainty bound of the same parameter. The magnitude of the effect varies from more than 70% for $\bar{\pi}, \delta$ and λ_w to 1% or less for γ, ρ_a, ρ_g, ρ_w and the standard deviations of the seven shocks.[5] In many cases there is also a substantial effect on the bounds of other parameters; for instance, an increase of 1% in the prior standard deviations of r_π leads to a .23% increase in the posterior bounds for ρ and r_y. Note that the patterns revealed in Table 21 are quite similar to what was discovered earlier about the collinearity relationships among the parameters' effects on the likelihood function. This is not surprising as both measures reflect the fact that information, whether in the likelihood or the prior, is shared among parameters with overlapping functions in the structural model.

A related question concerns the effect of fixing some parameters, i.e. reducing the prior uncertainty to zero. This is a common practice in the DSGE literature; for instance, Smets and Wouters (2007) fix three parameters - δ, λ_w and g_y. How this would affect the estimation results can be analyzed by measuring the change in the posterior bounds before and after a parameter is fixed. The effect of fixing any one of the parameters in the SW07 model is reported in Tables 22 and 23. We can see that the effects of fixing δ or g_y are relatively small, at most 4% and 1.1%, respectively. Fixing λ_w, on the other hand, has a significant effect for two parameters - 16% for ξ_w and 5% for λ, and weaker effects of about 1% for ϕ, σ_l, ρ, β and μ_w. There are a significant (between 16% and 21%) reductions of the bounds of the Taylor rule parameters (ρ, r_y and r_π) when any

[5]The columns for ρ_a, ρ_g, ρ_w and the standard deviations of the shocks are omitted from Table 21 because all sensitivities to these parameters are less than 1%.

of them is fixed. Other notable examples of a strong effect include λ and σ_c as well as some of the structural shock parameters (ρ_b and σ_b, ρ_I and σ_I, μ_p and ρ_p). Overall, the patterns are similar to what we have in Table 21.

6 Concluding Remarks

There are two main reasons why we should care about identification in DSGE models. First, using such models for policy analysis hinges upon having reliably estimated parameters. Obtaining such estimates is impossible when identification fails or is very weak. Second, identification failures often have their roots in the underlying model and the economic theory that motivated it. Thus, detecting identification problems and investigating the causes leading to them may provide useful insights to researchers who are not interested in estimation.

This paper develops a new framework for analyzing parameter identification in linearized DSGE models. By following the steps and applying the tools described here, researchers can assess how well identified the model parameters are, and determine the causes for identification problems when they occur. The main advantage of the methodology is that it does not involve simulation or estimation. This makes it suitable for analysis of large and complicated models prior to their empirical evaluation.

An important lesson learnt from the application of the methodology is that the identification properties of a model are strongly dependent on the parameter values, and may change quite dramatically across different regions in the parameter space. Therefore, it is a mistake to label a model as "weakly identified" or "strongly identified", unless it is determined that either one of this conclusions applies to the large majority of the theoretically plausible parameter values. Unfortunately, the results indicate that many parameters in the Smets and Wouters (2007) model are quite poorly identified in most of the parameter space. The analysis also shows that the identification problems are largely due to the structure of the model, and could not be resolved by extending the set of observed variables. Thus, it may be concluded that this and other similar models are indeed nearly overparameterized, as has been suggested in the literature.

One limitation of the approach in this paper is that it cannot detect certain types of global identification problems. It is possible that some parameters are well identified locally, and yet globally unidentifiable or poorly identified. Such identification failures are less common, but not impossible. Unfortunately, they are very difficult to discover in large and highly non-linear models as those estimated in the DSGE literature.

Table 1: Log-linearized equations of the SW07 model (sticky-price-wage economy)

(1)	$y_t = c_y c_t + i_y i_t + r^{kss} k_y z_t + \varepsilon_t^g$
(2)	$c_t = \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \frac{1}{1+\lambda/\gamma} \mathrm{E}_t\, c_{t+1} + \frac{w^{ss} l^{ss}(\sigma_c - 1)}{c^{ss}\sigma_c(1+\lambda/\gamma)}(l_t - \mathrm{E}_t\, l_{t+1})$
	$-\frac{1-\lambda/\gamma}{(1+\lambda/\gamma)\sigma_c}(r_t - \mathrm{E}_t\, \pi_{t+1}) - \frac{1-\lambda/\gamma}{(1+\lambda\lambda/\gamma)\sigma_c}\varepsilon_t^b$
(3)	$i_t = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1} + \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} \mathrm{E}_t\, i_{t+1} + \frac{1}{\varphi\gamma^2(1+\beta\gamma^{(1-\sigma_c)})} q_t + \varepsilon_t^i$
(4)	$q_t = \beta(1-\delta)\gamma^{-\sigma_c} \mathrm{E}_t\, q_{t+1} - r_t + \mathrm{E}_t\, \pi_{t+1} + (1 - \beta(1-\delta)\gamma^{-\sigma_c}) \mathrm{E}_t\, r_{t+1}^k - \varepsilon_t^b$
(5)	$y_t = \phi_p(\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a)$
(6)	$k_t^s = k_{t-1} + z_t$
(7)	$z_t = \frac{1-\psi}{\psi} r_t^k$
(8)	$k_t = (1-\delta)/\gamma k_{t-1} + (1-(1-\delta)/\gamma) i_t + (1-(1-\delta)/\gamma)\varphi\gamma^2(1+\beta\gamma^{(1-\sigma_c)})\varepsilon_t^i$
(9)	$\mu_t^p = \alpha(k_t^s - l_t) - w_t + \varepsilon_t^a$
(10)	$\pi_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\iota_p\beta\gamma^{(1-\sigma_c)}} \mathrm{E}_t\, \pi_{t+1} + \frac{\iota_p}{1+\beta\gamma^{1-\sigma_c}\iota_p}\pi_{t-1} - \frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_p)(1-\xi_p)}{(1+\iota_p\beta\gamma^{(1-\sigma_c)})(1+(\phi_p-1)\varepsilon_p)\xi_p}\mu_t^p + \varepsilon_t^p$
(11)	$r_t^k = l_t + w_t - k_t$
(12)	$\mu_t^w = w_t - \sigma_l l_t - \frac{1}{1-\lambda/\gamma}(c_t - \lambda/\gamma c_{t-1})$
(13)	$w_t = \frac{\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}}(\mathrm{E}_t\, w_{t+1} + \mathrm{E}_t\, \pi_{t+1}) + \frac{1}{1+\beta\gamma^{(1-\sigma_c)}}(w_{t-1} + \iota_w\pi_{t-1}) - \frac{1+\beta\gamma^{(1-\sigma_c)}\iota_w}{1+\beta\gamma^{(1-\sigma_c)}}\pi_t$
	$-\frac{(1-\beta\gamma^{(1-\sigma_c)}\xi_w)(1-\xi_w)}{(1+\beta\gamma^{(1-\sigma_c)})(1+(\phi_w-1)\varepsilon_w)\xi_w}\mu_t^w + \varepsilon_t^w$
(14)	$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y(y_t - y_t^*)) + r_{\triangle y}((y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)) + \varepsilon_t^r$
(15)	$\varepsilon_t^a = \rho_a \varepsilon_{t-1}^a + \eta_t^a$
(16)	$\varepsilon_t^b = \rho_a \varepsilon_{t-1}^b + \eta_t^b$
(17)	$\varepsilon_t^g = \rho_g \varepsilon_{t-1}^a + \rho_{ga}\eta_t^a + \eta_t^g$
(18)	$\varepsilon_t^i = \rho_I \varepsilon_{t-1}^I + \eta_t^I$
(19)	$\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \eta_t^r$
(20)	$\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p$
(21)	$\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w$

Note: The model variables are: output (y_t), consumption (c_t), investment (i_t), utilized and installed capital ($k_t^s,\ k_t$), capacity utilization (z_t), rental rate of capital (r_t^k), Tobin's q (q_t), price and wage markup ($\mu_t^p,\ \mu_t^w$), inflation rate(π_t), real wage (w_t), total hours worked (l_t), and nominal interest rate (r_t). The shocks are: total factor productivity (ε_t^a), investment-specific technology (ε_t^i), government purchases (ε_t^g), risk premium (ε_t^b), monetary policy (ε_t^r), wage markup (ε_t^w) and price markup (ε_t^p).

Table 2: Log-linearized equations of the SW07 model (flexible-price-wage economy)

(1)	$y_t^* = c_y c_t^* + i_y i_t^* + r^{kss} k_y z_t^* + \varepsilon_t^g$
(2)	$c_t^* = \dfrac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1}^* + \dfrac{1}{1+\lambda/\gamma} \mathrm{E}_t\, c_{t+1}^* + \dfrac{w^{ss} l^{ss}(\sigma_c - 1)}{c^{ss}\sigma_c(1+\lambda/\gamma)}(l_t^* - \mathrm{E}_t\, l_{t+1}^*)$
	$-\frac{1-\lambda/\gamma}{(1+\lambda/\gamma)\sigma_c} r_t^* - \frac{1-\lambda/\gamma}{(1+\lambda\lambda/\gamma)\sigma_c}\varepsilon_t^b$
(3)	$i_t^* = \frac{1}{1+\beta\gamma^{(1-\sigma_c)}} i_{t-1}^* + \frac{\beta\beta\gamma^{(1-\sigma_c)}}{1+\beta\gamma^{(1-\sigma_c)}} \mathrm{E}_t\, i_{t+1}^* + \frac{1}{\varphi\gamma^2(1+\beta\gamma^{(1-\sigma_c)})} q_t^* + \varepsilon_t^i$
(4)	$q_t^* = \beta(1-\delta)\gamma^{-\sigma_c} \mathrm{E}_t\, q_{t+1}^* - r_t^* + (1-\beta(1-\delta)\gamma^{-\sigma_c}) \mathrm{E}_t\, r_{t+1}^{k*} - \varepsilon_t^b$
(5)	$y_t^* = \phi_p(\alpha k_t^{s*} + (1-\alpha) l_t^* + \varepsilon_t^a)$
(6)	$k_t^{s*} = k_{t-1}^* + z_t^*$
(7)	$z_t^* = \frac{1-\psi}{\psi} r_t^{k*}$
(8)	$k_t^* = (1-\delta)/\gamma k_{t-1}^* + (1-(1-\delta)/\gamma) i_t^* + (1-(1-\delta)/\gamma)\varphi\gamma^2(1+\beta\gamma^{(1-\sigma_c)})\varepsilon_t^i$
(9)	$\mu_t^{p*} = \alpha(k_t^{s*} - l_t^*) - w_t^* + \varepsilon_t^a$
(10)	$\mu_t^{p*} = 1$
(11)	$r_t^{k*} = l_t^* + w_t^* - k_t^*$
(12)	$\mu_t^{w*} = -\sigma_l l_t^* - \frac{1}{1-\lambda/\gamma}(c_t^* + \lambda/\gamma c_{t-1}^*)$
(13)	$w_t^* = \mu_t^{w*}$

Note: The model variables are: output (y_t^*), consumption (c_t^*), investment (i_t^*), utilized and installed capital ($k_t^{s*},\ k_t^*$), capacity utilization (z_t^*), rental rate of capital (r_t^{k*}), Tobin's q (q_t^*), price and wage markup ($\mu_t^{p*},\ \mu_t^{w*}$), real wage (w_t^*), and total hours worked (l_t^*).

Table 3: Prior and posterior distribution

par.	interpretation	prior			posterior			
		density	mean	std.	mean	std.	mean ± std.	
φ	invest. adj. cost	$\mathcal{N}$	4.000	1.500	5.744	1.029	4.715	6.773
σ_c	inv. elast. intert. subst.	$\mathcal{N}$	1.500	0.375	1.380	0.131	1.249	1.511
λ	habit	$\mathcal{B}$	0.700	0.100	0.714	0.041	0.673	0.755
ξ_w	wage rigidity	$\mathcal{B}$	0.500	0.100	0.701	0.071	0.630	0.771
σ_l	inv. elast. hours	$\mathcal{N}$	2.000	0.750	1.837	0.619	1.217	2.456
ξ_p	price rigidity	$\mathcal{B}$	0.500	0.100	0.650	0.058	0.592	0.709
ι_w	wage indexation	$\mathcal{B}$	0.500	0.150	0.589	0.133	0.456	0.722
ι_p	price indexation	$\mathcal{B}$	0.500	0.150	0.244	0.092	0.152	0.336
ψ	cap. utilization cost	$\mathcal{B}$	0.500	0.150	0.546	0.115	0.431	0.662
Φ	fixed cost	$\mathcal{N}$	1.250	0.125	1.604	0.078	1.527	1.682
r_π	response to inflation	$\mathcal{N}$	1.500	0.250	2.045	0.181	1.864	2.227
ρ	int rate smoothing	$\mathcal{B}$	0.750	0.100	0.808	0.024	0.784	0.833
r_y	response to output	$\mathcal{N}$	0.125	0.050	0.088	0.022	0.065	0.110
$r_{\triangle y}$	response to output growth	$\mathcal{N}$	0.125	0.050	0.224	0.027	0.196	0.251
$\bar{\pi}$	st.state infl.	$\mathcal{G}$	0.625	0.100	0.785	0.098	0.687	0.883
β'	discount factor*	$\mathcal{G}$	0.250	0.100	0.166	0.060	0.106	0.227
$\bar{l}$	st. state hours	$\mathcal{N}$	0.000	2.000	0.542	0.605	-0.063	1.147
γ	trend growth rate	$\mathcal{N}$	0.400	0.100	0.431	0.014	0.417	0.445
α	capital share	$\mathcal{N}$	0.300	0.050	0.191	0.018	0.173	0.208
δ	depreciation rate†	$\mathcal{B}$	0.025	0.005	n.a.	n.a.	n.a.	n.a.
λ_w	wage markup†	$\mathcal{N}$	1.500	0.250	n.a.	n.a.	n.a.	n.a.
g_y	government/output†	$\mathcal{N}$	0.180	0.050	n.a.	n.a.	n.a.	n.a.
ρ_a	AR prod. shock	$\mathcal{B}$	0.500	0.200	0.958	0.010	0.947	0.968
ρ_b	AR risk premium	$\mathcal{B}$	0.500	0.200	0.217	0.084	0.133	0.301
ρ_g	AR government	$\mathcal{B}$	0.500	0.200	0.976	0.008	0.968	0.985
ρ_I	AR investment	$\mathcal{B}$	0.500	0.200	0.711	0.059	0.652	0.770
ρ_r	AR mon. policy	$\mathcal{B}$	0.500	0.200	0.151	0.065	0.086	0.217
ρ_p	AR price markup	$\mathcal{B}$	0.500	0.200	0.891	0.046	0.845	0.938
ρ_w	AR wage markup	$\mathcal{B}$	0.500	0.200	0.968	0.013	0.955	0.981
μ_p	MA price markup	$\mathcal{B}$	0.500	0.200	0.699	0.087	0.612	0.786
μ_w	MA wage markup	$\mathcal{B}$	0.500	0.200	0.841	0.051	0.790	0.893
ρ_{ga}	prod. shock in G	$\mathcal{B}$	0.500	0.200	0.521	0.089	0.432	0.610
σ_a	st.dev. prod. shock	$\mathcal{IG}$	0.100	2.000	0.460	0.027	0.432	0.487
σ_b	st.dev. risk premium	$\mathcal{IG}$	0.100	2.000	0.240	0.023	0.217	0.264
σ_g	st.dev. government	$\mathcal{IG}$	0.100	2.000	0.529	0.030	0.499	0.559
σ_I	st.dev. investment	$\mathcal{IG}$	0.100	2.000	0.453	0.048	0.405	0.502
σ_r	st.dev. mon. policy	$\mathcal{IG}$	0.100	2.000	0.245	0.015	0.231	0.260
σ_p	st.dev. price markup	$\mathcal{IG}$	0.100	2.000	0.140	0.017	0.123	0.157
σ_w	st.dev. wage markup	$\mathcal{IG}$	0.100	2.000	0.244	0.022	0.222	0.266

Note: $\mathcal{N}$ is Normal distribution, $\mathcal{B}$ is Beta-distribution, $\mathcal{G}$ is Gamma distribution, $\mathcal{IG}$ is Inverse Gamma distribution.

* $\beta' := 100(\beta^{-1} - 1)$ where β is the discount factor.

† these parameters are assumed known in SW07.

Table 4: Identification strength at the posterior mean

Par.	$\hat{\theta}_i$	$\hat{\theta}_i \pm \mathrm{std}(\hat{\theta}_i)$		$\boldsymbol{s}_i^r(\hat{\boldsymbol{\theta}})$	**sens.**	**col.**	$\boldsymbol{\varrho}_i$
φ	5.744	3.432	8.056	2.5	5.5	0.45	0.894
σ_c	1.380	1.086	1.675	4.7	21.9	0.21	0.977
λ	0.714	0.651	0.777	11.4	33.1	0.34	0.939
ξ_w	0.701	0.515	0.886	3.8	40.3	0.09	0.996
σ_l	1.837	0.767	2.906	1.7	4.4	0.39	0.920
ξ_p	0.650	0.590	0.711	10.8	28.3	0.38	0.924
ι_w	0.589	0.379	0.799	2.8	4.0	0.70	0.716
ι_p	0.244	0.110	0.377	1.8	4.1	0.44	0.898
ψ	0.546	0.373	0.719	3.2	5.4	0.58	0.812
Φ	1.604	1.467	1.742	11.7	24.7	0.47	0.882
r_π	2.045	1.628	2.463	4.9	16.4	0.30	0.955
ρ	0.808	0.762	0.854	17.6	50.2	0.35	0.937
r_y	0.088	0.047	0.128	2.2	6.2	0.35	0.937
$r_{\triangle y}$	0.224	0.174	0.273	4.5	9.3	0.48	0.876
$\bar{\pi}$	0.785	0.555	1.016	3.4	5.9	0.57	0.818
β'	0.166	0.006	0.326	1.0	1.9	0.55	0.835
$\bar{l}$	0.542	-0.966	2.050	0.4	0.6	0.58	0.815
γ	0.431	0.420	0.442	39.7	40.4	0.98	0.180
α	0.191	0.168	0.213	8.6	14.0	0.61	0.789
δ	0.025	0.014	0.036	2.4	5.3	0.44	0.897
λ_w	1.500	0.707	2.293	1.9	20.2	0.09	0.996
g_y	0.180	0.109	0.251	2.5	4.0	0.64	0.769
ρ_a	0.958	0.942	0.974	60.0	95.2	0.63	0.777
ρ_b	0.217	0.127	0.307	2.4	5.1	0.47	0.881
ρ_g	0.976	0.965	0.988	87.5	120.0	0.73	0.684
ρ_I	0.711	0.643	0.778	10.6	24.6	0.43	0.903
ρ_r	0.151	0.056	0.247	1.6	2.1	0.76	0.650
ρ_p	0.891	0.832	0.951	15.0	66.6	0.22	0.974
ρ_w	0.968	0.952	0.984	61.4	124.4	0.49	0.870
μ_p	0.699	0.548	0.850	4.6	29.8	0.16	0.988
μ_w	0.841	0.781	0.902	14.0	51.5	0.27	0.963
ρ_{ga}	0.521	0.416	0.626	5.0	5.5	0.90	0.438
σ_a	0.460	0.424	0.495	12.9	17.3	0.75	0.666
σ_b	0.240	0.213	0.268	8.7	17.3	0.50	0.865
σ_g	0.529	0.493	0.564	14.9	17.3	0.86	0.513
σ_I	0.453	0.402	0.505	8.8	17.3	0.51	0.862
σ_r	0.245	0.228	0.263	13.9	17.3	0.81	0.591
σ_p	0.140	0.117	0.163	6.2	17.3	0.36	0.934
σ_w	0.244	0.216	0.272	8.7	17.4	0.50	0.865

Note: The values of $\mathrm{std}(\hat{\theta}_i) = 1/\boldsymbol{s}_i(\hat{\boldsymbol{\theta}})$ are computed using the Cramér-Rao lower bounds for θ_i at the posterior mean $\hat{\boldsymbol{\theta}}$. Under the labels **sens.** and **col.** are shown the sensitivity and collinearity components of the relative strength of identification ($\boldsymbol{s}_i^r(\hat{\boldsymbol{\theta}}) := |\theta_i|/\,\mathrm{std}(\hat{\theta}_i) =$ **sens.** $\times$ **col.**); $\boldsymbol{\varrho}_i$ is the coefficient of multiple correlation between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}$.

Table 5: Maximal multiple correlation coefficients at the posterior mean

Par.	$\boldsymbol{\varrho}_i$	$\boldsymbol{\varrho}_{i(1)}$	$\boldsymbol{\varrho}_{i(2)}$	$\boldsymbol{\varrho}_{i(3)}$	$\boldsymbol{\varrho}_{i(4)}$
φ	0.894	0.753 (ρ_I)	0.806 (λ,ρ_I)	0.812 (λ,Φ,ρ_I)	0.825 (λ,ξ_w,ρ_I,λ_w)
σ_c	0.977	0.739 (λ)	0.849 (ξ_w,λ_w)	0.898 (β',ξ_w,λ_w)	0.930 (β',λ,ξ_w,λ_w)
λ	0.939	0.739 (σ_c)	0.794 (σ_c,g_y)	0.822 (β',σ_c,g_y)	0.833 (β',σ_c,ρ_b,g_y)
ξ_w	0.996	0.944 (λ_w)	0.982 (σ_c,λ_w)	0.987 (μ_w,σ_c,λ_w)	0.990 (β',μ_w,σ_c,λ_w)
σ_l	0.920	0.790 (λ_w)	0.815 (μ_w,ξ_w)	0.848 (λ,$r_{\triangle y}$,λ_w)	0.858 (μ_w,λ,$r_{\triangle y}$,λ_w)
ξ_p	0.924	0.796 (ρ_p)	0.828 (ξ_w,ρ_p)	0.853 (ξ_w,ρ_p,σ_w)	0.872 (ξ_w,r_π,ρ_p,σ_w)
ι_w	0.716	0.440 (σ_w)	0.572 (σ_p,σ_w)	0.652 (ξ_w,σ_p,σ_w)	0.686 (ξ_w,ρ_p,σ_p,σ_w)
ι_p	0.898	0.813 (μ_p)	0.851 (μ_p,σ_p)	0.867 (μ_p,ρ_p,σ_p)	0.880 (μ_w,μ_p,ρ_p,σ_p)
ψ	0.812	0.535 (δ)	0.564 ($r_{\triangle y}$,δ)	0.606 (λ,$r_{\triangle y}$,δ)	0.645 (Φ,ξ_p,σ_a,δ)
Φ	0.882	0.461 (ξ_p)	0.559 (ξ_p,σ_a)	0.634 (α,ξ_p,σ_a)	0.678 (α,ξ_p,σ_a,δ)
r_π	0.955	0.553 (ρ)	0.866 (r_y,ρ)	0.912 (σ_c,r_y,ρ)	0.924 (σ_c,$r_{\triangle y}$,r_y,ρ)
ρ	0.937	0.553 (r_π)	0.824 (r_π,r_y)	0.870 (σ_c,r_π,r_y)	0.898 (σ_c,r_π,r_y,ρ_r)
r_y	0.937	0.517 (r_π)	0.813 (r_π,ρ)	0.886 (σ_c,r_π,ρ)	0.901 (σ_c,r_π,ρ,ρ_I)
$r_{\triangle y}$	0.876	0.521 (λ)	0.606 (λ,σ_r)	0.665 (λ,r_π,σ_r)	0.721 (λ,r_π,ρ_r,σ_r)
$\bar{\pi}$	0.818	0.809 ($\bar{l}$)	0.815 ($\bar{l}$,β')	0.818 ($\bar{l}$,β',γ)	0.818 ($\bar{l}$,β',α,γ)
β'	0.835	0.361 ($\bar{\pi}$)	0.432 ($\bar{\pi}$,α)	0.562 (σ_c,ξ_w,λ_w)	0.623 (σ_c,λ,ξ_w,λ_w)
$\bar{l}$	0.815	0.809 ($\bar{\pi}$)	0.814 ($\bar{\pi}$,γ)	0.815 ($\bar{\pi}$,β',γ)	0.815 ($\bar{\pi}$,β',α,γ)
γ	0.180	0.114 ($\bar{l}$)	0.150 ($\bar{l}$,$\bar{\pi}$)	0.178 ($\bar{l}$,$\bar{\pi}$,β')	0.180 ($\bar{l}$,$\bar{\pi}$,β',α)
α	0.789	0.601 (δ)	0.665 (Φ,δ)	0.694 (Φ,σ_a,δ)	0.714 (β',Φ,σ_a,δ)
δ	0.897	0.601 (α)	0.708 (α,g_y)	0.756 (α,ρ_a,g_y)	0.784 (α,ψ,ρ_a,g_y)
λ_w	0.996	0.944 (ξ_w)	0.984 (σ_c,ξ_w)	0.989 (β',σ_c,ξ_w)	0.991 (β',σ_c,ξ_w,r_y)
g_y	0.769	0.461 (δ)	0.542 ($r_{\triangle y}$,δ)	0.587 ($r_{\triangle y}$,r_y,δ)	0.602 (Φ,$r_{\triangle y}$,r_y,δ)
ρ_a	0.777	0.358 (ρ_g)	0.534 (ρ_g,δ)	0.650 (σ_c,ρ_g,δ)	0.671 (σ_c,λ,ρ_g,δ)
ρ_b	0.881	0.834 (σ_b)	0.868 (λ,σ_b)	0.870 (σ_c,λ,σ_b)	0.870 (σ_c,λ,$r_{\triangle y}$,σ_b)
ρ_g	0.684	0.358 (ρ_a)	0.448 (ρ_a,δ)	0.554 (σ_c,ρ_a,δ)	0.575 (σ_c,λ,ρ_a,δ)
ρ_I	0.903	0.840 (σ_I)	0.882 (φ,σ_I)	0.886 (φ,σ_I,δ)	0.888 (φ,$r_{\triangle y}$,σ_I,δ)
ρ_r	0.650	0.451 (ρ)	0.547 ($r_{\triangle y}$,ρ)	0.580 ($r_{\triangle y}$,ρ,σ_r)	0.587 ($r_{\triangle y}$,ρ,ρ_b,σ_r)
ρ_p	0.974	0.964 (μ_p)	0.968 (μ_p,ξ_p)	0.969 (μ_p,ι_p,ξ_p)	0.973 (μ_p,ι_p,ξ_p,σ_p)
ρ_w	0.870	0.756 (ξ_w)	0.809 (ξ_w,δ)	0.822 (ξ_w,r_y,δ)	0.828 (μ_w,ξ_w,r_y,δ)
μ_p	0.988	0.964 (ρ_p)	0.976 (ρ_p,σ_p)	0.986 (ι_p,ρ_p,σ_p)	0.987 (μ_w,ι_p,ρ_p,σ_p)
μ_w	0.963	0.904 (ξ_w)	0.933 (ξ_w,σ_w)	0.941 (ξ_w,r_π,σ_w)	0.946 (ξ_w,σ_l,r_π,σ_w)
ρ_{ga}	0.438	0.233 (Φ)	0.263 (Φ,ξ_p)	0.290 (Φ,ξ_p,g_y)	0.311 (Φ,ξ_p,r_y,g_y)
σ_a	0.666	0.316 (Φ)	0.376 (ψ,Φ)	0.431 (ψ,Φ,ξ_p)	0.485 (α,ψ,Φ,δ)
σ_b	0.865	0.834 (ρ_b)	0.842 (λ,ρ_b)	0.848 (σ_c,λ,ρ_b)	0.850 (σ_c,λ,ρ_b,g_y)
σ_g	0.513	0.214 (Φ)	0.261 (ψ,Φ)	0.327 (ψ,Φ,g_y)	0.370 (ψ,Φ,ξ_p,g_y)
σ_I	0.862	0.840 (ρ_I)	0.841 (ρ_I,δ)	0.843 (α,ρ_I,δ)	0.845 (σ_l,ρ_I,ρ_w,δ)
σ_r	0.591	0.318 ($r_{\triangle y}$)	0.370 (r_π,$r_{\triangle y}$)	0.414 (r_π,$r_{\triangle y}$,ρ_r)	0.454 (r_π,$r_{\triangle y}$,r_y,ρ_r)
σ_p	0.934	0.880 (μ_p)	0.904 (μ_p,ι_p)	0.918 (μ_p,ι_w,ι_p)	0.924 (μ_p,ι_w,ι_p,ρ_p)
σ_w	0.865	0.725 (μ_w)	0.802 (μ_w,ι_w)	0.819 (μ_w,ι_w,ξ_p)	0.827 (μ_w,ι_w,ξ_p,ρ_w)

Note: For $\boldsymbol{\theta}$ equal to the posterior mean and n between 1 and 4, the table shows the values of $\boldsymbol{\varrho}_{i(n)}$, defined as largest among all coefficients of multiple correlation between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}(n)$ for θ_i in the first column and all possible combinations of n parameters from $\boldsymbol{\theta}_{-i}$. The selected parameters are shown in parentheses. $\boldsymbol{\varrho}_i$ in the second column is the coefficient of multiple correlation between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}$

Table 6: Sensitivity and collinearity in the model at the posterior mean

Par.	sens.	collinearity, ♯ of parameters all	1	2	3	4
φ	233	0.9364	0.522 (Φ)	0.734 (Φ,ρ_I)	0.803 (ρ,ρ_I,ρ_p)	0.8187 (β',ρ,ρ_I,ρ_p)
σ_c	595	0.9983	0.918 (λ)	0.956 (α,λ)	0.977 (α,λ,ρ_a)	0.9888 (λ,σ_l,$r_{\triangle y}$,λ_w)
λ	1753	0.9971	0.918 (σ_c)	0.952 (α,σ_c)	0.977 (σ_c,σ_b,δ)	0.9853 (σ_c,σ_l,$r_{\triangle y}$,λ_w)
ξ_w	765	0.9997	0.992 (λ_w)	0.995 (r_y,λ_w)	0.997 (ι_w,r_y,λ_w)	0.9986 (ι_w,σ_l,r_y,λ_w)
σ_l	289	0.9917	0.876 (λ_w)	0.931 (ρ_a,λ_w)	0.951 (σ_c,ξ_w,σ_b)	0.9774 (σ_c,λ,$r_{\triangle y}$,λ_w)
ξ_p	1721	0.9961	0.831 (ι_p)	0.945 (Φ,ρ_w)	0.964 (Φ,ι_p,λ_w)	0.9728 (Φ,ι_w,ρ_p,λ_w)
ι_w	148	0.9712	0.508 (ρ_p)	0.746 (ξ_p,ρ_p)	0.838 (ξ_p,ρ_p,δ)	0.9119 (ξ_w,ξ_p,ρ_p,ρ_w)
ι_p	186	0.9673	0.831 (ξ_p)	0.894 (ξ_p,λ_w)	0.925 (ξ_p,δ,λ_w)	0.9292 (ξ_p,ρ,δ,λ_w)
ψ	229	0.4165	0.221 (β')	0.263 (r_y,ρ_w)	0.330 (α,σ_l,δ)	0.3554 (α,σ_l,ρ_p,δ)
Φ	1969	0.9893	0.796 (ξ_p)	0.961 (ξ_p,ρ_w)	0.969 (ξ_p,σ_l,ρ_w)	0.9745 (ξ_w,ξ_p,$r_{\triangle y}$,λ_w)
r_π	481	0.9993	0.987 (ρ)	0.998 (r_y,ρ)	0.998 ($r_{\triangle y}$,r_y,ρ)	0.9985 ($r_{\triangle y}$,r_y,ρ,σ_b)
ρ	551	0.9984	0.987 (r_π)	0.997 (r_π,r_y)	0.997 (ι_p,r_π,r_y)	0.9975 (ι_p,ξ_p,r_π,r_y)
r_y	169	0.9950	0.845 (r_π)	0.963 (r_π,ρ)	0.969 (ξ_w,r_π,ρ)	0.9812 (ξ_p,r_π,ρ,ρ_p)
$r_{\triangle y}$	163	0.9935	0.979 (r_π)	0.983 (r_π,λ_w)	0.987 (r_π,σ_r,λ_w)	0.9897 (α,ι_w,r_π,σ_r)
$\bar{\pi}$	1	0.6756	0.364 (β')	0.411 (β',α)	0.502 (β',α,δ)	0.5787 (β',α,δ,λ_w)
β'	33	0.9428	0.555 (γ)	0.755 (α,γ)	0.791 (α,ξ_w,γ)	0.8585 ($\bar{\pi}$,α,δ,λ_w)
$\bar{l}$	1	0.0000	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
γ	51	0.7825	0.611 (δ)	0.730 (β',δ)	0.748 ($\bar{\pi}$,β',δ)	0.7607 ($\bar{\pi}$,β',λ,δ)
α	333	0.9634	0.512 ($r_{\triangle y}$)	0.746 (σ_l,$r_{\triangle y}$)	0.859 (β',δ,λ_w)	0.8991 (β',σ_l,$r_{\triangle y}$,δ)
δ	166	0.9819	0.703 (ρ_w)	0.888 (ξ_w,ρ_w)	0.916 (α,ξ_w,ρ_w)	0.9408 (α,ξ_w,ρ_a,ρ_w)
λ_w	586	0.9998	0.992 (ξ_w)	0.995 (ξ_w,r_y)	0.997 (ξ_w,σ_l,r_y)	0.9988 (ι_w,ξ_w,σ_l,r_y)
g_y	189	0.9003	0.730 (r_π)	0.777 (r_π,δ)	0.811 (r_π,ρ_a,δ)	0.8392 (α,Φ,ξ_p,r_π)
ρ_a	3124	0.9523	0.710 (σ_a)	0.756 (ρ_{ga},σ_a)	0.815 (ξ_w,σ_l,σ_a)	0.8360 (α,σ_c,λ,σ_a)
ρ_b	22	0.8222	0.546 (λ)	0.621 (λ,σ_r)	0.652 (λ,ρ_w,σ_r)	0.6774 (λ,ρ_w,σ_b,σ_r)
ρ_g	13241	0.7272	0.302 (σ_g)	0.401 (σ_a,σ_g)	0.467 (ρ_{ga},σ_a,σ_g)	0.5183 (σ_l,ρ_a,σ_g,λ_w)
ρ_I	231	0.8600	0.800 (σ_I)	0.818 (φ,σ_I)	0.827 (φ,Φ,σ_I)	0.8342 (φ,ρ,ρ_p,σ_I)
ρ_r	24	0.3119	0.280 (σ_r)	0.280 (σ_b,σ_r)	0.299 ($r_{\triangle y}$,ρ,σ_r)	0.3052 (ι_p,$r_{\triangle y}$,ρ,σ_r)
ρ_p	1965	0.9955	0.821 (ξ_p)	0.924 (ι_w,ξ_p)	0.953 (ι_w,ξ_p,δ)	0.9751 (ι_w,ξ_p,ρ_w,λ_w)
ρ_w	390	0.9946	0.851 (λ_w)	0.965 (ξ_w,δ)	0.971 (ρ_a,δ,λ_w)	0.9745 (α,ξ_w,ρ_a,δ)
μ_p	567	0.8629	0.477 (ξ_p)	0.542 (ι_w,ξ_p)	0.585 (ξ_p,$r_{\triangle y}$,r_y)	0.6253 (ξ_p,$r_{\triangle y}$,r_y,ρ_p)
μ_w	220	0.7073	0.556 (ξ_w)	0.602 (ξ_w,ρ_w)	0.633 (ξ_w,ρ_w,δ)	0.6510 (ξ_w,ρ_w,σ_w,δ)
ρ_{ga}	458	0.6516	0.536 (ρ_a)	0.600 (ρ_a,ρ_g)	0.625 (ρ_a,ρ_g,σ_g)	0.6336 (ρ_a,ρ_g,σ_a,σ_g)
σ_a	1182	0.7528	0.710 (ρ_a)	0.724 (ρ_a,ρ_g)	0.732 (ρ_{ga},ρ_a,ρ_g)	0.7329 (ρ_{ga},ρ_a,ρ_g,σ_g)
σ_b	65	0.9835	0.840 (λ)	0.936 (λ,ρ_w)	0.952 (λ,σ_l,δ)	0.9571 (λ,ξ_w,δ,λ_w)
σ_g	1146	0.4797	0.351 (ρ_{ga})	0.421 (ρ_{ga},ρ_g)	0.429 (ρ_{ga},ρ_g,σ_a)	0.4338 (ρ_{ga},α,ρ_g,σ_a)
σ_I	332	0.8589	0.800 (ρ_I)	0.804 (α,ρ_I)	0.806 (α,ρ_I,g_y)	0.8090 (β',φ,ρ_I,g_y)
σ_r	44	0.7358	0.570 (σ_b)	0.624 (ρ_b,σ_b)	0.651 (ρ_b,σ_b,σ_p)	0.6688 (ρ_b,ρ_r,σ_b,σ_p)
σ_p	264	0.5094	0.317 (σ_w)	0.404 (σ_r,σ_w)	0.418 (μ_p,σ_r,σ_w)	0.4318 (μ_p,ξ_p,σ_r,σ_w)
σ_w	79	0.3854	0.317 (σ_p)	0.348 (μ_w,σ_p)	0.364 (μ_w,ξ_w,σ_p)	0.3702 (μ_w,ι_w,ξ_w,σ_p)

Note: Sensitivity is measured by the square root of $\sum \left(\frac{\partial \tau_j}{\partial \theta_i}\frac{\theta_i}{\tau_j}\right)^2$. Collinearity is measured by the cosine of the angle between $\partial\boldsymbol{\tau}/\partial\theta_i$ and $\partial\boldsymbol{\tau}/\partial\boldsymbol{\theta}_{-i}$. Also shown are subsets of 1 to 4 elements of $\boldsymbol{\theta}_{-i}$ having the strongest collinearity with θ_i among all subsets of that size.

Table 7: Observables and identification strength at the posterior mean

	the observables include $\triangle y_t, \triangle c_t, \triangle i_t, l_t, \pi_t, \triangle w_t, r_t$ except:						
Par.	$\triangle y_t$	$\triangle c_t$	$\triangle i_t$	l_t	π_t	$\triangle w_t$	r_t
φ	0.9175	0.6979	0.3236	0.8432	0.6811	0.9308	0.5405
σ_c	0.9121	0.2445	0.7113	0.8406	0.4865	0.8989	0.3338
λ	0.8746	0.5580	0.7333	0.7737	0.8924	0.8821	0.5659
ξ_w	0.6062	0.2256	0.6419	0.4985	0.4608	0.6874	0.3554
σ_l	0.8482	0.5258	0.8662	0.4549	0.7735	0.7993	0.7277
ξ_p	0.2994	0.8582	0.7301	0.2375	0.4429	0.4177	0.8178
ι_w	0.9688	0.9572	0.9626	0.9526	0.3448	0.1226	0.9571
ι_p	0.9388	0.9622	0.9812	0.8730	0.2605	0.3383	0.9715
ψ	0.6011	0.7836	0.5492	0.7855	0.7986	0.7637	0.7286
Φ	0.1494	0.8367	0.6064	0.1108	0.9048	0.7300	0.8098
r_π	0.9565	0.8046	0.9133	0.8221	0.3534	0.8752	0.3477
ρ	0.9618	0.9231	0.9545	0.8152	0.2755	0.8938	0.4263
r_y	0.6299	0.8162	0.9228	0.7884	0.4108	0.8877	0.4255
$r_{\triangle y}$	0.3836	0.6153	0.8856	0.8659	0.7429	0.9027	0.2656
$\bar{\pi}$	0.9995	0.9819	0.9978	0.9993	0.1485	0.9970	0.9970
β'	0.9399	0.2993	0.7729	0.8840	0.1138	0.9266	0.1026
$\bar{l}$	0.9975	0.9940	0.9957	0	0.9983	0.9971	0.9980
γ	0.9510	0.6022	0.5798	0.9745	0.9960	0.8920	0.7600
α	0.2157	0.4822	0.1895	0.7773	0.2963	0.9123	0.2526
δ	0.7813	0.6654	0.5670	0.6936	0.7600	0.8806	0.6058
λ_w	0.5803	0.2271	0.6198	0.4691	0.4247	0.8994	0.3443
g_y	0.1586	0.1291	0.4374	0.8766	0.8614	0.8765	0.8765
ρ_a	0.6654	0.7134	0.6799	0.5937	0.9500	0.9323	0.7644
ρ_b	0.9810	0.4628	0.8894	0.9599	0.9753	0.9590	0.5670
ρ_g	0.5040	0.4693	0.6799	0.7352	0.9642	0.9534	0.7889
ρ_I	0.9672	0.8290	0.3691	0.9275	0.9413	0.9768	0.8564
ρ_r	0.9538	0.8472	0.9726	0.7943	0.6401	0.9658	0.0186
ρ_p	0.7883	0.9681	0.9828	0.7730	0.7265	0.1979	0.9716
ρ_w	0.9614	0.7960	0.8279	0.7516	0.9117	0.5463	0.9027
μ_p	0.9015	0.9403	0.9799	0.8342	0.5569	0.2710	0.9787
μ_w	0.9508	0.8159	0.9309	0.9011	0.7818	0.1166	0.8876
ρ_{ga}	0.0573	0.6349	0.5892	0.2364	0.8550	0.9455	0.8096
σ_a	0.1067	0.8499	0.5556	0.1340	0.8861	0.8560	0.7687
σ_b	0.9808	0.1355	0.9542	0.9515	0.9833	0.9777	0.4700
σ_g	0.0610	0.3711	0.4565	0.5855	0.8308	0.8824	0.8466
σ_I	0.9753	0.9055	0.1282	0.9463	0.9523	0.9658	0.8515
σ_r	0.9418	0.8562	0.9425	0.7715	0.3946	0.9640	0.0097
σ_p	0.9272	0.9948	0.9875	0.8728	0.2123	0.3072	0.9654
σ_w	0.9661	0.9641	0.9542	0.9367	0.6986	0.0433	0.9586

Note: Each column reports the strength of identification of θ_i when the variable in the first row is unobserved, relative to when all seven variables are observed.

Table 8: Identification strength in $\boldsymbol{\Theta}$

Par.	mean	CoV	deciles of $\boldsymbol{s}_i^r(\boldsymbol{\theta})$ 1	2	3	4	5	6	7	8	9
φ	4.6	0.88	2.4	3.5	4.0	4.4	4.8	5.4	6.1	6.9	9.0
σ_c	5.0	0.90	2.5	3.7	4.1	4.7	5.3	5.8	6.7	8.2	10.3
λ	48.1	3.05	10.0	16.3	19.6	23.7	28.5	35.8	49.2	65.4	113.1
ξ_w	3.0	1.46	0.5	1.2	1.7	2.2	2.8	3.5	4.2	5.5	7.7
σ_l	5.2	8.90	1.2	2.2	3.0	3.5	4.2	5.5	6.6	8.6	12.6
ξ_p	18.6	0.98	7.2	12.4	14.9	17.2	19.3	22.5	26.9	30.4	39.0
ι_w	7.0	1.12	1.8	3.2	4.1	5.2	6.3	7.7	9.7	12.2	16.9
ι_p	10.0	1.26	2.2	4.1	5.5	7.1	9.1	11.0	13.9	17.5	25.0
ψ	10.1	10.50	2.9	5.2	6.0	8.1	9.5	11.1	13.5	16.6	27.2
Φ	31.1	6.16	11.0	17.9	20.8	23.7	27.0	35.1	40.8	54.0	74.3
r_π	9.2	1.66	2.6	4.3	5.3	7.0	7.5	8.9	11.6	17.6	24.0
ρ	29.3	1.30	10.1	16.0	18.6	21.5	25.3	30.6	35.4	45.3	65.4
r_y	2.2	1.16	0.6	1.2	1.4	1.7	2.0	2.4	2.9	4.1	5.1
$r_{\triangle y}$	7.3	1.25	1.8	3.2	4.2	5.2	6.4	7.7	9.5	12.2	17.4
$\bar{\pi}$	6.6	0.84	1.7	3.1	3.9	4.9	5.8	7.4	9.4	12.1	16.8
β'	1.5	1.12	0.6	1.0	1.1	1.4	1.5	2.0	2.0	2.5	3.2
$\bar{l}$	8.0	1.10	0.8	2.0	3.4	4.5	6.4	8.3	11.2	15.1	22.1
γ	302.7	0.64	121.9	283.0	349.9	399.6	468.2	612.7	627.9	761.4	946.3
α	17.0	5.12	6.4	11.8	13.7	16.3	19.6	24.1	26.7	30.1	45.9
δ	5.6	2.15	1.7	3.5	4.1	4.4	5.3	6.5	7.9	10.2	14.2
λ_w	2.8	1.50	0.5	1.1	1.6	2.1	2.7	3.3	4.1	5.3	6.9
g_y	7.6	2.13	2.7	4.4	5.5	6.5	7.5	8.8	11.4	12.7	19.5
ρ_a	8.8	1.06	2.7	4.8	6.0	7.2	8.7	10.3	12.3	15.3	20.4
ρ_b	17.3	4.73	2.5	4.6	5.8	7.2	9.1	11.8	15.4	22.1	39.2
ρ_g	8.9	1.03	2.8	4.9	6.1	7.4	8.7	10.3	12.3	15.3	20.4
ρ_I	7.7	0.91	2.4	4.2	5.3	6.4	7.5	8.9	10.9	13.2	17.9
ρ_r	25.8	7.60	2.4	4.3	5.7	7.3	9.4	12.8	17.5	27.2	53.1
ρ_p	3.5	2.25	0.3	0.8	1.2	1.7	2.2	2.9	3.9	5.5	9.3
ρ_w	3.8	2.25	0.3	0.8	1.2	1.8	2.4	3.1	4.2	5.8	10.0
μ_p	4.2	1.34	0.3	0.7	1.0	1.4	2.1	2.8	4.9	10.5	20.3
μ_w	4.5	1.36	0.3	0.7	1.1	1.5	2.1	3.1	4.9	9.0	19.8
ρ_{ga}	6.1	1.00	0.8	1.9	2.9	3.9	5.2	6.8	8.8	11.8	17.7
σ_a	12.0	0.14	9.6	13.6	14.8	15.1	15.6	15.8	16.3	16.7	17.0
σ_b	8.7	0.20	6.7	9.0	9.4	9.9	10.3	11.0	11.6	12.3	13.6
σ_g	12.8	0.08	11.3	15.0	15.6	15.8	16.0	16.5	16.6	16.8	17.0
σ_I	8.7	0.12	7.6	9.7	10.1	10.3	10.5	10.8	11.1	11.4	12.2
σ_r	11.2	0.23	7.5	11.0	12.2	13.4	14.6	15.4	16.2	16.7	17.0
σ_p	9.4	0.21	6.7	9.8	10.7	11.6	12.0	12.5	13.0	13.6	14.1
σ_w	9.6	0.19	7.1	10.0	10.9	11.6	12.2	12.8	13.3	13.9	14.4

Note: The table shows the mean, the coefficient of variation, and the deciles of the relative strength of identification measure ($\boldsymbol{s}_i^r(\hat{\boldsymbol{\theta}}) := |\theta_i| / \operatorname{std}(\hat{\theta}_i)$, where $\operatorname{std}(\hat{\theta}_i)$ is the Cramér-Rao lower bound for θ_i). The results are based on $100,000$ draws from $\boldsymbol{\Theta}$.

Table 9: Distributions of the sensitivity components

			deciles								
Par.	mean	CoV	1	2	3	4	5	6	7	8	9
φ	43.3	2.11	8.6	11.6	14.9	18.7	23.5	29.9	39.0	54.0	87.2
σ_c	93.4	2.71	27.3	32.7	38.1	44.5	52.5	63.6	79.7	107.1	170.3
λ	327.6	3.91	33.8	48.4	65.1	86.2	115.0	156.1	220.1	333.6	625.4
ξ_w	173.4	2.23	25.8	35.8	47.7	62.3	81.4	107.7	146.6	212.0	363.9
σ_l	52.1	3.00	5.7	8.5	11.9	16.3	22.0	30.2	42.3	63.1	111.6
ξ_p	133.7	2.96	22.0	27.8	34.8	44.1	56.9	75.5	103.7	154.8	271.7
ι_w	21.7	2.36	3.2	4.7	6.3	8.3	10.6	13.8	18.6	26.5	44.7
ι_p	28.9	2.60	6.0	7.6	9.2	11.3	14.0	17.8	23.8	34.1	57.8
ψ	48.4	3.68	9.7	13.6	17.5	21.8	26.9	33.5	43.0	58.5	92.7
Φ	210.9	3.72	48.4	63.8	78.9	96.2	117.0	144.4	183.6	250.9	398.1
r_π	59.0	3.92	14.2	17.6	21.0	25.0	30.0	36.7	46.6	63.7	104.9
ρ	301.4	5.90	41.5	56.4	72.1	90.4	114.4	148.2	200.1	294.7	530.3
r_y	11.0	3.28	2.2	3.1	3.8	4.7	5.7	7.1	9.1	12.6	20.4
$r_{\triangle y}$	36.1	5.46	4.9	7.2	9.5	12.1	15.4	19.7	26.2	37.2	64.7
$\bar{\pi}$	24.2	0.90	6.4	9.1	11.7	14.6	17.9	21.9	27.1	34.8	49.3
β'	10.5	11.32	3.1	4.1	5.0	6.0	7.0	8.4	10.1	12.6	17.9
$\bar{l}$	26.5	1.26	2.4	4.9	7.8	11.3	15.5	21.0	28.6	40.3	63.2
γ	312.0	0.68	127.0	164.9	198.1	231.6	267.2	308.0	357.5	425.8	540.9
α	84.0	12.32	17.7	23.6	29.7	36.7	45.0	55.9	71.7	97.4	155.3
δ	17.6	12.86	3.3	4.5	5.7	7.0	8.7	10.9	14.2	19.6	31.9
λ_w	195.0	2.37	25.1	36.0	49.0	65.0	86.0	115.3	160.3	236.0	412.5
g_y	38.4	3.82	6.8	9.5	12.3	15.3	19.1	24.1	31.4	43.8	72.8
ρ_a	12.5	1.83	2.9	4.2	5.3	6.5	7.9	9.6	12.0	15.5	23.1
ρ_b	139.9	8.25	5.9	9.4	13.4	18.5	25.9	37.4	58.1	101.9	233.9
ρ_g	12.5	3.21	3.0	4.2	5.4	6.6	8.0	9.7	11.9	15.3	22.6
ρ_I	16.1	2.01	3.9	5.8	7.4	9.0	10.9	13.0	15.7	19.5	27.3
ρ_r	120.0	10.51	3.2	5.0	7.1	10.1	14.6	22.1	35.7	65.8	163.0
ρ_p	18.7	4.56	2.8	5.1	7.2	9.3	11.5	14.2	17.5	22.4	32.3
ρ_w	18.0	3.68	2.8	4.9	6.8	8.8	10.9	13.2	16.2	20.7	29.9
μ_p	16.1	0.97	3.8	5.8	7.7	9.8	12.4	15.7	20.1	24.4	30.2
μ_w	15.3	1.14	3.5	5.3	7.2	9.2	11.5	14.6	18.7	23.2	29.0
ρ_{ga}	7.6	1.56	0.9	1.7	2.5	3.4	4.5	5.8	7.6	10.3	16.1
σ_a	14.1	0.01	14.0	14.0	14.0	14.1	14.1	14.1	14.1	14.1	14.1
σ_b	14.2	0.01	14.1	14.1	14.1	14.2	14.2	14.2	14.2	14.3	14.3
σ_g	14.1	0.01	14.0	14.0	14.1	14.1	14.1	14.1	14.1	14.1	14.2
σ_I	14.1	0.00	14.0	14.1	14.1	14.1	14.1	14.1	14.2	14.2	14.2
σ_r	14.1	0.01	14.1	14.1	14.1	14.1	14.1	14.1	14.2	14.2	14.2
σ_p	14.1	0.01	14.0	14.0	14.0	14.1	14.1	14.1	14.1	14.2	14.2
σ_w	14.1	0.01	14.0	14.0	14.0	14.1	14.1	14.1	14.1	14.1	14.2

Note: The sensitivity component of the measure of identification strength is defined as the square root of $\mathrm{E}\left(\theta_i \partial \ell(\boldsymbol{\theta})/\partial \theta_i\right)^2$. The table shows the mean, the coefficient of variation and the deciles of the sensitivity components computed on the basis of $100,000$ draws from $\boldsymbol{\Theta}$.

Table 10: Distributions of the multiple correlation coefficients

Par.	mean	std.	deciles								
			1	2	3	4	5	6	7	8	9
φ	0.971	0.36	0.922	0.9512	0.9678	0.9785	0.9857	0.99067	0.99411	0.99654	0.99833
σ_c	0.996	0.05	0.990	0.9935	0.9952	0.9963	0.9972	0.99790	0.99848	0.99898	0.99943
λ	0.967	0.31	0.926	0.9487	0.9616	0.9705	0.9772	0.98255	0.98697	0.99087	0.99464
ξ_w	0.998	0.04	0.996	0.9982	0.9991	0.9995	0.9997	0.99984	0.99992	0.99997	0.99999
σ_l	0.976	0.31	0.941	0.9630	0.9747	0.9824	0.9877	0.99152	0.99443	0.99666	0.99839
ξ_p	0.947	0.53	0.868	0.9054	0.9308	0.9499	0.9650	0.97666	0.98556	0.99211	0.99673
ι_w	0.816	1.64	0.560	0.6749	0.7575	0.8197	0.8673	0.90601	0.93673	0.96130	0.98060
ι_p	0.868	0.88	0.746	0.7894	0.8228	0.8523	0.8796	0.90619	0.93146	0.95452	0.97587
ψ	0.941	0.75	0.855	0.9100	0.9374	0.9547	0.9670	0.97591	0.98305	0.98889	0.99400
Φ	0.964	0.50	0.909	0.9454	0.9634	0.9743	0.9817	0.98701	0.99122	0.99461	0.99734
r_π	0.966	0.40	0.916	0.9457	0.9614	0.9720	0.9796	0.98545	0.99011	0.99394	0.99710
ρ	0.973	0.32	0.933	0.9564	0.9689	0.9773	0.9835	0.98835	0.99220	0.99529	0.99780
r_y	0.936	0.68	0.845	0.8953	0.9248	0.9450	0.9600	0.97145	0.98058	0.98811	0.99433
$r_{\triangle y}$	0.914	0.79	0.802	0.8543	0.8883	0.9147	0.9367	0.95490	0.97001	0.98241	0.99219
$\bar{\pi}$	0.874	1.75	0.620	0.7853	0.8728	0.9229	0.9529	0.97169	0.98370	0.99142	0.99645
β'	0.974	0.34	0.938	0.9608	0.9722	0.9794	0.9847	0.98868	0.99186	0.99463	0.99710
$\bar{l}$	0.848	1.84	0.559	0.7219	0.8213	0.8864	0.9287	0.95606	0.97398	0.98606	0.99389
γ	0.217	1.14	0.085	0.1165	0.1446	0.1719	0.2003	0.23049	0.26530	0.30726	0.36881
α	0.933	0.65	0.846	0.8916	0.9188	0.9377	0.9524	0.96453	0.97502	0.98413	0.99216
δ	0.867	1.06	0.719	0.7948	0.8393	0.8714	0.8964	0.91710	0.93513	0.95228	0.97010
λ_w	0.999	0.03	0.997	0.9988	0.9993	0.9996	0.9998	0.99987	0.99993	0.99997	0.99999
g_y	0.913	0.96	0.779	0.8532	0.8973	0.9264	0.9479	0.96407	0.97667	0.98652	0.99395
ρ_a	0.405	2.61	0.096	0.1494	0.2089	0.2761	0.3552	0.44536	0.54920	0.66671	0.80784
ρ_b	0.941	0.50	0.868	0.8960	0.9186	0.9376	0.9538	0.96771	0.97880	0.98720	0.99369
ρ_g	0.416	2.45	0.119	0.1800	0.2408	0.3052	0.3763	0.45557	0.54644	0.65387	0.78761
ρ_I	0.816	0.71	0.739	0.7709	0.7903	0.8052	0.8189	0.83262	0.84758	0.86514	0.89308
ρ_r	0.758	2.24	0.406	0.5688	0.6805	0.7664	0.8324	0.88502	0.92450	0.95420	0.97796
ρ_p	0.928	1.49	0.773	0.9337	0.9615	0.9753	0.9840	0.99017	0.99461	0.99756	0.99938
ρ_w	0.916	1.71	0.731	0.9213	0.9536	0.9700	0.9804	0.98786	0.99332	0.99702	0.99925
μ_p	0.950	0.91	0.837	0.9431	0.9656	0.9770	0.9845	0.99017	0.99446	0.99749	0.99937
μ_w	0.940	1.02	0.813	0.9282	0.9541	0.9689	0.9792	0.98713	0.99301	0.99692	0.99924
ρ_{ga}	0.360	2.10	0.139	0.1807	0.2194	0.2598	0.3064	0.36207	0.43138	0.52800	0.68229
σ_a	0.468	1.97	0.199	0.2819	0.3497	0.4109	0.4693	0.52731	0.58746	0.65378	0.73305
σ_b	0.774	1.15	0.628	0.7097	0.7543	0.7836	0.8059	0.82402	0.84084	0.85835	0.87930
σ_g	0.378	1.58	0.174	0.2316	0.2793	0.3241	0.3691	0.41524	0.46464	0.52045	0.59603
σ_I	0.783	0.66	0.711	0.7495	0.7696	0.7834	0.7950	0.80562	0.81633	0.82815	0.84437
σ_r	0.528	2.53	0.165	0.2620	0.3603	0.4585	0.5512	0.63702	0.71213	0.78095	0.85045
σ_p	0.725	1.13	0.576	0.6206	0.6571	0.6899	0.7207	0.75224	0.78609	0.82639	0.88183
σ_w	0.706	1.20	0.539	0.5875	0.6318	0.6717	0.7079	0.74223	0.77812	0.81747	0.86652

Note: The table shows the mean, the standard deviation (multiplied by 10) and the nine deciles of $\boldsymbol{\varrho}_i := \text{corr}\left(\frac{\partial\ell(\boldsymbol{\theta})}{\partial\theta_i}, \frac{\partial\ell(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_{-i}}\right)$ computed on the basis of 100,000 draws from $\boldsymbol{\Theta}$.

Table 11: Sensitivity in the model

Par.	mean	CoV	deciles								
			1	2	3	4	5	6	7	8	9
φ	652	34	51	67	83	103	127	161	214	311	578
σ_c	1750	22	119	162	207	261	330	427	579	865	1666
λ	2585	17	186	263	347	446	575	753	1032	1544	2913
ξ_w	884	51	54	74	96	121	154	199	268	398	746
σ_l	442	28	35	46	58	71	89	112	149	218	411
ξ_p	1298	51	84	113	144	179	223	286	385	567	1082
ι_w	137	33	6	9	12	16	22	29	41	62	119
ι_p	300	61	20	27	34	42	53	68	91	134	249
ψ	526	19	55	73	91	112	138	175	231	338	620
Φ	2134	39	147	198	251	315	395	509	683	1014	1906
r_π	1130	86	63	85	108	135	171	221	299	446	866
ρ	1896	39	106	159	216	284	374	499	694	1058	2023
r_y	259	66	13	19	25	32	41	54	73	109	206
$r_{\triangle y}$	200	38	14	19	25	33	42	55	75	112	212
$\bar{\pi}$	1	0	1	1	1	1	1	1	1	1	1
β'	50	28	3	5	6	8	10	13	17	25	47
$\bar{l}$	1	0	1	1	1	1	1	1	1	1	1
γ	51	35	4	5	7	8	11	14	18	27	51
α	751	25	73	94	116	142	174	219	288	418	785
δ	217	41	18	22	27	34	42	53	70	103	199
λ_w	1089	33	76	102	131	165	209	270	365	542	1030
g_y	212	20	17	24	30	37	47	60	80	118	223
ρ_a	420	32	20	31	43	58	76	101	138	207	402
ρ_b	435	46	15	24	34	45	60	80	113	178	370
ρ_g	235	28	12	19	25	33	43	58	80	121	233
ρ_I	530	35	17	28	42	58	79	107	151	231	451
ρ_r	256	18	10	14	20	27	38	55	82	135	288
ρ_p	318	29	10	16	23	32	45	63	93	154	321
ρ_w	529	120	10	17	24	33	46	64	94	150	309
μ_p	218	26	8	11	15	21	30	42	62	101	211
μ_w	331	85	8	11	15	20	28	40	58	95	201
ρ_{ga}	335	37	7	13	20	29	40	55	80	130	271
σ_a	675	33	27	47	65	85	109	144	200	304	607
σ_b	233	45	19	23	24	26	31	39	55	85	175
σ_g	526	31	19	34	49	66	87	116	162	250	503
σ_I	654	43	29	42	54	69	87	114	156	237	480
σ_r	129	24	9	10	12	16	20	25	35	56	117
σ_p	155	40	12	13	15	18	24	31	44	68	141
σ_w	134	62	11	12	13	15	17	20	27	41	85

Note: The sensitivity in the model is measured by the square root of $\sum\left(\frac{\partial\tau_j}{\partial\theta_i}\frac{\theta_i}{\tau_j}\right)^2$. The table shows the mean, the coefficient of variation and the deciles of the measure computed on the basis of $100,000$ draws from $\boldsymbol{\Theta}$.

Table 12: Collinearity in the model

			deciles								
Par.	mean	std.	1	2	3	4	5	6	7	8	9
φ	0.887	0.088	0.763	0.812	0.846	0.8749	0.9004	0.9252	0.9490	0.97139	0.99049
σ_c	0.998	0.003	0.995	0.997	0.998	0.9988	0.9992	0.9996	0.9998	0.99990	0.99998
λ	0.996	0.007	0.990	0.995	0.997	0.9982	0.9989	0.9994	0.9997	0.99988	0.99997
ξ_w	0.997	0.006	0.992	0.996	0.998	0.9988	0.9993	0.9996	0.9998	0.99993	0.99999
σ_l	0.979	0.018	0.955	0.967	0.974	0.9791	0.9835	0.9874	0.9910	0.99464	0.99805
ξ_p	0.970	0.028	0.933	0.953	0.964	0.9725	0.9786	0.9838	0.9882	0.99231	0.99662
ι_w	0.896	0.081	0.782	0.827	0.858	0.8839	0.9070	0.9298	0.9530	0.97550	0.99290
ι_p	0.912	0.068	0.818	0.854	0.879	0.9007	0.9205	0.9405	0.9609	0.97971	0.99412
ψ	0.334	0.121	0.195	0.228	0.258	0.2865	0.3161	0.3490	0.3860	0.43084	0.49697
Φ	0.946	0.034	0.900	0.919	0.932	0.9427	0.9517	0.9603	0.9686	0.97672	0.98530
r_π	0.981	0.028	0.949	0.970	0.981	0.9874	0.9919	0.9950	0.9973	0.99878	0.99969
ρ	0.962	0.049	0.897	0.935	0.956	0.9705	0.9806	0.9879	0.9932	0.99684	0.99914
r_y	0.971	0.037	0.923	0.952	0.967	0.9771	0.9844	0.9899	0.9939	0.99698	0.99908
$r_{\triangle y}$	0.823	0.150	0.601	0.703	0.770	0.8207	0.8631	0.8989	0.9291	0.95633	0.98027
$\bar{\pi}$	0.682	0.027	0.658	0.673	0.680	0.6849	0.6887	0.6918	0.6946	0.69724	0.70028
β'	0.905	0.054	0.833	0.857	0.875	0.8898	0.9040	0.9183	0.9350	0.95667	0.98447
$\bar{l}$	0.000	0.000	0.000	0.000	0.000	0.0000	0.0000	0.0000	0.0000	0.00000	0.00000
γ	0.678	0.153	0.494	0.551	0.592	0.6277	0.6609	0.6963	0.7380	0.80171	0.92215
α	0.916	0.058	0.843	0.872	0.890	0.9055	0.9207	0.9368	0.9546	0.97194	0.98785
δ	0.875	0.090	0.750	0.805	0.840	0.8672	0.8908	0.9126	0.9341	0.95604	0.97706
λ_w	0.998	0.004	0.993	0.996	0.998	0.9987	0.9993	0.9996	0.9998	0.99993	0.99998
g_y	0.887	0.060	0.812	0.836	0.854	0.8697	0.8846	0.9006	0.9191	0.94308	0.97360
ρ_a	0.660	0.139	0.479	0.555	0.603	0.6413	0.6754	0.7064	0.7376	0.77175	0.81571
ρ_b	0.944	0.100	0.849	0.923	0.953	0.9701	0.9810	0.9883	0.9935	0.99708	0.99917
ρ_g	0.560	0.180	0.327	0.406	0.463	0.5128	0.5603	0.6078	0.6562	0.71170	0.78947
ρ_I	0.460	0.232	0.173	0.245	0.308	0.3679	0.4285	0.4955	0.5764	0.67751	0.80729
ρ_r	0.706	0.259	0.302	0.444	0.569	0.6800	0.7754	0.8545	0.9158	0.95969	0.98771
ρ_p	0.783	0.176	0.517	0.632	0.714	0.7785	0.8301	0.8734	0.9103	0.94230	0.97084
ρ_w	0.594	0.206	0.317	0.397	0.466	0.5321	0.5967	0.6600	0.7233	0.78980	0.87117
μ_p	0.640	0.172	0.393	0.496	0.566	0.6203	0.6668	0.7081	0.7470	0.78743	0.83897
μ_w	0.468	0.157	0.251	0.329	0.387	0.4359	0.4790	0.5197	0.5605	0.60357	0.65764
ρ_{ga}	0.862	0.187	0.587	0.778	0.865	0.9136	0.9432	0.9624	0.9756	0.98525	0.99247
σ_a	0.790	0.216	0.443	0.638	0.750	0.8244	0.8747	0.9115	0.9391	0.96073	0.97862
σ_b	0.926	0.082	0.802	0.859	0.901	0.9328	0.9579	0.9759	0.9876	0.99467	0.99840
σ_g	0.814	0.183	0.532	0.687	0.778	0.8379	0.8816	0.9142	0.9396	0.96067	0.97826
σ_I	0.408	0.180	0.217	0.262	0.299	0.3342	0.3709	0.4114	0.4603	0.52861	0.65535
σ_r	0.860	0.089	0.734	0.782	0.816	0.8448	0.8707	0.8947	0.9182	0.94224	0.96909
σ_p	0.533	0.153	0.351	0.408	0.449	0.4854	0.5204	0.5573	0.5984	0.65193	0.73699
σ_w	0.331	0.135	0.195	0.230	0.257	0.2816	0.3063	0.3329	0.3640	0.40648	0.48542

Note: The collinearity in the model is measured by $\cos\left(\frac{\partial \boldsymbol{\tau}}{\partial \theta_i}, \frac{\partial \boldsymbol{\tau}}{\partial \boldsymbol{\theta}_{-i}}\right)$. The table shows the mean, the standard deviation and the deciles of the measure computed on the basis of 100,000 draws from $\boldsymbol{\Theta}$.

Table 13: Maximal multiple correlation coefficients (first decile of $\boldsymbol{\varrho}_i$)

Par.	$\boldsymbol{\varrho}_i$	$\boldsymbol{\varrho}_{i(1)}$	$\boldsymbol{\varrho}_{i(2)}$	$\boldsymbol{\varrho}_{i(3)}$	$\boldsymbol{\varrho}_{i(4)}$
φ	0.922	0.377 (λ)	0.519 (λ,δ)	0.636 (λ,ξ_w,λ_w)	0.771 (λ,ξ_w,r_y,λ_w)
σ_c	0.990	0.698 (β')	0.948 (β',λ)	0.959 (β',λ,σ_I)	0.970 (β',λ,σ_b,σ_I)
λ	0.926	0.700 (φ)	0.828 (φ,ρ_b)	0.846 (φ,ρ_b,σ_b)	0.880 (β',σ_c,ξ_w,λ_w)
ξ_w	0.996	0.960 (σ_l)	0.983 (ι_w,σ_l)	0.988 (ι_w,σ_l,σ_w)	0.989 (μ_p,ι_w,σ_l,σ_w)
σ_l	0.941	0.853 (ξ_w)	0.870 (ι_w,ξ_w)	0.884 (ι_w,ξ_w,σ_w)	0.890 (ι_w,ξ_w,ξ_p,σ_w)
ξ_p	0.868	0.284 (μ_p)	0.716 (ι_p,ρ_p)	0.740 (μ_p,ι_p,ρ_p)	0.754 (μ_p,ι_p,ρ_p,σ_p)
ι_w	0.560	0.520 (σ_l)	0.524 (ι_p,σ_l)	0.529 (μ_p,σ_l,ρ_p)	0.534 (λ,σ_l,ρ_b,σ_b)
ι_p	0.746	0.332 (ρ_p)	0.420 (r_π,ρ_p)	0.480 (λ,r_π,ρ_p)	0.541 (λ,$r_{\triangle y}$,r_y,ρ_p)
ψ	0.855	0.377 (g_y)	0.489 (Φ,g_y)	0.589 (α,Φ,δ)	0.699 (α,Φ,σ_a,δ)
Φ	0.909	0.506 (ψ)	0.704 (α,ψ)	0.769 (α,ψ,g_y)	0.823 (α,ψ,σ_c,g_y)
r_π	0.916	0.669 (σ_c)	0.809 (σ_c,r_y)	0.838 (σ_c,ξ_p,r_y)	0.853 (σ_c,ξ_p,r_y,ρ_w)
ρ	0.933	0.765 (r_π)	0.862 (r_π,ρ_b)	0.913 (r_π,$r_{\triangle y}$,ρ_b)	0.924 (ξ_p,r_π,$r_{\triangle y}$,ρ_b)
r_y	0.845	0.371 (ρ_r)	0.527 (ξ_w,ρ)	0.692 (ξ_w,r_π,ρ)	0.741 (ξ_w,r_π,ρ,ρ_b)
$r_{\triangle y}$	0.802	0.613 (ρ)	0.692 (r_π,ρ)	0.711 (r_π,ρ,ρ_r)	0.726 (ι_w,r_π,ρ,ρ_r)
$\bar{\pi}$	0.620	0.591 (β')	0.603 ($\bar{l}$,β')	0.612 ($\bar{l}$,β',δ)	0.616 ($\bar{l}$,β',γ,δ)
β'	0.938	0.770 ($\bar{\pi}$)	0.864 ($\bar{\pi}$,α)	0.893 ($\bar{l}$,$\bar{\pi}$,α)	0.901 ($\bar{l}$,$\bar{\pi}$,α,g_y)
$\bar{l}$	0.559	0.337 (β')	0.500 ($\bar{\pi}$,β')	0.555 ($\bar{\pi}$,β',γ)	0.558 ($\bar{\pi}$,β',α,γ)
γ	0.085	0.068 ($\bar{\pi}$)	0.083 ($\bar{l}$,$\bar{\pi}$)	0.084 ($\bar{l}$,$\bar{\pi}$,σ_c)	0.084 ($\bar{l}$,$\bar{\pi}$,σ_c,ρ)
α	0.846	0.500 (Φ)	0.717 (ψ,δ)	0.756 (ψ,ι_w,δ)	0.777 (ψ,ι_w,ι_p,δ)
δ	0.719	0.310 (σ_c)	0.475 (σ_c,r_y)	0.531 (α,σ_c,r_y)	0.601 (α,ψ,σ_c,r_y)
λ_w	0.997	0.897 (ξ_w)	0.932 (σ_c,ξ_w)	0.959 (β',σ_c,ξ_w)	0.983 (β',φ,σ_c,ξ_w)
g_y	0.779	0.559 (σ_c)	0.627 (α,σ_c)	0.670 (α,ξ_w,λ_w)	0.687 (α,ξ_w,ρ_g,λ_w)
ρ_a	0.096	0.092 (ρ_g)	0.095 (ρ_g,σ_a)	0.096 (ρ_{ga},ρ_g,σ_a)	0.096 (ρ_{ga},Φ,ρ_g,σ_a)
ρ_b	0.868	0.767 (σ_b)	0.812 (λ,σ_b)	0.818 (λ,$r_{\triangle y}$,σ_b)	0.842 (λ,r_y,ρ,σ_b)
ρ_g	0.119	0.096 (ρ_a)	0.104 (ρ_a,g_y)	0.106 (ρ_a,σ_g,g_y)	0.108 (Φ,ρ_a,σ_g,g_y)
ρ_I	0.739	0.737 (σ_I)	0.737 (φ,σ_I)	0.738 (φ,σ_c,σ_I)	0.738 (φ,σ_c,λ,σ_I)
ρ_r	0.406	0.216 (ρ)	0.332 (ι_w,ρ)	0.347 (ι_w,ρ,δ)	0.360 (ι_w,$r_{\triangle y}$,ρ,σ_r)
ρ_p	0.773	0.702 (μ_p)	0.771 (μ_p,σ_p)	0.771 (μ_w,μ_p,σ_p)	0.772 (μ_w,μ_p,ρ_w,σ_p)
ρ_w	0.731	0.489 (μ_w)	0.723 (μ_w,σ_w)	0.726 (μ_w,μ_p,σ_w)	0.728 (μ_w,μ_p,ρ_p,σ_w)
μ_p	0.837	0.744 (ρ_p)	0.813 (ρ_p,σ_p)	0.814 (ι_p,ρ_p,σ_p)	0.821 (ι_p,ξ_p,ρ_p,σ_p)
μ_w	0.813	0.592 (σ_w)	0.791 (ρ_w,σ_w)	0.799 (ρ_w,σ_w,λ_w)	0.808 (ξ_w,σ_l,ρ_w,σ_w)
ρ_{ga}	0.139	0.028 (α)	0.037 (α,ψ)	0.046 (α,ψ,δ)	0.060 ($\bar{l}$,$\bar{\pi}$,β',α)
σ_a	0.199	0.026 (ψ)	0.055 (ψ,g_y)	0.093 (β',α,ψ)	0.117 ($\bar{\pi}$,β',α,ψ)
σ_b	0.628	0.301 (ρ_b)	0.356 (σ_c,ρ_b)	0.386 (σ_c,ξ_w,ρ_b)	0.439 (σ_c,ξ_p,ρ_a,ρ_b)
σ_g	0.174	0.074 (ψ)	0.115 (ψ,Φ)	0.124 (α,ψ,Φ)	0.136 (α,ψ,Φ,ξ_p)
σ_I	0.711	0.295 (ρ_I)	0.393 (ρ_I,δ)	0.451 (φ,ξ_p,ρ_I)	0.498 (φ,ξ_p,σ_l,ρ_I)
σ_r	0.165	0.120 ($r_{\triangle y}$)	0.146 (r_π,$r_{\triangle y}$)	0.153 (r_π,$r_{\triangle y}$,r_y)	0.157 (r_π,$r_{\triangle y}$,r_y,ρ)
σ_p	0.576	0.254 (μ_p)	0.450 (μ_p,ρ_p)	0.450 (μ_p,r_π,ρ_p)	0.451 (β',μ_p,r_π,ρ_p)
σ_w	0.539	0.535 (μ_w)	0.539 (μ_w,ρ_w)	0.539 (μ_w,ξ_p,ρ_w)	0.539 (μ_w,$r_{\triangle y}$,ρ_w,λ_w)

Note: The table shows the 1th decile of $\boldsymbol{\varrho}_i := \text{corr}\left(\frac{\partial \ell(\boldsymbol{\theta})}{\partial \theta_i}, \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_{-i}}\right)$ and the corresponding values of $\boldsymbol{\varrho}_{i(n)}$, defined as the largest (in absolute value) among all coefficients of multiple correlation between $\partial \ell(\boldsymbol{\theta})/\partial \theta_i$ and $\partial \ell(\boldsymbol{\theta})/\partial \boldsymbol{\theta}_{-i}(n)$ for θ_i in the first column and all possible combinations of $n = 1, \ldots, 4$ parameters from $\boldsymbol{\theta}_{-i}$. The selected parameters are shown in parentheses.

Table 14: Maximal multiple correlation coefficients (median of $\boldsymbol{\varrho}_i$)

Par.	$\boldsymbol{\varrho}_i$	$\boldsymbol{\varrho}_{i(1)}$	$\boldsymbol{\varrho}_{i(2)}$	$\boldsymbol{\varrho}_{i(3)}$	$\boldsymbol{\varrho}_{i(4)}$
φ	0.9857	0.559 (σ_l)	0.685 (ξ_p,λ_w)	0.861 (λ,ξ_p,λ_w)	0.883 (λ,ξ_p,ρ_b,λ_w)
σ_c	0.9972	0.847 (β')	0.894 (β',λ)	0.974 (β',ξ_w,λ_w)	0.985 (β',φ,ξ_w,λ_w)
λ	0.9772	0.770 (σ_c)	0.834 (φ,σ_c)	0.859 (φ,σ_c,σ_I)	0.892 (β',σ_c,ξ_w,λ_w)
ξ_w	0.9997	0.965 (σ_l)	0.988 (ι_w,σ_l)	0.989 (ι_w,σ_l,σ_w)	0.993 (β',φ,σ_c,λ_w)
σ_l	0.9877	0.885 ($r_{\triangle y}$)	0.939 ($r_{\triangle y}$,λ_w)	0.956 (ξ_w,$r_{\triangle y}$,r_y)	0.968 (λ,$r_{\triangle y}$,r_y,λ_w)
ξ_p	0.9650	0.647 (Φ)	0.701 (Φ,ι_w)	0.744 (α,ψ,Φ)	0.799 (α,ψ,Φ,λ_w)
ι_w	0.8673	0.498 (ρ_b)	0.643 (λ,ρ_b)	0.707 (λ,ρ_b,λ_w)	0.760 (σ_l,ρ_b,σ_w,λ_w)
ι_p	0.8796	0.723 (μ_p)	0.792 (μ_p,ξ_p)	0.818 (μ_p,ξ_p,r_π)	0.832 (μ_p,ξ_p,r_π,$r_{\triangle y}$)
ψ	0.9670	0.493 (g_y)	0.637 (Φ,g_y)	0.763 (Φ,ξ_p,λ_w)	0.830 (Φ,ξ_p,λ_w,g_y)
Φ	0.9817	0.663 (ψ)	0.931 (ψ,g_y)	0.952 (α,ψ,g_y)	0.958 (α,ψ,ξ_p,g_y)
r_π	0.9796	0.811 (ρ)	0.911 (ρ_r,δ)	0.952 (r_y,ρ,δ)	0.965 (ξ_p,σ_l,ρ,δ)
ρ	0.9835	0.745 (r_y)	0.895 ($r_{\triangle y}$,r_y)	0.939 (r_π,$r_{\triangle y}$,r_y)	0.961 (r_π,$r_{\triangle y}$,r_y,σ_r)
r_y	0.9600	0.920 (r_π)	0.926 (r_π,$r_{\triangle y}$)	0.939 (r_π,$r_{\triangle y}$,ρ_b)	0.942 (ι_w,r_π,$r_{\triangle y}$,ρ_b)
$r_{\triangle y}$	0.9367	0.793 (σ_l)	0.873 (r_π,ρ)	0.886 (λ,r_π,ρ)	0.904 (α,λ,r_π,ρ)
$\bar{\pi}$	0.9529	0.916 (β')	0.923 ($\bar{l}$,β')	0.936 (β',α,g_y)	0.946 ($\bar{l}$,β',α,g_y)
β'	0.9847	0.777 ($\bar{\pi}$)	0.799 ($\bar{\pi}$,σ_c)	0.849 (σ_c,ξ_w,λ_w)	0.899 ($\bar{\pi}$,σ_c,ξ_w,λ_w)
$\bar{l}$	0.9287	0.890 ($\bar{\pi}$)	0.911 ($\bar{\pi}$,β')	0.917 ($\bar{\pi}$,β',γ)	0.921 ($\bar{\pi}$,β',α,g_y)
γ	0.2003	0.187 ($\bar{l}$)	0.197 ($\bar{l}$,β')	0.199 ($\bar{l}$,$\bar{\pi}$,β')	0.200 ($\bar{l}$,$\bar{\pi}$,β',δ)
α	0.9524	0.707 (Φ)	0.806 (Φ,δ)	0.883 (ψ,Φ,δ)	0.924 (ψ,Φ,ξ_p,δ)
δ	0.8964	0.478 (ψ)	0.745 (r_π,r_y)	0.784 (σ_c,r_y,ρ)	0.818 (ξ_w,r_π,r_y,λ_w)
λ_w	0.9998	0.918 (ξ_w)	0.965 (σ_c,ξ_w)	0.997 (φ,σ_c,ξ_w)	0.999 (β',φ,σ_c,ξ_w)
g_y	0.9479	0.569 (ψ)	0.719 (ψ,Φ)	0.851 (α,ψ,δ)	0.884 (ψ,Φ,ξ_p,δ)
ρ_a	0.3552	0.080 (ξ_p)	0.140 (Φ,ξ_p)	0.203 (α,ψ,g_y)	0.211 (α,ψ,Φ,g_y)
ρ_b	0.9538	0.602 (ξ_w)	0.786 (r_π,ρ)	0.833 (r_π,ρ,σ_b)	0.873 (ξ_w,r_π,ρ,σ_b)
ρ_g	0.3763	0.294 (ρ_a)	0.309 (α,ρ_a)	0.314 (α,σ_l,ρ_a)	0.319 (α,σ_l,ρ_a,ρ_b)
ρ_I	0.8189	0.777 (σ_I)	0.785 (φ,σ_I)	0.787 (φ,σ_I,δ)	0.788 (φ,σ_l,σ_I,λ_w)
ρ_r	0.8324	0.381 (ξ_p)	0.636 (ξ_p,ρ)	0.663 (ξ_p,ρ,σ_r)	0.696 (ι_p,ξ_p,r_π,σ_r)
ρ_p	0.9840	0.970 (μ_p)	0.974 (μ_p,σ_p)	0.975 (μ_p,ξ_p,σ_p)	0.976 (μ_p,φ,ξ_w,σ_p)
ρ_w	0.9804	0.976 (μ_w)	0.977 (μ_w,σ_w)	0.977 (μ_w,r_π,σ_w)	0.978 (μ_w,r_π,r_y,σ_w)
μ_p	0.9845	0.980 (ρ_p)	0.984 (ρ_p,σ_p)	0.984 (ι_p,ρ_p,σ_p)	0.984 (ι_p,ξ_p,ρ_p,σ_p)
μ_w	0.9792	0.960 (ρ_w)	0.975 (ρ_w,σ_w)	0.975 (r_y,ρ_w,σ_w)	0.976 (ξ_p,$r_{\triangle y}$,ρ_w,σ_w)
ρ_{ga}	0.3064	0.219 (ψ)	0.239 (α,ψ)	0.259 (α,ψ,r_π)	0.269 (α,ψ,r_y,δ)
σ_a	0.4693	0.089 (ψ)	0.126 (ψ,Φ)	0.191 (ψ,Φ,g_y)	0.273 (α,ψ,Φ,g_y)
σ_b	0.8059	0.554 (ρ_b)	0.622 (λ,ρ_b)	0.670 (σ_c,λ,ρ_b)	0.726 (λ,ξ_w,ρ_b,λ_w)
σ_g	0.3691	0.055 (Φ)	0.134 (ψ,Φ)	0.159 (ψ,Φ,ξ_p)	0.202 (ψ,Φ,ξ_p,δ)
σ_I	0.7950	0.690 (ρ_I)	0.694 (ρ_I,δ)	0.698 (β',σ_c,ρ_I)	0.715 (α,ρ_I,δ,g_y)
σ_r	0.5512	0.190 (r_π)	0.254 (ι_w,r_π)	0.327 (ι_w,r_π,ρ_r)	0.388 (ι_w,r_π,r_y,ρ_r)
σ_p	0.7207	0.632 (μ_p)	0.690 (μ_p,ρ_p)	0.699 (μ_p,ι_p,ρ_p)	0.702 (μ_p,ι_p,ξ_p,ρ_p)
σ_w	0.7079	0.626 (μ_w)	0.686 (μ_w,ρ_w)	0.690 (μ_w,ξ_w,ρ_w)	0.696 (μ_w,ξ_w,σ_l,ρ_w)

Note: The table shows the median of $\boldsymbol{\varrho}_i := \text{corr}\left(\frac{\partial \ell(\boldsymbol{\theta})}{\partial \theta_i}, \frac{\partial \ell(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}_{-i}}\right)$ and the corresponding values of $\boldsymbol{\varrho}_{i(n)}$, defined as the largest (in absolute value) among all coefficients of multiple correlation between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}(n)$ for θ_i in the first column and all possible combinations of $n = 1, \ldots, 4$ parameters from $\boldsymbol{\theta}_{-i}$. The selected parameters are shown in parentheses.

Table 15: Maximal multiple correlation coefficients (ninth decile of $\boldsymbol{\varrho}_i$)

Par.	$\boldsymbol{\varrho}_i$	$\boldsymbol{\varrho}_{i(1)}$	$\boldsymbol{\varrho}_{i(2)}$	$\boldsymbol{\varrho}_{i(3)}$	$\boldsymbol{\varrho}_{i(4)}$
φ	0.99833	0.81145 (λ)	0.98506 (λ,ξ_w)	0.99207 (λ,ξ_w,ρ)	0.99297 (λ,ξ_w,ρ,σ_I)
σ_c	0.99943	0.89038 (σ_l)	0.96498 (ξ_w,λ_w)	0.98345 (φ,ξ_w,λ_w)	0.99202 (φ,ξ_w,ρ,λ_w)
λ	0.99464	0.81240 (σ_c)	0.98556 (φ,σ_c)	0.98976 (φ,ξ_w,λ_w)	0.99136 (φ,ξ_w,ξ_p,λ_w)
ξ_w	0.99999	0.99974 (λ_w)	0.99986 (σ_c,λ_w)	0.99988 (β',σ_c,λ_w)	0.99991 (φ,σ_c,ι_w,λ_w)
σ_l	0.99839	0.94778 (σ_c)	0.97158 (σ_c,λ)	0.98572 (σ_c,λ,ξ_w)	0.98938 (φ,σ_c,λ,ξ_w)
ξ_p	0.99673	0.94541 (Φ)	0.96691 (Φ,λ_w)	0.97858 (Φ,λ_w,g_y)	0.98559 (Φ,ξ_w,λ_w,g_y)
ι_w	0.98060	0.85876 (λ)	0.95268 (σ_l,λ_w)	0.95571 (σ_c,λ,ι_p)	0.96547 (λ,ι_p,σ_l,λ_w)
ι_p	0.97587	0.85476 (ι_w)	0.89863 ($r_{\triangle y}$,ρ_r)	0.93783 (ξ_p,r_π,r_y)	0.95123 (φ,ξ_p,r_π,r_y)
ψ	0.99400	0.82614 (Φ)	0.89645 (Φ,ξ_p)	0.95783 (Φ,ξ_p,g_y)	0.97411 (α,Φ,ξ_p,δ)
Φ	0.99734	0.68871 (ξ_p)	0.86201 (ξ_p,g_y)	0.90494 (α,ξ_p,g_y)	0.94724 (ρ_{ga},α,ξ_p,g_y)
r_π	0.99710	0.93258 (ρ)	0.99185 ($r_{\triangle y}$,ρ)	0.99290 ($r_{\triangle y}$,ρ,σ_r)	0.99439 (σ_l,ρ,ρ_b,λ_w)
ρ	0.99780	0.95205 (r_π)	0.96706 (r_π,$r_{\triangle y}$)	0.99248 (r_π,$r_{\triangle y}$,r_y)	0.99385 (r_π,$r_{\triangle y}$,r_y,σ_r)
r_y	0.99433	0.73895 (ρ)	0.94225 (r_π,ρ)	0.98481 (r_π,$r_{\triangle y}$,ρ)	0.98721 (σ_c,r_π,$r_{\triangle y}$,ρ)
$r_{\triangle y}$	0.99219	0.93367 (σ_l)	0.97368 (r_y,ρ)	0.98087 (ι_w,r_y,ρ)	0.98648 (σ_l,r_y,ρ,λ_w)
$\bar{\pi}$	0.99645	0.99133 ($\bar{l}$)	0.99202 ($\bar{l}$,β')	0.99302 ($\bar{l}$,β',δ)	0.99567 ($\bar{l}$,β',α,g_y)
β'	0.99710	0.90748 ($\bar{l}$)	0.97199 ($\bar{l}$,δ)	0.98497 ($\bar{l}$,α,g_y)	0.98593 ($\bar{l}$,α,ρ,g_y)
$\bar{l}$	0.99389	0.99313 ($\bar{\pi}$)	0.99324 ($\bar{\pi}$,β')	0.99334 ($\bar{\pi}$,β',δ)	0.99373 ($\bar{\pi}$,β',α,g_y)
γ	0.36881	0.23413 ($\bar{l}$)	0.32814 ($\bar{l}$,$\bar{\pi}$)	0.33485 ($\bar{l}$,$\bar{\pi}$,β')	0.34041 ($\bar{l}$,$\bar{\pi}$,β',g_y)
α	0.99216	0.93565 (β')	0.96944 (β',δ)	0.98670 ($\bar{l}$,β',δ)	0.98848 ($\bar{l}$,β',δ,g_y)
δ	0.97010	0.83430 (β')	0.95203 (α,g_y)	0.95439 (α,ψ,g_y)	0.95688 (α,ρ_I,σ_I,g_y)
λ_w	0.99999	0.99924 (ξ_w)	0.99940 (φ,ξ_w)	0.99974 (φ,σ_c,ξ_w)	0.99988 (φ,σ_c,ξ_w,ρ)
g_y	0.99395	0.85649 (δ)	0.90031 (ψ,δ)	0.96347 (β',ψ,Φ)	0.97188 (β',ψ,Φ,δ)
ρ_a	0.80784	0.67293 (ρ_g)	0.67606 (α,ρ_g)	0.68167 (ψ,ρ_g,ρ_r)	0.69204 ($r_{\triangle y}$,r_y,ρ,ρ_g)
ρ_b	0.99369	0.96676 (r_π)	0.98713 (r_π,ρ)	0.98871 (λ,r_π,ρ)	0.98952 (σ_c,r_π,r_y,ρ)
ρ_g	0.78761	0.66588 (ρ_a)	0.67389 (α,ρ_a)	0.69044 (α,ρ_a,δ)	0.70724 (α,ρ_a,δ,g_y)
ρ_I	0.89308	0.79356 (σ_I)	0.80223 (φ,σ_I)	0.80750 (φ,σ_I,δ)	0.81581 (r_y,ρ,ρ_b,σ_I)
ρ_r	0.97796	0.70338 (Φ)	0.90007 (ξ_p,r_π)	0.91201 (ι_p,ξ_p,r_π)	0.94076 (ξ_p,σ_l,ρ,λ_w)
ρ_p	0.99938	0.99929 (μ_p)	0.99938 (μ_p,σ_p)	0.99938 (μ_p,ι_p,σ_p)	0.99938 (μ_w,μ_p,ι_p,σ_p)
ρ_w	0.99925	0.99918 (μ_w)	0.99924 (μ_w,σ_w)	0.99924 (μ_w,σ_l,σ_w)	0.99924 (μ_w,σ_l,ρ_b,σ_w)
μ_p	0.99937	0.99914 (ρ_p)	0.99928 (ρ_p,σ_p)	0.99932 (ι_p,ρ_p,σ_p)	0.99934 (ι_p,ξ_p,ρ_p,σ_p)
μ_w	0.99924	0.99912 (ρ_w)	0.99922 (ρ_w,σ_w)	0.99923 (σ_l,ρ_w,σ_w)	0.99923 (ξ_w,σ_l,ρ_w,σ_w)
ρ_{ga}	0.68229	0.43952 (Φ)	0.54756 (Φ,ξ_p)	0.58745 (Φ,ξ_p,$r_{\triangle y}$)	0.61203 (Φ,ξ_p,$r_{\triangle y}$,σ_a)
σ_a	0.73305	0.18253 (α)	0.22426 (α,ψ)	0.32075 (ψ,Φ,ξ_p)	0.39813 (α,ψ,Φ,ξ_p)
σ_b	0.87930	0.60097 (ρ_b)	0.71635 (ξ_w,ρ_b)	0.77775 (ξ_w,ρ_b,λ_w)	0.79927 (λ,ξ_w,ρ_b,λ_w)
σ_g	0.59603	0.19834 (α)	0.29379 (α,ψ)	0.42390 (ψ,Φ,ξ_p)	0.47155 (ψ,Φ,ξ_p,δ)
σ_I	0.84437	0.43444 (ρ_I)	0.61363 (φ,σ_c)	0.66783 (φ,σ_c,ρ_I)	0.72918 (φ,σ_c,λ,ρ_I)
σ_r	0.85045	0.45770 ($r_{\triangle y}$)	0.53670 (r_π,$r_{\triangle y}$)	0.66280 (r_π,$r_{\triangle y}$,ρ)	0.72346 (r_π,$r_{\triangle y}$,ρ,ρ_r)
σ_p	0.88183	0.81031 (μ_p)	0.82076 (μ_p,ι_w)	0.83493 (μ_p,ι_w,ξ_p)	0.84210 (μ_p,ι_w,ξ_p,ρ_p)
σ_w	0.86652	0.85882 (μ_w)	0.86052 (μ_w,ι_w)	0.86166 (μ_w,ι_w,ρ_w)	0.86476 (μ_w,ι_w,ξ_w,σ_l)

Note: The table shows the 9th decile of $\boldsymbol{\varrho}_i := \text{corr}\left(\frac{\partial\ell(\boldsymbol{\theta})}{\partial\theta_i}, \frac{\partial\ell(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_{-i}}\right)$ and the corresponding values of $\boldsymbol{\varrho}_{i(n)}$, defined as the largest (in absolute value) among all coefficients of multiple correlation between $\partial\ell(\boldsymbol{\theta})/\partial\theta_i$ and $\partial\ell(\boldsymbol{\theta})/\partial\boldsymbol{\theta}_{-i}(n)$ for θ_i in the first column and all possible combinations of $n = 1, \ldots, 4$ parameters from $\boldsymbol{\theta}_{-i}$. The selected parameters are shown in parentheses.

Table 16: Identification strength and structural features of the model

Par.	φ	σ_c	λ	ξ_w	σ_l	ξ_p	ι_w	ι_p	ψ	Φ	r_π	ρ	r_y	$r_{\triangle y}$	$\bar{\pi}$	β'	$\bar{l}$	γ	α	δ	λ_w	g_y	ρ_a	ρ_b	ρ_g	ρ_I	ρ_r	ρ_p	ρ_w	μ_p	μ_w	ρ_{ga}	σ_a	σ_b	σ_g	σ_I	σ_r	σ_p	σ_w
φ	−	+	+	.	.	.	.	.	+	.	−	+	−	.	.	.	.	.	.	.	−	.	.	±	.	−	+	+	.	.	+	.	+	.	.	−	+	+	-
σ_c	−	+	−	-	.	.	.	.	+	.	−	+	.	.	.	.	.	.	+	.	−	.	.	±	.	+	+	+	−	+	+	.	+	−	+	±	+	+	-
λ	−	+	+	−	.	.	.	.	.	.	−	+	.	.	.	.	.	.	+	.	−	.	.	+	.	−	+	.	−	.	+	.	+	+	+	−	+	.	-
ξ_w	.	+	.	+	.	.	.	.	.	.	−	+	.	.	.	.	.	.	.	.	+	.	.	+	.	.	+	.	−	.	+	.	.	.	.	.	+	+	-
σ_l	.	−	.	−	+	+	.	.	.	+	.	+	.	.	.	.	.	.	.	.	−	.	.	+	.	+	+	+	−	-	+	.	.	+	+	+	.	+	-
ξ_p	.	+	+	.	.	+	.	.	−	+	−	+	.	.	.	.	.	.	.	+	.	.	.	+	−	+	+	−	.	+	.	.	−	+	−	+	+	−	.
ι_w	.	+	+	+	.	−	+	.	.	.	−	+	.	.	.	.	.	.	.	.	.	.	.	+	.	.	+	+	−	-	+	.	.	+	.	.	+	+	-
ι_p	.	+	+	−	+	−	.	+	.	−	-	+	.	.	.	.	.	.	.	.	−	.	.	+	.	.	+	−	.	+	.	.	.	+	.	.	+	−	.
ψ	.	−	+	+	.	.	.	.	+	+	−	+	.	.	.	.	.	.	+	+	.	.	−	+	−	+	+	+	.	.	.	−	-	+	−	+	+	+	+
Φ	.	−	+	+	−	.	.	.	−	.	−	+	.	.	.	.	.	.	.	+	+	.	−	+	−	+	+	.	+	.	.	.	−	+	−	+	+	.	+
r_π	.	+	+	.	.	.	.	.	+	.	−	-	.	.	.	.	.	.	+	.	.	.	.	+	.	+	+	.	.	+	+	.	.	+	.	+	−	+	-
ρ	.	+	+	−	.	.	.	.	+	.	−	+	.	.	.	.	.	.	.	.	−	.	.	+	.	+	+	.	.	.	+	.	.	+	.	+	−	+	-
r_y	−	+	+	+	.	+	.	.	.	+	−	-	+	.	.	.	.	.	+	.	.	.	.	+	.	.	+	+	.	.	.	.	.	+	.	.	−	+	.
$r_{\triangle y}$	−	+	+	.	.	.	.	.	−	.	−	+	.	+	.	.	.	.	+	.	.	.	.	+	.	.	+	.	.	.	.	.	+	+	.	.	−	+	.
$\bar{\pi}$	.	−	-	.	.	+	.	.	.	+	+	−	.	.	+	.	.	.	.	.	+	.	.	−	.	.	−	-	−	+	.	.	.	−	.	.	−	-	.
β'	−	-	−	.	.	.	.	.	.	.	.	.	.	.	.	+	.	−	+	.	.	.	.	−	.	.	.	.	.	.	+	.	.	−	.	.	−	+	.
$\bar{l}$	.	−	-	−	+	−	.	.	.	.	+	−	+	.	.	.	±	.	.	.	−	.	.	−	.	.	−	-	−	.	.	.	.	−	.	.	−	.	.
γ	−	.	.	.	.	−	.	.	−	-	.	.	.	.	.	.	.	+	−	.	.	.	−	-	−	-	.	−	-	+	+	.	−	.	−	-	.	−	-
α	.	−	+	+	.	.	.	.	−	+	−	+	.	.	.	.	.	.	+	+	.	.	−	+	−	+	.	.	.	.	.	.	−	+	−	+	.	.	.
δ	.	−	+	+	.	+	.	.	.	+	−	+	.	.	.	.	.	.	+	+	.	.	−	+	−	+	+	.	.	+	+	.	−	+	−	+	+	.	.
λ_w	.	+	.	.	.	.	.	.	.	.	.	+	.	.	.	.	.	.	.	.	.	.	.	+	.	.	+	.	.	.	.	.	.	.	.	.	+	+	.
g_y	.	+	+	+	−	.	.	.	.	+	−	+	.	.	.	.	.	.	.	+	+	+	.	+	−	+	+	+	+	.	.	−	-	+	−	+	+	+	+
ρ_a	.	+	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	.	.	.	.	.	.	.	+	+	.	−	.	.	.	.
ρ_b	.	+	+	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−	.	.	+	.	.	+	.	−	.	+	.	.	+	.	.	.	.	-
ρ_g	.	+	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	.	.	−	.	.	+	+	.	.	.	.	.	-
ρ_I	.	+	+	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	.	.	.	+	.	.	.	.	+	.	.	-
ρ_r	.	.	.	−	.	.	.	.	.	.	.	+	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	−	.	+	.	.	.	.	.	+	.	-
ρ_p	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	+	.	.	.	.	.	.	.	.	.
ρ_w	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	+	.	.	.	.	.	.	.	+
μ_p	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−	.	+	.	.	.	.	.	.	.	.	.
μ_w	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−	.	+	.	.	.	.	.	.	.	.
ρ_{ga}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	+	+	.	−	.	.	.	.
σ_a	.	−	+	+	−	.	.	.	−	.	−	+	.	.	.	.	.	.	−	+	+	.	−	+	−	+	+	.	.	+	.	−	-	+	−	+	+	.	+
σ_b	−	+	+	−	+	.	.	.	.	.	.	.	.	.	.	.	.	.	+	.	−	.	.	+	.	−	+	.	−	+	+	.	+	+	+	−	+	−	-
σ_g	.	−	+	+	−	.	.	.	−	+	−	+	.	.	.	.	.	.	−	+	+	.	−	+	−	+	+	+	+	.	.	−	-	+	−	+	+	+	+
σ_I	−	+	+	−	+	.	.	.	+	−	.	.	.	.	.	.	.	.	+	.	−	.	.	+	.	−	.	−	-	+	+	.	.	+	.	+	.	−	-
σ_r	.	+	+	.	.	.	.	.	.	.	.	−	.	.	.	.	.	.	.	.	.	.	.	+	.	.	−	.	.	.	+	.	+	+	.	.	−	.	-
σ_p	−	+	.	−	+	−	.	+	.	−	.	+	−	.	.	.	.	.	.	.	.	.	.	+	.	.	.	−	.	+	.	.	.	+	.	−	.	.	-
σ_w	.	.	.	−	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	−	.	.	.	.	.	.	.	−	.	+	.	.	.	.	.	.	.	-

Note: The table shows the relationship between the values of the parameters and the strength of identification measured. Positive (negative) sign in cell (i, j) means that θ_j tends to be larger (smaller) in the region of $\boldsymbol{\Theta}$ where θ_i is better identified, than in the region where θ_i is worse identified. The sign $\pm$ indicates that there is a systematic effect whose sign changes above certain value of θ_j. An empty cell indicates the absence of a systematic effect.

Table 17: Identification strength and time series properties of the model

	persistence							volatility						
Par.	$\triangle y_t$	$\triangle c_t$	$\triangle i_t$	l_t	π_t	$\triangle w_t$	r_t	$\triangle y_t$	$\triangle c_t$	$\triangle i_t$	l_t	π_t	$\triangle w_t$	r_t
φ	+	+	+	+	+	+	±	+	+	+	+	+	+	+
σ_c	+	+	+	+	+	+	+	+	+	+	+	+	+	+
λ	+	+	.	+	+	+	+	+	+	+	+	+	+	+
ξ_w	+	+	+	+	+	+	+	+	+	+	+	+	+	+
σ_l	.	.	+	.	+	+	+	+	+	+	+	+	+	+
ξ_p	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ι_w	+	+	.	+	±	+	+	+	+	+	+	+	+	+
ι_p	+	+	−	.	+	+	+	+	+	+	+	+	+	+
ψ	+	+	+	+	+	+	+	+	+	+	+	+	+	+
Φ	+	+	+	+	+	+	+	+	+	+	+	+	+	+
r_π	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ρ	+	+	+	+	+	+	+	+	+	+	+	+	+	+
r_y	+	+	+	+	+	+	+	+	+	+	+	+	+	+
$r_{\triangle y}$	+	+	.	+	+	+	+	+	+	+	+	+	+	+
$\bar{\pi}$	−	−	−	−	−	−	−	−	−	−	−	−	−	−
β'	.	−	+	+	−	−	−	−	−	−	−	−	−	−
$\bar{l}$	−	−	−	−	−	−	−	−	−	−	−	−	−	−
γ	−	−	−	−	.	−	−	−	−	−	−	−	−	−
α	+	+	+	+	+	+	+	+	+	+	+	+	+	+
δ	+	+	+	+	+	+	+	+	+	+	+	+	+	+
λ_w	+	+	.	+	+	+	+	+	+	+	+	+	+	+
g_y	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ρ_a	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ρ_b	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ρ_g	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ρ_I	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ρ_r	+	+	+	+	+	+	+	+	+	+	+	+	+	+
ρ_p	+	+	+	+	.	+	.	+	+	+	+	+	+	+
ρ_w	.	.	+	+	+	.	.	.	.	.	+	.	.	.
μ_p	.	.	.	.	.	.	.	.	.	.	.	.	.	.
μ_w	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ρ_{ga}	+	.	.	+	.	.	.	.	.	.	.	.	.	.
σ_a	+	+	+	+	+	+	+	+	+	+	+	+	+	+
σ_b	+	+	−	.	+	+	+	+	+	+	+	+	+	+
σ_g	+	+	+	+	+	+	+	+	+	+	+	+	+	+
σ_I	.	+	−	−	+	+	+	+	+	+	+	+	+	+
σ_r	.	+	−	−	.	+	+	+	+	+	+	+	+	+
σ_p	−	.	−	−	+	−	+	+	+	+	+	+	+	+
σ_w	−	.	−	−	.	.	.	+	.	.	.	.	+	.

Note: The table shows the relationship between the the persistence and the volatility of the observables and the strength of identification of the parameters. Positive (negative) sign in cell (i, j) means that the variable in column j tends to be more (less) persistent/volatile in the region of $\boldsymbol{\Theta}$ where θ_i is better identified, than in the region where θ_i is worse identified. The sign ± indicates that there is a systematic effect whose sign changes with the degree of persistence/volatility. An empty cell indicates the absence of a systematic effect.

Table 18: Identification strength and correlation structure (positive correlations)

Par.	(Y,C)	(Y,I)	(Y,L)	(Y,π)	(Y,w)	(Y,r)	(C,I)	(C,L)	(C,π)	(C,w)	(C,r)	(I,L)	(I,π)	(I,w)	(I,r)	(L,π)	(L,w)	(L,r)	(π,w)	(π,r)	(w,r)
φ	+	+	−	+	+	n.a.	+	+	+	+	+	.	+	+	n.a.	+	.	+	+	+	.
σ_c	±	+	−	+	+	n.a.	±	±	+	+	+	+	+	+	n.a.	+	+	+	+	+	.
λ	+	+	−	+	+	n.a.	−	+	+	+	n.a.	−	−	+	n.a.	+	+	+	+	+	.
ξ_w	+	+	−	+	+	n.a.	−	+	+	+	+	−	.	+	n.a.	+	+	+	+	+	.
σ_l	±	+	+	+	+	n.a.	±	−	.	+	.	+	+	+	n.a.	+	+	+	+	+	+
ξ_p	+	+	±	+	+	n.a.	+	+	+	+	n.a.	+	+	+	n.a.	+	+	+	+	+	n.a.
ι_w	+	+	−	+	+	n.a.	+	+	+	+	.	.	.	+	n.a.	+	+	+	+	+	+
ι_p	+	+	+	+	+	n.a.	+	+	+	+	.	−	.	+	n.a.	+	+	+	.	+	n.a.
ψ	+	+	−	+	+	n.a.	±	±	+	.	+	+	+	+	n.a.	+	.	+	+	+	.
Φ	+	+	−	+	+	n.a.	±	−	+	.	+	+	+	+	n.a.	+	.	+	+	+	n.a.
r_π	+	+	.	.	+	n.a.	−	+	+	+	+	.	.	+	n.a.	+	+	+	+	+	n.a.
ρ	+	+	−	+	+	n.a.	+	+	+	+	.	−	+	+	n.a.	+	+	+	+	+	.
r_y	.	+	−	.	+	n.a.	.	+	+	.	+	.	.	+	n.a.	−	+	+	+	+	n.a.
$r_{\triangle y}$	+	+	.	+	+	n.a.	+	+	+	+	n.a.	.	.	+	n.a.	+	+	+	+	+	n.a.
$\bar{\pi}$	−	−	+	−	−	n.a.	−	−	−	−	−	+	.	−	n.a.	−	−	−	−	−	.
β'	−	−	−	.	−	n.a.	−	−	−	−	.	+	+	−	n.a.	−	−	−	.	−	.
$\bar{l}$	−	−	+	−	−	n.a.	−	−	−	−	.	+	.	−	n.a.	−	−	−	−	−	.
γ	+	.	+	.	.	n.a.	+	.	.	.	.	.	.	.	n.a.	+	.	.	.	−	.
α	.	+	−	+	+	n.a.	−	±	+	.	+	+	+	.	n.a.	+	.	+	+	+	.
δ	+	+	−	+	+	n.a.	±	±	+	+	+	+	+	+	n.a.	+	+	+	+	+	+
λ_w	+	.	−	+	+	n.a.	−	+	+	+	+	−	−	+	n.a.	+	+	+	+	+	.
g_y	+	+	−	+	+	n.a.	+	−	+	.	+	+	+	+	n.a.	+	.	+	+	+	.
ρ_a	.	.	−	.	+	n.a.	.	.	+	+	.	−	.	.	n.a.	+	.	.	.	.	.
ρ_b	+	+	−	+	+	n.a.	+	+	+	+	.	.	.	+	n.a.	+	+	+	+	+	.
ρ_g	.	.	−	.	+	n.a.	.	+	+	+	.	.	.	.	n.a.	+	+	+	.	.	.
ρ_I	−	−	.	+	+	n.a.	−	.	+	+	.	.	+	.	n.a.	+	+	+	+	+	.
ρ_r	+	+	−	+	+	n.a.	+	.	+	+	.	.	.	+	n.a.	+	+	+	+	+	.
ρ_p	.	+	−	.	.	n.a.	.	.	.	.	.	.	.	+	n.a.	−	−	.	.	+	.
ρ_w	.	.	−	.	−	n.a.	.	.	.	−	.	.	.	−	n.a.	−	+	.	.	.	.
μ_p	.	.	.	.	.	n.a.	.	.	.	.	.	.	.	.	n.a.	+	+	.	.	.	.
μ_w	.	.	.	.	.	n.a.	.	.	.	.	.	.	.	.	n.a.	.	.	.	.	.	.
ρ_{ga}	+	+	−	.	.	n.a.	.	.	.	.	.	.	.	.	n.a.	+	−	.	.	.	.
σ_a	+	+	−	+	+	n.a.	±	−	+	.	+	+	+	+	n.a.	+	.	+	+	+	.
σ_b	+	+	.	+	+	n.a.	+	+	+	+	n.a.	−	.	+	n.a.	+	+	+	+	+	.
σ_g	+	+	−	+	+	n.a.	+	−	+	−	+	+	+	+	n.a.	+	−	+	+	+	.
σ_I	+	+	+	+	+	n.a.	−	+	+	+	+	−	−	+	n.a.	+	+	+	+	+	.
σ_r	+	+	+	+	+	n.a.	−	+	+	+	n.a.	−	−	+	n.a.	+	+	+	+	+	n.a.
σ_p	+	.	+	.	+	n.a.	+	+	+	+	.	.	.	+	n.a.	+	+	+	.	+	.
σ_w	.	.	+	+	+	n.a.	.	+	+	+	.	.	.	+	n.a.	+	+	+	+	.	.

Note: The table shows the relationship between the degree of positive correlation among observables and the strength of identification. Positive (negative) sign in cell (i,j) means that correlation between the j-th pair of variables tends to be stronger (weaker) *and* positive in the region of $\boldsymbol{\Theta}$ where θ_i is better identified, than in the region where θ_i is worse identified. The sign "±" indicates that there is a systematic effect whose sign changes with the degree of correlation. An empty cell indicates the absence of a systematic effect. With "na" are indicated the cases where there are not enough points with positive correlation in the two regions to test for difference in the distributions.

Table 19: Identification strength and correlation structure (negative correlations)

Par.	(Y,C)	(Y,I)	(Y,L)	(Y,π)	(Y,w)	(Y,r)	(C,I)	(C,L)	(C,π)	(C,w)	(C,r)	(I,L)	(I,π)	(I,w)	(I,r)	(L,π)	(L,w)	(L,r)	(π,w)	(π,r)	(w,r)
φ	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	n.a.	−	n.a.	−	n.a.	n.a.	n.a.	−	n.a.	−
σ_c	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	−	−	n.a.	−	n.a.	n.a.	n.a.	−	n.a.	−
λ	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	n.a.	n.a.	+	n.a.	−	n.a.	−	n.a.	n.a.	n.a.	.	n.a.	+
ξ_w	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	n.a.	−	n.a.	−	n.a.	n.a.	.	−	n.a.	−
σ_l	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	.	n.a.	+	.	−	n.a.	+	n.a.	n.a.	.	−	n.a.	−
ξ_p	n.a.	n.a.	n.a.	n.a.	n.a.	+	n.a.	n.a.	n.a.	n.a.	+	−	+	n.a.	+	n.a.	n.a.	n.a.	n.a.	n.a.	+
ι_w	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	.	−	n.a.	−	n.a.	n.a.	.	−	n.a.	−
ι_p	n.a.	n.a.	n.a.	n.a.	n.a.	+	n.a.	n.a.	n.a.	n.a.	+	−	+	n.a.	.	n.a.	n.a.	n.a.	n.a.	n.a.	+
ψ	n.a.	n.a.	n.a.	−	n.a.	.	n.a.	n.a.	−	n.a.	.	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
Φ	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	.	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
r_π	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	+	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
ρ	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	+	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
r_y	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	+	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
$r_{\triangle y}$	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	.	n.a.	+	.	.	n.a.	+	n.a.	n.a.	n.a.	.	n.a.	+
$\bar{\pi}$	n.a.	n.a.	n.a.	+	n.a.	−	n.a.	n.a.	n.a.	n.a.	−	.	.	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	−
β'	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	−	−	n.a.	−	n.a.	n.a.	n.a.	−	n.a.	−
$\bar{l}$	n.a.	n.a.	n.a.	+	n.a.	.	n.a.	n.a.	+	n.a.	.	.	.	n.a.	.	n.a.	n.a.	.	.	n.a.	.
γ	n.a.	n.a.	n.a.	.	n.a.	.	n.a.	n.a.	.	n.a.	.	.	+	n.a.	.	n.a.	n.a.	.	.	n.a.	.
α	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	.	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
δ	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	.	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
λ_w	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	n.a.	−	n.a.	−	n.a.	n.a.	.	−	n.a.	−
g_y	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	.	.	−	n.a.	+	n.a.	n.a.	−	−	n.a.	+
ρ_a	n.a.	n.a.	n.a.	.	n.a.	.	n.a.	n.a.	.	n.a.	.	.	.	n.a.	.	n.a.	n.a.	n.a.	.	n.a.	.
ρ_b	n.a.	n.a.	n.a.	+	n.a.	+	n.a.	n.a.	n.a.	n.a.	+	.	+	n.a.	+	n.a.	n.a.	n.a.	+	n.a.	+
ρ_g	n.a.	n.a.	n.a.	.	n.a.	.	n.a.	n.a.	.	n.a.	.	−	.	n.a.	.	n.a.	n.a.	n.a.	.	n.a.	.
ρ_I	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	.	n.a.	+	.	.	n.a.	+	n.a.	n.a.	n.a.	.	n.a.	+
ρ_r	n.a.	n.a.	n.a.	.	n.a.	.	n.a.	n.a.	.	n.a.	.	.	+	n.a.	−	n.a.	n.a.	n.a.	+	n.a.	−
ρ_p	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	.	n.a.	.	.	−	n.a.	−	n.a.	n.a.	n.a.	−	n.a.	−
ρ_w	n.a.	n.a.	n.a.	.	n.a.	.	n.a.	n.a.	.	n.a.	.	.	.	n.a.	.	n.a.	n.a.	n.a.	.	n.a.	.
μ_p	n.a.	n.a.	n.a.	+	n.a.	.	n.a.	n.a.	.	n.a.	.	.	+	n.a.	.	n.a.	n.a.	n.a.	+	n.a.	+
μ_w	n.a.	n.a.	n.a.	.	n.a.	.	n.a.	n.a.	.	n.a.	.	.	.	n.a.	.	n.a.	n.a.	n.a.	.	n.a.	.
ρ_{ga}	n.a.	n.a.	n.a.	.	n.a.	−	n.a.	n.a.	.	n.a.	.	.	.	n.a.	.	n.a.	n.a.	n.a.	.	n.a.	.
σ_a	n.a.	n.a.	n.a.	−	n.a.	+	n.a.	n.a.	−	n.a.	.	−	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
σ_b	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	n.a.	n.a.	+	n.a.	−	n.a.	−	n.a.	n.a.	n.a.	.	n.a.	+
σ_g	n.a.	n.a.	n.a.	−	n.a.	−	n.a.	n.a.	−	n.a.	−	.	−	n.a.	+	n.a.	n.a.	n.a.	−	n.a.	+
σ_I	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	n.a.	n.a.	+	n.a.	.	n.a.	−	n.a.	n.a.	n.a.	.	n.a.	+
σ_r	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	.	n.a.	+	−	−	n.a.	+	n.a.	n.a.	n.a.	.	n.a.	+
σ_p	n.a.	n.a.	n.a.	+	n.a.	+	n.a.	n.a.	+	n.a.	+	−	+	n.a.	.	n.a.	n.a.	n.a.	+	n.a.	+
σ_w	n.a.	n.a.	n.a.	.	n.a.	+	n.a.	n.a.	.	n.a.	+	.	.	n.a.	.	n.a.	n.a.	n.a.	.	n.a.	.

Note: The table shows the relationship between the degree of negative correlation among observables and the strength of identification. Positive (negative) sign in cell (i,j) means that correlation between the j-th pair of variables tends to be stronger (weaker) *and* negative in the region of $\boldsymbol{\Theta}$ where θ_i is better identified, than in the region where θ_i is worse identified. The sign "±" indicates that there is a systematic effect whose sign changes with the degree of correlation. An empty cell indicates the absence of a systematic effect. With "na" are indicated the cases where there are not enough points with negative correlation in the two regions to test for difference in the distributions.

Table 20: Bayesian and frequentist bounds at the posterior mean

	posterior			a priori bounds			
Par.	$\hat{\theta}_i$	$\hat{\theta}_i \pm \text{std}(\hat{\theta}_i)$		bayesian		frequentist	
φ	5.744	4.715	6.773	4.571	6.917	3.432	8.056
σ_c	1.380	1.249	1.511	1.247	1.514	1.086	1.675
λ	0.714	0.673	0.755	0.674	0.754	0.651	0.777
ξ_w	0.701	0.630	0.771	0.638	0.763	0.515	0.886
σ_l	1.837	1.217	2.456	1.268	2.405	0.767	2.906
ξ_p	0.650	0.592	0.709	0.604	0.696	0.590	0.711
ι_w	0.589	0.456	0.722	0.468	0.710	0.379	0.799
ι_p	0.244	0.152	0.336	0.150	0.337	0.110	0.377
ψ	0.546	0.431	0.662	0.444	0.648	0.373	0.719
Φ	1.604	1.527	1.682	1.521	1.688	1.467	1.742
r_π	2.045	1.864	2.227	1.850	2.241	1.628	2.463
ρ	0.808	0.784	0.833	0.782	0.835	0.762	0.854
r_y	0.088	0.065	0.110	0.065	0.110	0.047	0.128
$r_{\triangle y}$	0.224	0.196	0.251	0.193	0.255	0.174	0.273
$\bar{\pi}$	0.785	0.687	0.883	0.696	0.875	0.555	1.016
β'	0.166	0.106	0.227	0.091	0.242	0.006	0.326
$\bar{l}$	0.542	-0.063	1.147	-0.375	1.458	-0.966	2.050
γ	0.431	0.417	0.445	0.420	0.442	0.420	0.442
α	0.191	0.173	0.208	0.174	0.208	0.168	0.213
δ	na	na	na	0.021	0.029	0.014	0.036
λ_w	na	na	na	1.289	1.711	0.707	2.293
g_y	na	na	na	0.141	0.219	0.109	0.251
ρ_a	0.958	0.947	0.968	0.945	0.971	0.942	0.974
ρ_b	0.217	0.133	0.301	0.137	0.296	0.127	0.307
ρ_g	0.976	0.968	0.985	0.966	0.986	0.965	0.988
ρ_I	0.711	0.652	0.770	0.654	0.767	0.643	0.778
ρ_r	0.151	0.086	0.217	0.070	0.232	0.056	0.247
ρ_p	0.891	0.845	0.938	0.842	0.941	0.832	0.951
ρ_w	0.968	0.955	0.981	0.954	0.982	0.952	0.984
μ_p	0.699	0.612	0.786	0.590	0.808	0.548	0.850
μ_w	0.841	0.790	0.893	0.791	0.892	0.781	0.902
ρ_{ga}	0.521	0.432	0.610	0.433	0.610	0.416	0.626
σ_a	0.460	0.432	0.487	0.430	0.489	0.424	0.495
σ_b	0.240	0.217	0.264	0.215	0.266	0.213	0.268
σ_g	0.529	0.499	0.559	0.497	0.561	0.493	0.564
σ_I	0.453	0.405	0.502	0.405	0.501	0.402	0.505
σ_r	0.245	0.231	0.260	0.230	0.261	0.228	0.263
σ_p	0.140	0.123	0.157	0.121	0.159	0.117	0.163
σ_w	0.244	0.222	0.266	0.220	0.268	0.216	0.272

Note: The values of $\text{std}(\hat{\theta}_i)$ are estimated from the posterior sample. The a priori bounds are computed using the (Bayesian) information matrix and are based on the (Bayesian) Cramér-Rao lower bound.

Table 21: Sensitivity to prior uncertainty

Par.	φ	σ_c	λ	ξ_w	σ_l	ξ_p	ι_w	ι_p	ψ	Φ	r_π	ρ	r_y	$r_{\triangle y}$	$\bar{\pi}$	β'	$\bar{l}$	γ	α	δ	λ_w	g_y	ρ_b	ρ_I	ρ_r	ρ_p	μ_p	μ_w	ρ_{ga}
φ	61	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	.	.	.
σ_c	.	13	5	1	1	.	.	.	.	.	2	.	1	.	.	9	.	.	.	2	7	1	.	.	.	.	.	.	.
λ	3	4	16	.	4	.	.	.	1	1	.	.	.	3	.	3	.	.	.	1	1	1	2	.	.	.	.	.	.
ξ_w	.	.	.	39	4	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	21	.	.	.	.	.	.	1	.
σ_l	.	.	1	3	57	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.
ξ_p	.	.	.	2	.	21	.	.	2	3	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.
ι_w	.	.	.	.	.	.	65	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ι_p	.	.	.	.	.	.	.	39	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	8	.	.
ψ	.	.	.	.	.	1	.	.	47	1	.	.	.	1	.	.	.	.	.	1	.	1	.	.	.	.	.	.	.
Φ	.	.	.	.	.	1	.	.	2	45	.	.	.	1	.	.	.	.	1	1	.	2	.	.	.	.	.	.	1
r_π	.	.	.	.	.	.	.	.	.	.	61	3	8	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ρ	.	1	.	.	1	.	.	.	.	.	23	7	6	.	.	.	.	.	.	.	1	.	.	.	3	.	.	.	.
r_y	.	1	.	.	.	.	.	.	.	.	23	2	21	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
$r_{\triangle y}$	.	.	1	.	4	.	.	.	1	1	1	.	.	38	.	.	.	.	.	.	.	1	.	.	1	.	.	.	.
$\bar{\pi}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	80	.	4	.	.	.	.	.	.	.	.	.	.	.	.
β'	.	2	1	.	.	.	.	.	.	.	.	.	.	.	.	57	.	.	1	.	1	.	.	.	.	.	.	.	.
$\bar{l}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	14	1	21	.	.	.	.	.	.	.	.	.	.	.	.
γ	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
α	.	.	.	.	.	.	.	.	.	4	.	.	.	.	.	3	.	.	11	5	.	.	.	.	.	.	.	.	.
δ	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1	77	.	.	.	.	.	.	.	.	.
λ_w	1	1	.	11	1	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	71	.	.	.	.	.	.	.	.
g_y	.	.	.	.	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	.	.	61	.	.	.	.	.	.	.
ρ_a	.	1	.	.	.	.	.	.	1	2	.	.	.	.	.	1	.	.	1	6	.	.	.	.	.	.	.	.	.
ρ_b	.	.	2	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	16	.	.	.	.	.	.
ρ_g	.	1	.	.	.	.	.	.	1	.	.	.	.	.	.	1	.	.	.	2	.	1	.	.	.	.	.	.	.
ρ_I	6	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	8	.	.	.	.	.
ρ_r	.	.	.	.	.	.	.	.	.	.	.	1	.	3	.	.	.	.	.	.	.	.	.	.	16	.	.	.	.
ρ_p	.	.	.	.	.	2	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	6	16	.	.
ρ_w	.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	.	.	.	.	3	.	.	.	.	.	.	.	1	.
μ_p	.	.	.	.	.	.	.	10	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	3	30	.	.
μ_w	.	.	.	8	1	1	.	2	.	.	1	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	6	.
ρ_{ga}	.	.	.	.	.	.	.	.	.	2	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	20
σ_a	.	.	.	.	.	.	.	.	2	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
σ_b	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	10	.	.	.	.	.	.
σ_g	.	.	.	.	.	.	.	.	1	3	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.
σ_I	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	4	.	.	.	.	.
σ_r	1	.	.	.	1	.	.	.	.	.	.	.	.	5	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.
σ_p	.	.	.	.	.	.	4	5	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	10	.	.
σ_w	.	.	.	.	.	1	9	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.

Note: Cell (i, j) of the table shows (100 $\times$) the elasticity of the Bayesian bound on the root mean squared error of parameter θ_i to the prior standard deviation of parameter θ_j.

Table 22: Effect of fixing parameters (part I)

Par.	φ	σ_c	λ	ξ_w	σ_l	ξ_p	ι_w	ι_p	ψ	Φ	r_π	ρ	r_y	$r_{\triangle y}$	$\bar{\pi}$	β'	$\bar{l}$	γ	α	δ	λ_w	g_y
φ	100	0.1	2.5	0.0	.	0.4	.	.	.	0.4	0.1	0.2	0.1	0.1	.	.	.	.	.	.	0.9	.
σ_c	0.1	100	16.6	0.9	1.1	0.3	.	.	.	.	1.7	2.2	3.2	0.1	.	7.8	0.2	.	1.5	1.2	5.0	1.1
λ	2.5	16.6	100	.	3.1	.	.	.	0.7	0.8	0.1	.	0.8	4.5	.	2.3	0.1	.	0.6	0.6	0.6	1.1
ξ_w	.	0.9	.	100	4.0	2.5	0.4	0.1	.	.	0.1	0.5	0.1	0.1	.	0.1	.	.	0.1	.	15.9	.
σ_l	.	1.1	3.1	4.0	100	.	.	.	0.3	0.2	.	0.5	.	3.1	.	0.1	.	.	0.1	.	0.9	0.2
ξ_p	0.4	0.3	.	2.5	.	100	0.1	.	2.2	3.2	0.7	0.4	0.6	0.5	.	0.1	.	.	0.2	.	0.1	0.3
ι_w	.	.	.	0.4	.	0.1	100	0.1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
ι_p	.	.	.	0.1	.	.	0.1	100	.	.	.	0.2	.	.	.	.	.	.	.	.	.	.
ψ	.	.	0.7	.	0.3	2.2	.	.	100	1.7	.	0.1	.	1.2	.	.	.	.	0.1	0.9	.	1.0
Φ	0.4	.	0.8	.	0.2	3.2	.	.	1.7	100	.	0.1	0.2	1.4	.	.	.	.	4.1	0.4	0.1	1.4
r_π	0.1	1.7	0.1	0.1	.	0.7	.	.	.	.	100	21.1	21.4	1.0	.	0.3	.	.	0.2	.	.	.
ρ	0.2	2.2	.	0.5	0.5	0.4	.	0.2	0.1	0.1	21.1	100	15.8	0.3	.	0.3	.	.	0.3	.	0.9	.
r_y	0.1	3.2	0.8	0.1	.	0.6	.	.	.	0.2	21.4	15.8	100	0.3	.	0.4	.	.	0.1	0.2	0.3	0.6
$r_{\triangle y}$	0.1	0.1	4.5	0.1	3.1	0.5	.	.	1.2	1.4	1.0	0.3	0.3	100	.	.	.	.	0.3	.	.	0.8
$\bar{\pi}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	100	0.2	9.4	0.1	.	.	.	.
β'	.	7.8	2.3	0.1	0.1	0.1	.	.	.	.	0.3	0.3	0.4	.	0.2	100	0.9	0.1	2.4	0.1	0.8	0.2
$\bar{l}$	.	0.2	0.1	.	.	.	.	.	.	.	.	.	.	.	9.4	0.9	100	0.4	.	.	.	.
γ	.	.	.	.	.	.	.	.	.	.	.	.	.	.	0.1	0.1	0.4	100	.	.	.	.
α	.	1.5	0.6	0.1	0.1	0.2	.	.	0.1	4.1	0.2	0.3	0.1	0.3	.	2.4	.	.	100	3.5	0.1	.
δ	.	1.2	0.6	.	.	.	.	.	0.9	0.4	.	.	0.2	.	.	0.1	.	.	3.5	100	.	0.1
λ_w	0.9	5.0	0.6	15.9	0.9	0.1	.	.	.	0.1	.	0.9	0.3	.	.	0.8	.	.	0.1	.	100	.
g_y	.	1.1	1.1	.	0.2	0.3	.	.	1.0	1.4	.	.	0.6	0.8	.	0.2	.	.	.	0.1	.	100
ρ_a	.	5.2	0.3	0.1	.	0.9	.	.	1.2	2.1	0.2	0.1	.	0.1	.	1.2	.	.	2.6	3.9	0.1	.
ρ_b	.	0.1	5.3	.	0.1	.	.	.	0.1	.	0.1	0.1	0.1	0.2	.	.	.	.	.	.	.	.
ρ_g	.	4.9	0.4	.	0.4	0.4	.	.	0.9	0.2	.	.	.	0.2	.	0.9	.	.	2.2	1.6	.	0.6
ρ_I	4.9	1.1	0.2	.	0.2	.	.	.	0.2	.	0.2	0.1	0.3	0.1	.	0.2	.	.	.	0.5	.	.
ρ_r	.	0.2	.	0.1	0.2	.	.	.	.	.	0.2	8.6	0.5	3.9	.	.	.	.	.	.	0.1	.
ρ_p	.	.	.	0.1	0.1	5.8	0.3	0.9	0.1	.	.	.	0.1	.	.	.	.	.	.	.	.	.
ρ_w	.	0.5	0.3	1.9	0.1	1.2	0.3	0.1	0.8	0.1	0.5	.	0.9	.	.	.	.	.	.	1.9	.	0.2
μ_p	.	.	.	.	0.1	.	0.1	14.3	.	0.1	.	.	.	.	.	.	.	.	.	.	.	.
μ_w	0.2	0.4	.	11.2	0.6	2.3	0.1	2.2	.	0.2	0.5	1.0	0.7	0.1	.	0.1	.	.	.	0.3	0.8	.
ρ_{ga}	.	.	0.1	.	.	0.1	.	.	0.2	2.7	.	.	.	0.2	.	.	.	.	.	.	.	0.5
σ_a	.	.	0.1	.	0.1	0.8	.	.	2.0	9.3	.	.	.	0.1	.	.	.	.	2.1	0.3	.	0.3
σ_b	.	1.4	3.3	.	0.2	.	.	.	.	0.1	.	.	.	0.3	.	0.2	.	.	0.1	0.1	.	0.1
σ_g	.	.	.	.	.	0.4	.	.	1.4	3.5	.	.	.	.	.	0.1	.	.	0.1	.	.	0.9
σ_I	0.7	.	0.2	.	0.1	0.2	.	.	.	.	0.2	.	0.2	0.2	.	.	.	.	0.1	0.3	0.1	.
σ_r	0.6	.	0.5	.	0.4	.	.	.	.	.	0.3	0.5	.	7.1	.	.	.	.	.	.	.	0.1
σ_p	0.1	.	.	0.1	0.2	0.2	3.3	6.0	.	.	0.1	.	.	.	.	.	.	.	.	.	.	.
σ_w	0.1	.	.	0.3	0.3	2.5	6.7	1.6	0.1	.	.	0.3	0.1	.	.	.	.	.	.	.	.	.

Note: Cell (i, j) of the table shows the percentage reduction of the Bayesian bound on the root mean squared error of parameter θ_i due to fixing parameter θ_j.

Table 23: Effect of fixing parameters (part II)

Par.	ρ_a	ρ_b	ρ_g	ρ_I	ρ_r	ρ_p	ρ_w	μ_p	μ_w	ρ_{ga}	σ_a	σ_b	σ_g	σ_I	σ_r	σ_p	σ_w
φ	.	.	.	4.9	.	.	.	.	0.2	.	.	.	.	0.7	0.6	0.1	0.1
σ_c	5.2	0.1	4.9	1.1	0.2	.	0.5	.	0.4	.	.	1.4	.	.	.	.	.
λ	0.3	5.3	0.4	0.2	.	.	0.3	.	.	0.1	0.1	3.3	.	0.2	0.5	.	.
ξ_w	0.1	.	.	.	0.1	0.1	1.9	.	11.2	.	.	.	.	.	.	0.1	0.3
σ_l	.	0.1	0.4	0.2	0.2	0.1	0.1	0.1	0.6	.	0.1	0.2	.	0.1	0.4	0.2	0.3
ξ_p	0.9	.	0.4	.	.	5.8	1.2	.	2.3	0.1	0.8	.	0.4	0.2	.	0.2	2.5
ι_w	.	.	.	.	.	0.3	0.3	0.1	0.1	.	.	.	.	.	.	3.3	6.7
ι_p	.	.	.	.	.	0.9	0.1	14.3	2.2	.	.	.	.	.	.	6.0	1.6
ψ	1.2	0.1	0.9	0.2	.	0.1	0.8	.	.	0.2	2.0	.	1.4	.	.	.	0.1
Φ	2.1	.	0.2	.	.	.	0.1	0.1	0.2	2.7	9.3	0.1	3.5	.	.	.	.
r_π	0.2	0.1	.	0.2	0.2	.	0.5	.	0.5	.	.	.	.	0.2	0.3	0.1	.
ρ	0.1	0.1	.	0.1	8.6	.	.	.	1.0	.	.	.	.	.	0.5	.	0.3
r_y	.	0.1	.	0.3	0.5	0.1	0.9	.	0.7	.	.	.	.	0.2	.	.	0.1
$r_{\triangle y}$	0.1	0.2	0.2	0.1	3.9	.	.	.	0.1	0.2	0.1	0.3	.	0.2	7.1	.	.
$\bar{\pi}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
β'	1.2	.	0.9	0.2	.	.	.	.	0.1	.	.	0.2	0.1	.	.	.	.
$\bar{l}$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
γ	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
α	2.6	.	2.2	.	.	.	.	.	.	.	2.1	0.1	0.1	0.1	.	.	.
δ	3.9	.	1.6	0.5	.	.	1.9	.	0.3	.	0.3	0.1	.	0.3	.	.	.
λ_w	0.1	.	.	.	0.1	.	.	.	0.8	.	.	.	.	0.1	.	.	.
g_y	.	.	0.6	.	.	.	0.2	.	.	0.5	0.3	0.1	0.9	.	0.1	.	.
ρ_a	100	0.2	13.8	.	.	.	0.4	.	0.5	.	0.9	0.1	0.1	.	.	.	.
ρ_b	0.2	100	0.1	.	0.1	.	.	.	.	.	.	39.5	.	.	.	.	.
ρ_g	13.8	0.1	100	.	.	.	0.2	.	0.1	.	0.2	0.1	.	.	.	.	.
ρ_I	.	.	.	100	.	.	0.6	.	.	.	.	0.1	.	31.7	.	.	.
ρ_r	.	0.1	.	.	100	.	.	.	0.1	.	.	.	.	.	1.8	.	.
ρ_p	.	.	.	.	.	100	0.3	32.4	0.2	.	.	.	.	.	.	0.8	0.2
ρ_w	0.4	.	0.2	0.6	.	0.3	100	0.1	5.3	.	0.1	.	.	0.2	.	0.1	1.7
μ_p	.	.	.	.	.	32.4	0.1	100	0.4	.	.	.	.	.	.	18.6	0.6
μ_w	0.5	.	0.1	.	0.1	0.2	5.3	0.4	100	.	.	.	.	0.1	.	1.0	10.0
ρ_{ga}	.	.	.	.	.	.	.	.	.	100	0.3	.	0.2	.	.	.	.
σ_a	0.9	.	0.2	.	.	.	0.1	.	.	0.3	100	.	0.7	.	.	.	.
σ_b	0.1	39.5	0.1	0.1	.	.	.	.	.	.	.	100	.	.	.	.	.
σ_g	0.1	.	.	.	.	.	.	.	.	0.2	0.7	.	100	.	.	.	.
σ_I	.	.	.	31.7	.	.	0.2	.	0.1	.	.	.	.	100	0.1	0.1	.
σ_r	.	.	.	.	1.8	.	.	.	.	.	.	.	.	0.1	100	.	.
σ_p	.	.	.	.	.	0.8	0.1	18.6	1.0	.	.	.	.	0.1	.	100	0.8
σ_w	.	.	.	.	.	0.2	1.7	0.6	10.0	.	.	.	.	.	.	0.8	100

Note: Cell (i, j) of the table shows the percentage reduction of the Bayesian bound on the root mean squared error of parameter θ_i due to fixing parameter θ_j.

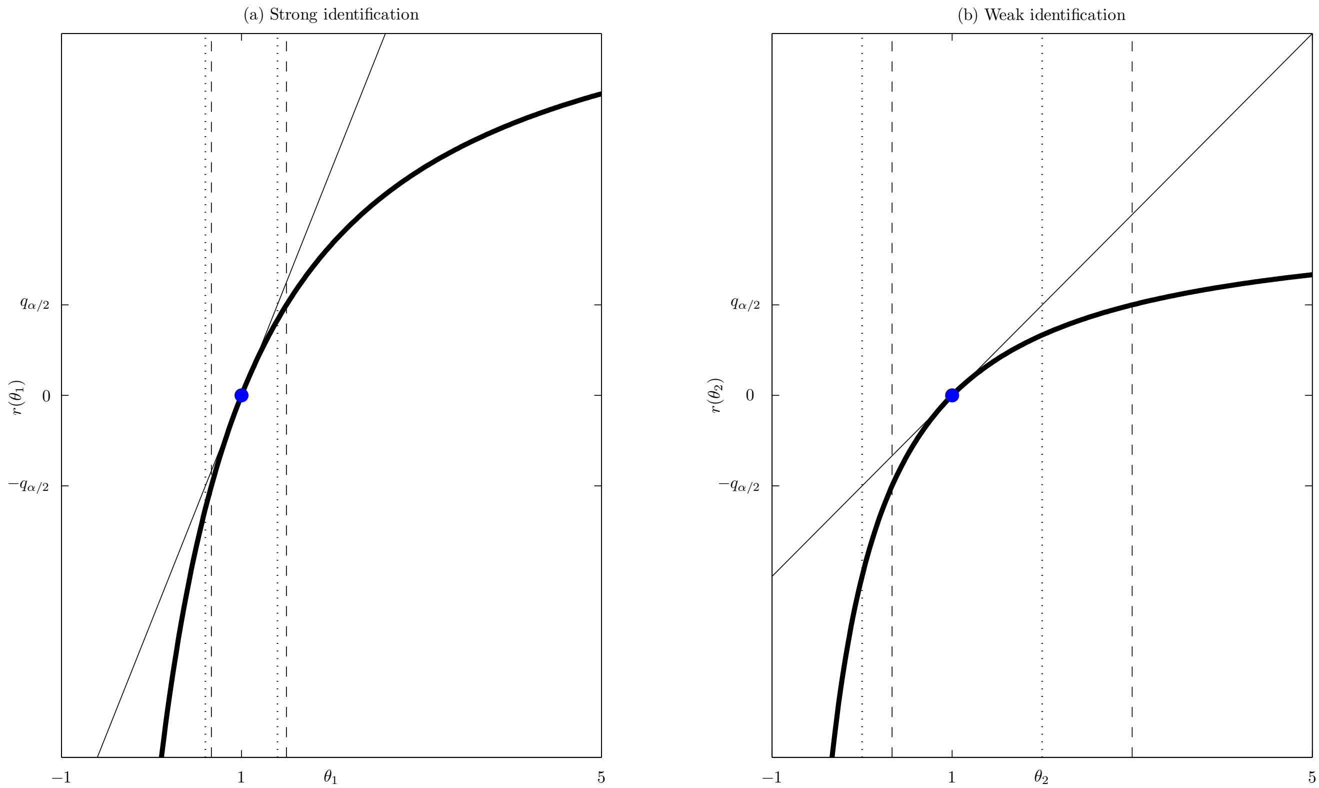

Figure 1: Likelihood-based (dashed lines) and linearized (dotted lines) confidence intervals.

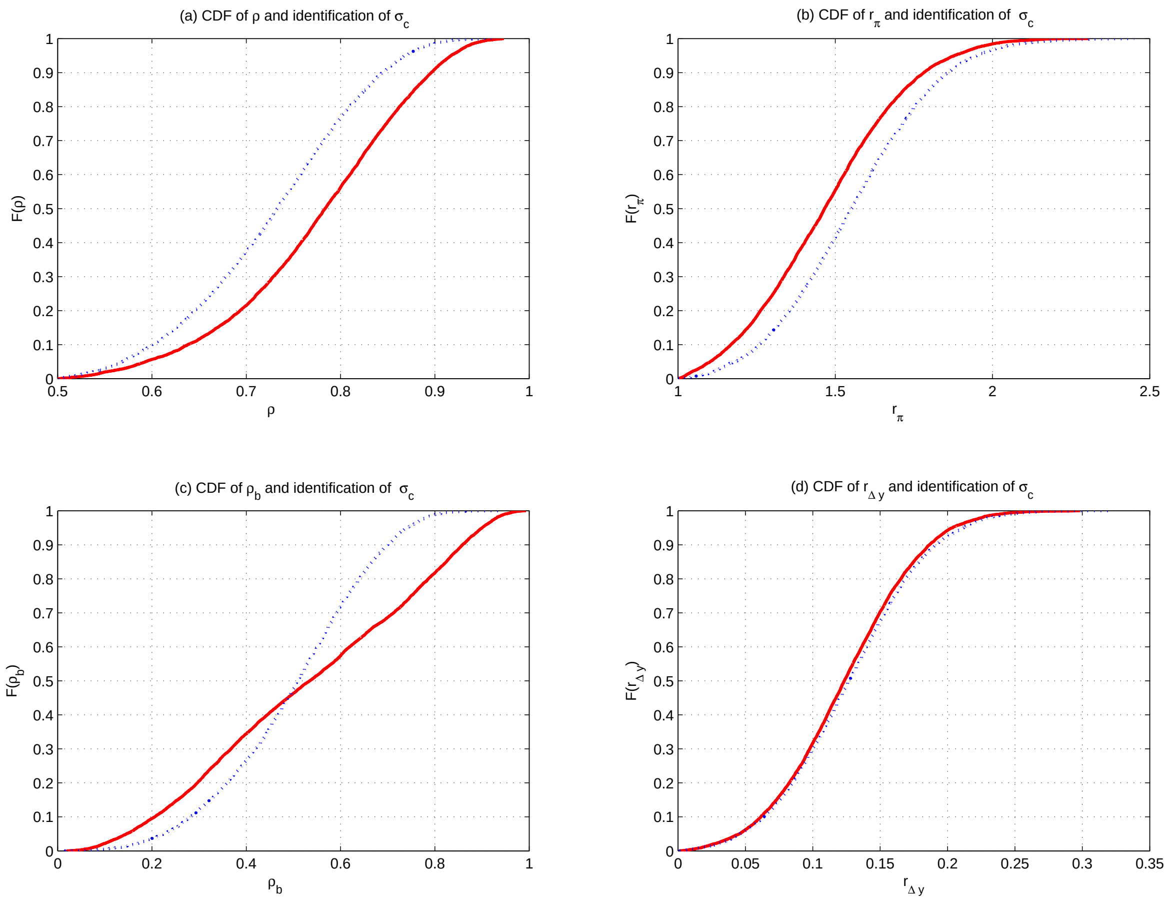

Figure 2: Cumulative distribution functions of four parameters in regions of the parameter space where the identification of σ_c is stronger (solid line) and weaker (dotted line). The two regions consist of point $\boldsymbol{\theta}$ for which the relative CRLB for σ_c are below the second decile and above the 8 decile.

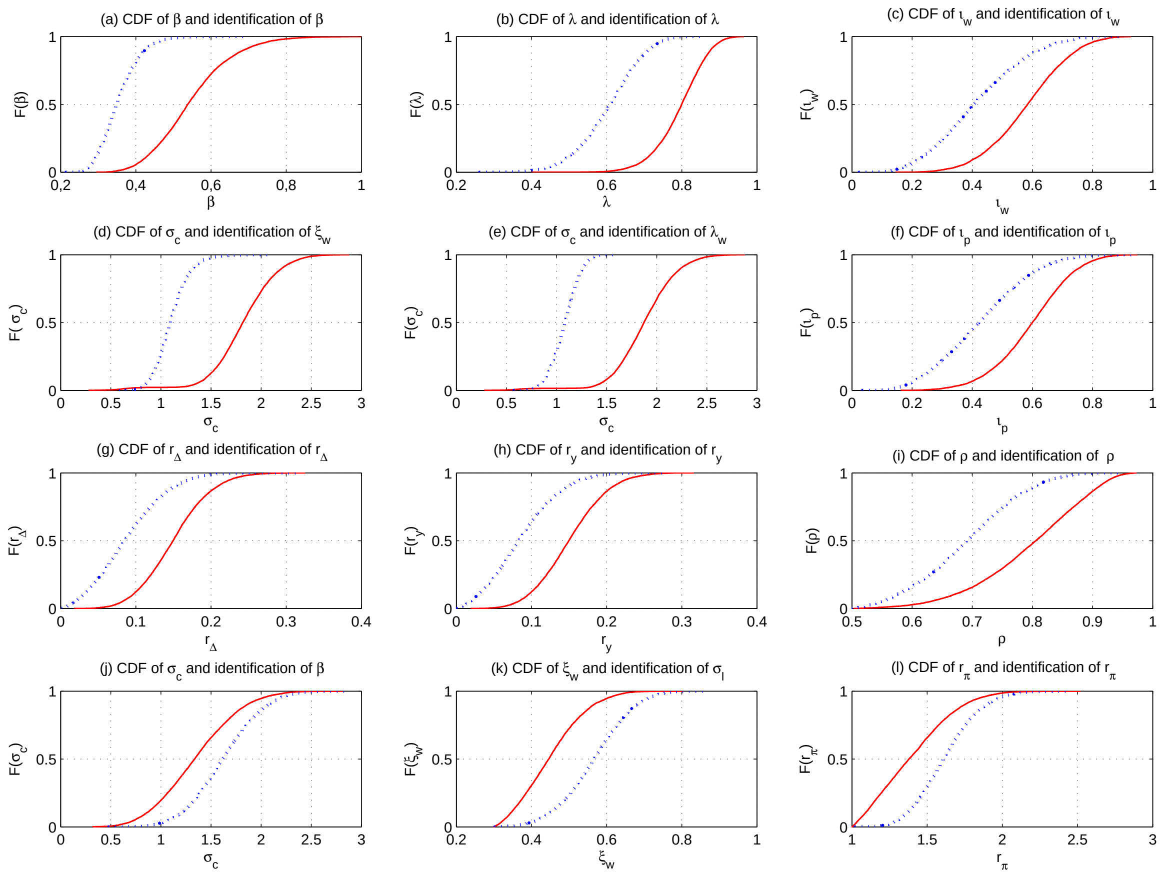

Figure 3: Cumulative distribution functions in regions of the parameter space where the identification is stronger (solid line) and weaker (dotted line).

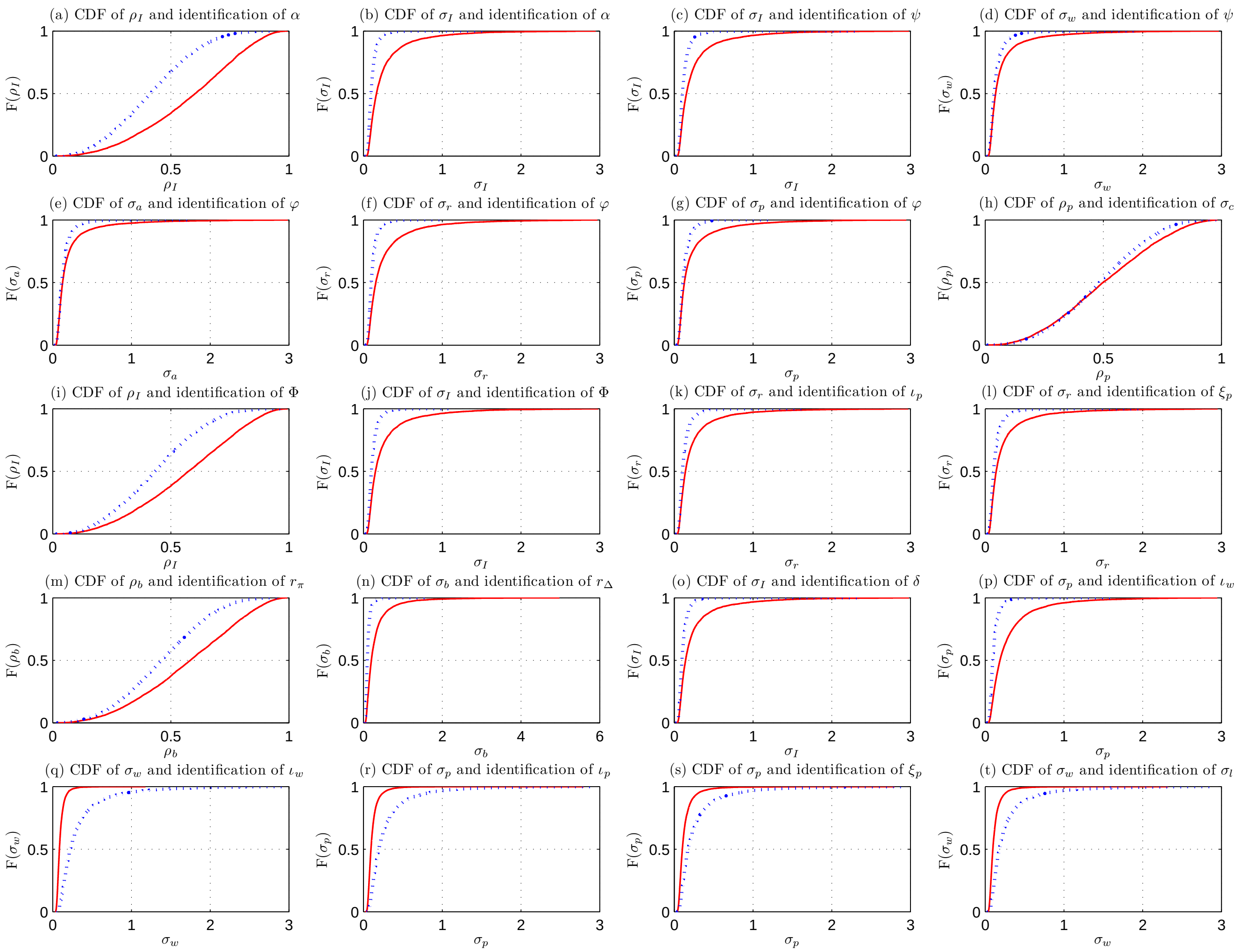

Figure 4: Cumulative distribution functions in regions of the parameter space where the identification is stronger (solid line) and weaker (dotted line).

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