Public Goods Provision and Redistributive Taxation

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Abstract

This paper studies the relationship between redistributive taxation and tax-deductible charitable contributions. Redistribution has two opposite effects on voluntary giving. The price of charitable giving decreases with the degree of redistribution, and this has a positive effect on the total amount of giving (substitution effect). However, redistribution leads to lower consumption for the contributors and therefore has a negative effect on contributions to the charity (income effect). The theoretical model developed in this paper demonstrates that, under a general class of utility functions, the substitution effect dominates the income effect. Hence, charitable giving increases with the tax rate. In purely egalitarian societies, the public good is provided efficiently and the total welfare is maximized independent of the ex-ante income inequality. However, the positive impact of taxation on charitable giving and welfare may disappear if individuals generate their income levels in anticipation of taxation and redistribution does not take into account the cost of effort.

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1 Introduction

In the United States, starting from 1917, donations to many charities have become tax-deductible. This feature of the tax code implies that, for agents who can itemize their deductions, the cost of charitable contributions is inversely related to the tax rate. For example, if the tax payer faces a tax rate of $\gamma$, then the price/cost of giving will be $1 - \gamma$ for a dollar worth of contribution[1]. Therefore, tax affects charitable giving through two channels: through its effect on the after-tax net income and through its effect on the price of giving. The main questions this paper asks are: How are the voluntary contributions of individuals affected by redistributive taxation? Is there a trade-off between redistribution and welfare?

Although there is a large empirical literature on taxation and charitable giving (i.e., Reece, 1979; Clotfelter, 1985; Auten et al., 1992, 2002; Randolph, 1995), how redistributive policies affect charitable contributions remains an open question. In the early empirical studies, typical estimates for price elasticity are greater than one in absolute value and typical estimates for income elasticity are less than one, i.e., a tax cut implies a decrease in charitable giving (Clotfelter, 1990). However, Randolph (1995) finds evidence that charitable giving may be relatively insensitive to changes in the price of giving. He finds that permanent price changes have a small effect on voluntary contributions. In contrast, Auten et al. (2002) find a substantial permanent price elasticity. However, the net effect of a tax change on contributions is still not entirely understood in the absence of a theory, since it is difficult to make strong inferences using the naturally occurring data (Auten et al., 2002; Peloza and Steel, 2005; Vesterlund, 2006).

The aim of this paper is to provide a theoretical foundation for analyzing the effects of taxation on charitable giving. The theory builds on the standard public goods model of Bergstrom, Blume and Varian (1986). Consider a group of agents with known wealth levels that are making observable voluntary contributions in order to provide a public good. In this paper, I incorporate redistributive taxation that depends on income net of contributions to

[1]The price of giving is defined as the dollar amount forgone per dollar of contribution.
the public good: the government collects a flat-rate tax \((\gamma)\) on income net of contributions, and then redistributes the tax revenue equally.

The model can clearly distinguish the impact of taxation and pre-tax income inequality on charitable giving. In a one-shot game, the theoretical model demonstrates that while pre-tax equality has a negative impact on charitable contributions\(^2\) taxation has two opposite effects on voluntary giving. The marginal cost of providing one more unit to the charity is lower with tax, and this has a positive effect on the total amount of giving (substitution effect). However, taxation leads to lower consumption for the contributors and therefore has a negative effect on contributions to the charity (income effect). The paper shows that if the relative risk aversion coefficient is low enough, then the substitution effect dominates the income effect. Hence, voluntary giving strictly increases in the degree of redistribution.

This paper contributes to the literature on the trade-off between equality and efficiency. In a joint production context, Ray and Ueda (1996) show that production increases with the degree of egalitarianism embodied in the social welfare function. In addition, Cornes and Sandler (1996), Bardhan et al. (2002) and Ray et al. (2007) argue that the trade-off between equality and efficiency also disappears when individuals’ contributions are partially complementary or when there are complementarities between voluntary contributions and private inputs. In this paper, I show that purely egalitarian societies \((\gamma = 1)\) have efficient levels of charitable giving and the highest possible total welfare independent of the initial income distribution.

The findings of the paper are theoretically robust in that they hold for a large class of utility functions (including the standard Cobb-Douglas utility function). Section 3 discusses three important extensions\(^3\) First, I argue that public goods provision may not be higher in societies with high tax rates if pre-tax income inequality and tax rate are correlated. Second, the model is extended in order to incorporate the impact of taxation on effort

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\(^2\)The main result of Bergstrom et al. (1986) continues to hold. This finding is also consistent with the Olson’s argument that higher levels of (pure) public goods will be achieved with higher levels of inequality. However, the effect of income inequality on the level of provision may become ambiguous when more general collective goods are considered (i.e., Bardhan et al. (2002, 2007), Baland and Platteau (1997, 1998, 1999 and 2007)).

\(^3\)I thank an anonymous referee for suggesting these extensions.
choice. I show that welfare may decrease with redistribution, if redistribution does not take into account the cost of effort. Finally, I explore whether societies can decide on the efficient levels of taxes through political processes. In particular, the trade-offs of majority voting on redistribution and total welfare is considered. For simplicity, throughout the paper we assume that agents do not have utility from own contributions. Although, as Andreoni (1989,1990) demonstrates, this may play a role in individuals’ contribution decisions, we disregard that here in order to focus on the effects of redistribution.

A paper that is closely related to this one is Falkinger (1996). The aim of Falkinger (1996) is to introduce a mechanism to efficiently provide public goods. The following incentive scheme is proposed: each individual receives a subsidy if her contribution is greater than the mean contribution and pays a tax if her contribution is lower than the mean contribution. The mean contribution is defined as either the average contribution of the whole population or the average contribution of the income class to which the individual belongs. However, this mechanism uses a non-standard taxation scheme; it either does not generate transfers between different income classes (if the tax-subsidy is within the income class to which the individual belongs), or if it does then the transfers are generally from poorer income classes to higher income classes. In this paper, redistribution depends on the net income of individuals relative to the average net income instead of the difference between own contribution and the average contribution.

The rest of the paper is organized as follows. The theoretical framework and main results are presented in Section 2. Section 3 demonstrates the main findings under Cobb-Douglas preferences and extends the model in several dimensions. Section 4 discusses to what extent the results of this paper can explain the impact of informal sharing, observed in developing societies, on voluntary public goods provision. Section 5 concludes.

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4It has been shown that this mechanism is indeed very successful at increasing the levels of public good provision (Falkinger et al. (2000)).

5I consider net income, income net of contributions, since charitable giving is tax-deductible.
# The Model

There is one private good, one pure public good and \( n > 1 \) agents. The public good is provided through charitable contributions. Each agent \( i \) has an exogenous endowment, \( w_i \), and decides how much to contribute to the public good, \( g_i \), in a static one-shot game. The level of public good provision is equal to the total giving, \( G = \sum_{i=1}^{n} g_i \). Let \( g_{-i} = (g_1, ..., g_{i-1}, g_{i+1}, ..., g_n) \) denotes the vector of contributions by all individuals except \( i \).

Suppose the government collects a flat-rate tax on income net of contributions, and then redistributes the tax revenue equally. Hence consumption of each individual \( i \), \( y_i \), is equal to

\[
y_i = (1 - \gamma)(w_i - g_i) + \frac{\gamma \sum_{j=1}^{n} (w_j - g_j)}{n}.
\]

where \( \gamma \) is the tax rate. Equivalently, each individual makes or receives a transfer depending on their income level net of their voluntary contributions. More formally, the transfer that they will receive or make is determined by a function \( t : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+^{n-1} \rightarrow \mathbb{R} \) where

\[
t(w_i, g_i, g_{-i}) = \gamma [(w_i - g_i) - \frac{1}{n} \sum_{j=1}^{n} (w_j - g_j)]
\]

with \( 0 \leq \gamma \leq 1 \) determining the degree of redistribution. For example, \( \gamma = 1 \) enforces perfect ex-post equality across agents. As \( \gamma \) decreases, the amount of redistribution decreases (i.e., there are no transfers when \( \gamma = 0 \) and the model is equivalent to Bergstrom et al. (1986)).

An important observation is \( \sum_{i=1}^{n} t(w_i, g_i, g_{-i}) = 0 \). In other words, the budget is balanced. The budget constraint for individual \( i \) is:

\[
y_i + g_i + t(w_i, g_i, g_{-i}) = w_i
\]

Suppose agents all have the same additively separable utility function and solve the following optimization problem:
\[
\max_{y_i, g_i} \ u(y_i) + v(G) \\
\text{s.t.} \ y_i + g_i + t(w_i, g_i, g_{-i}) = w_i \\
g_i \geq 0
\]

where \(u(.)\) and \(v(.)\) are strictly increasing, strictly concave, twice continuously differentiable functions and satisfy Inada conditions\(^6\).

Before moving on to studying the equilibrium outcomes, I find the socially optimal public good provision and consumption levels. The social planner maximizes \(\sum_{i=1}^{n} [u(y_i) + v(G)]\), and therefore has to divide the ex-post wealth equally. So, the social planner has the following problem:

\[
\max_{G} \ u\left(\frac{W - G}{n}\right) + v(G)
\]

with the first order condition,

\[
u'(y) \frac{1}{n} = v'(G).
\]

(3)

where the individual private consumption is given by \(y = \frac{W - G}{n}\), and \(W = \sum_{i=1}^{n} w_i\).

Now I examine the equilibrium. Each agent satisfies the following first order condition:

\[
u'(w_i - g_i - t(w_i, g_i, g_{-i}))\left[1 - \frac{n - 1}{n}\gamma\right] \geq v'(G)
\]

(4)

This holds with equality if \(g_i > 0\). Result 1 states that there cannot be more than one equilibrium.

**Result 1:** There is a unique equilibrium\(^7\).

The proofs are provided in Appendix A. Note that existence follows directly from Kaku-tani’s fixed point theorem. To investigate the impact of taxation on the individual contributions, initially we focus our attention on an interior equilibrium (each agent contributes a positive amount to the public good). Later the results will be generalized. Assuming an

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\(^6\)These assumptions are not necessary for the results. They only simplify the analysis. For example, quasilinear preferences, where \(v(G) = G\), satisfy our results as well.

\(^7\)Except when \(\gamma = 1\), individual contributions cannot be uniquely determined.
interior solution in the equilibrium, the first order condition is:

\[ u'(y_i)[1 - \frac{n-1}{n}\gamma] = u'(G) \]  

(5)

If everyone contributes in equilibrium, then \( y_i = y_j = \frac{W-G}{n} \) for all \( i \) and \( j \). Therefore, \( (5) \) implies that public good provision increases as \( \gamma \) increases. Higher \( \gamma \) decreases free riding since the marginal cost of giving is lower due to the threat of redistribution.\(^8\)

**Result 2:** Suppose everyone contributes in the equilibrium, then public good provision and total welfare increases with the degree of redistribution (\( \gamma \)). The socially optimal level of public good provision is reached when \( \gamma = 1 \).

Since the welfare function is concave in \( G \), welfare increases as \( \gamma \) increases. Indeed, increasing the degree of redistribution creates a Pareto improvement. The intuition is that since everyone enjoys the same utility, as total welfare increases with egalitarianism, everyone has improved. In order to see \( \gamma = 1 \) maximizes the total welfare, note that \( (5) \) converges to \( (3) \) as \( \gamma \to 1 \). In contrast, increasing the initial (pre-tax) income inequality does not change the public good provision and welfare of individuals.

**Result 3:** In the interior equilibrium, total contributions and welfare do not depend on the degree of initial (pre-tax) income inequality.

Any ex-ante redistribution of wealth between the contributors, that does not change the set of contributors, will not change the total amount provided. In other words, the distribution-neutrality result continues to hold in this model [see, Warr (1983) and Bergstrom et al. (1986)]. To see this, let everyone change their contribution to the public good by exactly the same amount as the change in their wealth levels. Clearly the first order conditions

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\(^8\)In the interior equilibrium, redistribution does not occur. The only way ex-post consumption can be equal across all agents when \( \gamma < 1 \) is if \( w_i - g_i = w_j - g_j \) for all \( i \) and \( j \). Since income net of contributions is same for everyone, there will be no redistribution of wealth following the public good provision, i.e., \( y_i = w_i - g_i \) for all \( i \).

\(^9\)Redistribution of income takes place before contributions to the public good are made.
will continue to be satisfied. So neither the total provision nor the private consumption of
the agents will change.

If we assume everybody contributes in the equilibrium, we have already seen that \( G(\gamma) \)
increases in \( \gamma \). For example, if the ex-ante wealth distribution is perfectly equal, then
everybody contributes in the equilibrium, and public good provision increases due to the
threat of redistribution. However, it is important to know what will happen if there are
non-contributors since in the real world we observe many cases where poor individuals do
not have enough resources to contribute to the public good. Each contributor has to satisfy
the following condition:

\[
u'(\gamma) = v'(G) \tag{6}
\]

Note that, if \( w_j < w_k \), then \( g_j \leq g_k \) for any \( j \) and \( k \). From (6), it can be seen that
all contributors will have the same amount of consumption. Non-contributors with different
levels of initial wealth will consume different amounts. Hence, in equilibrium, agents enjoy
different levels of welfare.

**Result 4:** Suppose \( 0 < w_1 < w_2 < \ldots < w_n \) and \( j = \min\{k \in \{1, 2, \ldots, n\} : \text{agent } k \text{ satisfies } (6)\} \). Then there exists \( j \) different welfare classes in the equilibrium, with \( u(y_1) < u(y_2) < \ldots < u(y_j) = u(y_{j+1}) = \ldots = u(y_n) \).

When we allow for non-contributors in an equilibrium, increasing the degree of initial
income inequality has a positive impact on public good provision. For any \( \gamma \), transferring
wealth from non-contributors to contributors increases public goods provision (the proof is
provided in Appendix A). Another important observation is that any transfer from a con-
tributor to other contributors that decreases the set of contributors also increases the public
good provision. Think of it as a 2-step procedure. The first step is to transfer wealth from
a contributor until that contributor becomes indifferent between contributing and not con-
tributing. This will not change the level of public good. The second step is to transfer more;
this is same as transferring from a non-contributor, and therefore the total amount of public
good will increase. Hence, in a boundary equilibrium, public good provision increases with ex-ante inequality (for a given $\gamma$). Transferring all wealth to only one individual maximizes the total voluntary giving. The following summarizes these arguments.\[10\]

**Result 5:** For a given $\gamma$, public good provision increases with ex-ante inequality when transferring wealth from non-contributors to contributors or from contributors to contributors such that the set of contributors decreases.

Result 5 shows that the main finding of Bergstrom et al. (1986) can be extended to any $\gamma^{11}$. Now we investigate the impact of (ex-post) redistribution. As the degree of redistribution increases, the cost of contributing one more unit to the public good decreases, which implies a higher contribution than before (substitution effect). On the other hand, as $\gamma$ increases the private consumption of a contributor decreases. The second effect implies a lower contribution (income effect). In general, the net effect is ambiguous as demonstrated in the example below.

**Example 1:** Suppose $u(x) = v(x) = -\frac{1}{x}$, and there are 2 agents with wealth levels $w_1$ and $w_2$, where $w_2 >> w_1$ so that only agent 2 contributes to the public good. If $\gamma = 0$, agent 2 provides $G = \frac{w_2}{2}$. However, as $\gamma \to 1$, agent 2 provides $G = 0.41(w_1 + w_2)$. Since $w_2 >> w_1$, public good provision in a non-egalitarian society will be higher than in a purely egalitarian society ($\gamma = 1$). For example, if $w_2 = 45$ and $w_1 = 5$, then $G = 22.5$ when $\gamma = 0$ and $G = 20.711$ when $\gamma = 1$. See Appendix B for graphical demonstration.

Hence, the total effect of $\gamma$ on public good provision is positive if the substitution effect dominates the income effect. The following theorem provides a sufficient condition for contributions to be increasing with taxation.

**Theorem:** Suppose $u(x)$ satisfies the following condition: $-\frac{u''(x)}{u'(x)} \leq 1$. Then $G(\gamma)$ is strictly increasing in $\gamma$.\[10\]

Increasing inequality between agents may also raise welfare in the boundary equilibrium (see Itaya et al. (1997)). If inequality is not too high, Pareto improvements are possible (Cornes and Sandler, 2000; Olszewski and Rosenthal, 2003).\[11\]

When $\gamma = 0$, Result 5 becomes identical to the main finding of Bergstrom et al. (1986).
Hence, if the relative risk aversion coefficient is less than or equal to 1, the substitution effect dominates the income effect: contributions are increasing with redistribution. This condition is satisfied by standard utility functions such as Cobb-Douglas utility functions. Moreover this result is independent of the initial income distribution. Therefore for any given wealth distribution there is a positive relationship between the degree of redistribution and the degree of efficiency in public goods provision.

Note that public good provision can be increased either by decreasing the ex-ante equality or by increasing the ex-post equality. The interaction of ex-ante and ex-post equality is discussed in Section 3.1. Result 6 shows that the best outcome is reached by increasing the degree of ex-post equality.

**Result 6:** In purely egalitarian societies \((\gamma = 1)\), the public good is provided efficiently and total welfare is maximized independent of the ex-ante income inequality.

Total welfare is highest when \(\gamma = 1\), and this result does not depend on the pre-tax income distribution. The intuition behind this argument is that when there is perfect equality ex-post, an agents’ utility is maximized only when total welfare is maximized. Clearly this result relies on the assumption that total income is exogenous and is not affected by redistribution. This assumption is relaxed in Section 3.2.

### 3 Extensions

In order to demonstrate the main results and argue to what extent these results can be generalized, I consider Cobb-Douglas preferences. Section 3.1 displays the interaction of ex-ante and ex-post inequality, and argues that societies with high tax rates may not have high public goods provision if high tax rates are associated with more equal income distributions. Section 3.2 extends the model to a two-stage game where effort is endogenously chosen in the first-stage in anticipation of taxation and voluntary contributions. In Section 3.3, I explore the trade-offs of using political processes to determine the level of redistribution. In particular, I ask whether majority voting brings efficient outcomes.
3.1 Ex-ante versus ex-post equality

I start by demonstrating the interplay of ex-ante and ex-post equality. Suppose agents have Cobb-Douglas preferences. Then, agents solve the following problem:

$$\max_{y_i, g_i} \log y_i + \log G$$

$$\text{s.t. } y_i + g_i + t(w_i, g_i, g_{-i}) = w_i$$

$$g_i \geq 0 \tag{7}$$

Each agent has to satisfy:

$$y_i \leq (1 - \frac{n - 1}{n} \gamma)G \tag{8}$$

Equation (8) holds with equality if $g_i > 0$. Therefore, the contribution of individual $i$ is:

$$g_i = \max\{0, w_i + \frac{\gamma W}{(1 - \gamma)n} - (1 + \frac{2\gamma}{(1 - \gamma)n})G\} \tag{9}$$

Note that $g_i$ is positively related to $w_i$ and, therefore, $w_i < w_j$ implies that $g_i \leq g_j$. Let $0 < w_1 \leq w_2 \leq \ldots \leq w_n$ and $j = \min\{k : g_k > 0, k = 1, 2, \ldots, n\}$. The aggregate wealth of all contributors and non-contributors are given by

$$W_{c}(\gamma) = \sum_{i=j}^{n} w_i \text{ and } W_{nc}(\gamma) = \sum_{i=1}^{j-1} w_i,$$

respectively. Since the aggregate wealth of contributors has to be allocated between the net private consumption, transfers to the society and the public good, we have

$$W_{c}(\gamma) = G + \sum_{i=j}^{n} y_i + \sum_{i=j}^{n} t_i,$$

\footnote{For $\gamma = 1$, there is no unique equilibrium, and therefore individual contributions cannot be determined from this formula.}

\footnote{This set is non-empty since the richest individual definitely contributes to the public good.}
where \( t_i = t(w_i, g_i, g_{-i}) \). Therefore, the public good provision is given by:

\[
G(\gamma) = \frac{(1 - \gamma)Wc(\gamma) + \frac{2}{n}(n - j + 1)W}{(1 - \gamma) + (n - j + 1)(1 - \gamma + \frac{2}{n}\gamma)}
\]  

(10)

From (10), it is easy to see that the public good provision depends on the total wealth and the number of contributors. Total provision does not change with transfers from contributors to contributors that do not change the set of contributors. However, transfers from non-contributors to contributors or from contributors (that become non-contributors due to reduced wealth) to contributors necessarily increase public goods provision. The highest amount of public good provision will be reached when all wealth is transferred to one individual. In that case, (10) implies that \( G = \frac{W}{2} \).

If everybody contributes in the equilibrium, then (10) simplifies to:

\[
G(\gamma) = \frac{W}{(n + 1) - (n - 1)\gamma}
\]  

(11)

Clearly public good provision is increasing in the degree of redistribution. It can be seen from equation (10) that \( G(1) = \frac{W}{2} \). Since, at \( \gamma = 1 \), private consumption of each individual is equal to \( \frac{W}{2n} \), first best outcome is reached.

In general, the net effect of (ex-ante) inequality and (ex-post) redistribution on the total contributions to the public good depends on the magnitudes of these two effects. In the two extreme cases, however, the net effect is not ambiguous. If all wealth is accumulated in one agent, then \( \frac{W}{2} \) will be achieved independent of \( \gamma \), and the same amount of provision will be achieved if the society is purely egalitarian (independent of the ex-ante income distribution). Total welfare in the society, however, is highest in the second case. Example 2 demonstrates the interaction of pre-tax income inequality and taxation.

**Example 2:** Consider three agents with wealth levels: \( w_1 = 5 \), \( w_2 = 5 \) and \( w_3 = 50 \).

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\(^{14}\)This solution technique is adopted from Olszewski and Rosenthal (2004).

\(^{15}\)The only exception is that when all wealth is accumulated in one person, the contribution level is independent of \( \gamma \).

\(^{16}\)The first best provision level, \( G^* \), maximizes \( \log \frac{(W-G)}{n} + \log G \). And, therefore, total welfare is maximized when \( G = \frac{W}{2} \) and private consumption of each individual is equal to \( \frac{W}{2n} \).
In equilibrium, only agent 3 contributes to the public good. In order to see this, suppose all agents contribute to the public good. Then (11) implies that \( G(\gamma) = \frac{60}{4-2\gamma} \). First order conditions hold for all agents and, therefore,

\[
w_1 - g_1 = (1 - \frac{2\gamma}{3}) \frac{60}{4-2\gamma}.
\]

Since \((1 - \frac{2\gamma}{3}) \frac{60}{4-2\gamma} \geq 10\) for any \(0 \leq \gamma \leq 1\), \(w_1 - g_1 \geq 10\). This contradicts \(g_1 > 0\). Therefore, only agent 3 contributes with

\[
G(\gamma) = g_3 = \frac{(1 - \frac{2\gamma}{3})w_3 + \frac{\gamma}{3}(w_1 + w_2)}{2(1 - \frac{2\gamma}{3})}
\]

which, for any \(\gamma > 0\), is clearly greater than \(\frac{w_3}{2}\), the public good provision without social obligations. Indeed, public good provision increases with the degree of redistribution (\(\gamma\)).

In contrast, the relationship between initial income equality and public good provision is negative. Now, consider an extreme case, where all individuals have equal incomes: \(w_1 = w_2 = w_3 = 20\). Then, everybody contributes to the public good, and therefore \(G(\gamma) = \frac{60}{4-2\gamma}\).

Note that

\[
\frac{60}{4-2\gamma} < \frac{(1 - \frac{2\gamma}{3})w_3 + \frac{\gamma}{3}(w_1 + w_2)}{2(1 - \frac{2\gamma}{3})}
\]

for any \(\gamma < 1\). Therefore, public good contributions are lower when individuals have equal incomes. This shows that initial income equality decreases the level of public good provision, except when \(\gamma = 1\) (purely egalitarian case).

Example 2 implies that public good provision may not be higher in societies with high tax rates if they also have a more equal income distribution. However, since \(\gamma = 1\) gives the first best outcome, it is clear that for any level of ex-ante inequality and taxation in a society, there exists a society with higher levels of taxation and initial income equality that has at least as high contributions and welfare as the former society.

The current discussion shows that if agents interact repeatedly, then one has to consider whether redistribution and pre-tax inequality are correlated or not. If a higher level of redistribution creates a more equal income distribution next period, then one has to consider the net effect of a lower level of ex-ante income inequality and a higher degree of redistribution on public goods provision.
The next section provides another natural extension of the model. I consider a two-stage game where effort is endogenously chosen in the first stage in anticipation of taxation and voluntary contributions.\textsuperscript{18}

### 3.2 Endogenous effort choice

Consider the following two-stage game. In the first stage, agents decide on their effort levels. Higher effort generates a higher income but is more costly. In the second stage, agents decide on their contributions to the public good given that there is redistributive taxation. If the redistribution takes into account the cost of effort, then the results of Section 2 hold here as well. However, this requires that government observes the cost of effort for each agent. Therefore, we assume that agents are not compensated for effort and redistribution depends only on incomes net of contributions.\textsuperscript{19}

More formally, each agent $i$ picks an effort level $e_i$ in the first stage. Incomes are determined according to an income function $f(e) = \sqrt{e}$. Each agent $i$ pays a cost given by $c_i(e) = k_i e^2$, where $k_i > 0$ determines how costly the effort is. As in Section 2, transfers depend on the income net of contributions: $t(w_i, g_i, g_{-i}) = \gamma [(w_i - g_i) - \frac{1}{n} \sum_{j=1}^{n} (w_j - g_j)]$ where $w_i = f(e_i)$. The optimization problem is as follows:

$$\max_{e_i, g_i} y_i G$$

subject to

$$y_i + g_i + t(w_i, g_i, g_{-i}) = w_i - c_i(e_i)$$

$$g_i \geq 0, \quad e_i \geq 0$$

\textsuperscript{18}In contrast with the traditional view that argues redistribution is detrimental to investment and growth, recent literature provides evidence on the fact that redistribution may also have a positive impact on growth (i.e., Saint Paul and Verdier (1993, 1996), Easterly and Rebelo (1993a, 1993b), Galor and Zeira (1993), Sala-i Martin (1996), Perotti (1996). If redistribution increases the total income, then the results of Section 2 get even stronger.

\textsuperscript{19}This section can be easily extended to the case where agents are only partially compensated for their effort.
The net consumption of agent $i$ is given as

$$y_i = (1 - \gamma)(w_i - g_i) + \gamma \frac{1}{n} \sum_{j=1}^{n} (w_j - g_j) - c_i(e_i).$$

Effort choices are given by

$$e^*_i = \left(\frac{1}{4k_i}(1 - \frac{n-1}{n}\gamma)\right)^{2/3}.$$

Equation (13) is very intuitive. There is a negative relationship between the cost ($k_i$) and effort level. In addition, effort decreases with the number of people ($n$) since the marginal value of effort is lower with a higher number of people due to redistribution. More importantly, for all $i$, $e^*_i$ decreases with $\gamma$.

Suppose everyone contributes in the equilibrium. Then, each agent satisfies the following first order condition:

$$(1 - \frac{n-1}{n}\gamma)G = (1 - \gamma)(\sqrt{e_i} - g_i) + \gamma \frac{\sum_{j=1}^{n} (\sqrt{e_j} - g_j)}{n} - k_i e_i^2.$$

Summing over $n$ people, one can solve for $G$:

$$G^* = \frac{\sum_{j=1}^{n} (\sqrt{e_j^*} - k_j e_j^2)}{n + 1 - (n - 1)\gamma}.$$

The numerator is simply the sum of income net of cost paid for effort. Equation (14) is equivalent to (11), i.e., $W = \sum_{j=1}^{n} (\sqrt{e_j} - k_j e_j^2)$. However, there is a major difference now. $\gamma$ affects both the numerator and the denominator of (14): both of them decrease with $\gamma$. Result 7 shows that the net effect of $\gamma$ on public goods provision is positive.

**Result 7:** In the interior equilibrium, public good provision increases with $\gamma$.

Similarly, $\gamma$ has two opposing effects on total welfare. Although the free riding problem decreases with $\gamma$, total income also decreases with $\gamma$. Next I analyze the $\gamma$ that maximizes
welfare. In the interior equilibrium, each agent consumes

\[ y = \frac{\sum_{j=1}^{n}(\sqrt{e_j} - k_j e_j^2) - G}{n}. \]

Therefore, total welfare (TW) is equal to

\[ TW = (\sum_{j=1}^{n}(\sqrt{e_j} - k_j e_j^2) - G)G. \]

Taking the derivative of welfare with respect to \( \gamma \) and equating it to zero finally gives:

\[ 3(1 - \gamma)(3n + (n - 1)\gamma) - 8(n + 1 - (n - 1)\gamma)\gamma = 0 \]

This implies that in the interior equilibrium, welfare maximizing \( \gamma \) depends on the number of people, \( n \). Result 8 shows that when everyone is a contributor at \( \gamma = 0 \), optimal \( \gamma \) strictly increases with \( n \).

**Result 8:** In the interior equilibrium, \( \gamma'(n) > 0 \), and therefore welfare is maximized at \( \gamma = 1 \) in the limit (as \( n \to \infty \)).

When the number of people in a society increases, the inefficiency of effort selection created by redistribution is compensated by the decrease in free riding. In the limit, perfect equality maximizes welfare.

Figure 1 shows the relationship between the welfare maximizing \( \gamma \) and the number of people. When \( n = 2 \), \( \gamma \) is computed as 0.493; for \( n = 3 \), \( \gamma = 0.571 \), for \( n = 100 \), \( \gamma = 0.96 \).

This implies that in general quite high levels of redistributive taxation is optimal.

While a high level of redistribution is optimal when everyone is a contributor in the equilibrium, very low levels of redistribution may become optimal when there are non-contributors. Indeed, in the boundary equilibrium, it is easy to find examples where welfare strictly decreases with \( \gamma \) (see example 3), i.e., optimal \( \gamma \) becomes 0.

\(^{20}\)When effort is endogenous, there is redistribution even in the interior equilibrium. Redistribution guarantees that \( y_i = y_j \) for all \( i \) and \( j \).
Example 3: There are three agents with $c_1(e) = 0.2e^2$, and $c_2(e) = c_3(e) = 10e^2$. So effort is very costly for agents 2 and 3. When there is no redistribution, only agent 1 contributes to the public good with $G(0) = 0.4$, and $\sum f(e) = 1.66$. Each agent’s welfare is 0.163, 0.089, and 0.089, respectively. The total welfare is approximately 0.34. Figure 2 shows that total welfare strictly decreases with $\gamma$. Therefore, the socially optimal level of redistribution is given by $\gamma = 0$. 

In Example 3, the reason that any positive level of redistribution lowers the welfare of the
society is because now agents are very differentiated in terms of their costs. This implies that agent 1 is much wealthier than the other two agents ex-ante, and therefore redistribution decreases the private consumption of agent 1 substantially. In turn, agent 1 is not willing to contribute much to the public good.\footnote{In fact, public goods provision may decrease with $\gamma$ if $c_2$ and $c_3$ are significantly larger.}

When we allow for non-contributors, the positive relationship between $n$ and welfare maximizing $\gamma$ also disappears. Consider the following example.

**Example 4:** Suppose now $n = 100$, $c_1(e) = 0.02e^2$, and $c_i(e) = 1000e^2$ for all $i \neq 1$. As in example 3, when there is no redistribution, only agent 1 contributes to the public good. It can be shown that total welfare strictly decreases with $\gamma$, and therefore, welfare maximizing level of redistribution is still given by $\gamma = 0$.

A natural question that arises is whether the optimal degree of redistribution can be selected through voting. Next section discusses when political processes, in particular majority voting, lead to efficiency.

### 3.3 Optimal redistribution and voting

In this section, we study when political processes impose a trade-off between redistribution and efficiency. We have seen that redistribution decreases the total effort in the society. This creates the possibility that voting may enforce non-optimal outcomes. The following example shows that when the majority of agents have relatively high costs of effort, majority voting may imply higher redistribution than the optimal.

**Example 5:** As in Example 3, there are three agents with $c_1(e) = 0.2e^2$, and $c_2(e) = c_3(e) = 10e^2$. Therefore, when $\gamma = 0$, the total welfare is approximately 0.34. However, if there is a majority voting over the level of redistribution, $\gamma$ will be chosen as 0.58, with $G(\gamma) = 0.42$ and $\sum f(e) = 1.41$.\footnote{This decreases agent 1’s welfare to 0.109, and increases the other agents’ welfare to 0.109.} The total welfare decreases to 0.326.
Example 5 shows that although redistribution increases the welfare of the individuals with high cost, it decreases the welfare of the individual with low cost more than proportionately. While the example may suggest that voting decreases the welfare of the society, this is not always true. When everyone contributes in the equilibrium, their incomes move in the same direction: if redistribution increases the welfare of one individual, it also increases the welfare of others. Therefore, agents’ preferred level of redistribution corresponds to the level of redistribution that maximizes the total welfare.

Result 9: When everyone contributes in the equilibrium, the optimal level of redistribution will be the outcome in a majority vote.

Result 9 shows that when agents are not very heterogeneous, social welfare is maximized by voting. However, as Example 5 demonstrates, if the level of heterogeneity is high, then voting on the level of redistribution may decrease the welfare of the society.

4 Public Goods Provision and Informal Sharing

The results presented in this paper can also shed light on how “norms of redistribution” may affect voluntary public goods provision in developing countries. In developing countries informal sharing rules often press rich individuals to support their poorer relatives. Private wealth accumulation is recognized as an attempt to break away from the traditional solidarity networks. Redistributive norms prevent excessive wealth accumulation and preserve the mutual insurance mechanism (Platteau, 1991; Fafchamps, 1992). In addition, in societies with such norms success is attributed mostly to luck. Therefore, persistent success is believed to be unfair, and rich individuals are forced to share their income with others. Feelings of envy and jealousy could be involved in bringing about redistribution of income between presumably lucky and unlucky individuals (Rogers, 1969; Drijver and Van Zorge, 1995). Redistribution of income is ensured by powerful sanctions in the form of social pressure, violence (physical harm), economic losses such as loss of employment or destruction of

\[ ^{23}\text{Evidence presented in this section relies mostly on Platteau (2000).} \]
property, social ostracism, witchcraft practices or accusations of witchcraft.

The following two citations are taken from Platteau (2000):

\[
\text{Demand to provide jobs for a wide range of kin, irrespective of their qualifications, and requests for cash donations or gifts of varying amounts from a stream of ‘visitors’ are probably the most frequent claims. [Kennedy, 1988:169]}
\]

\[
\text{Where the extended family exists, any member of the family whose income increases may be besieged by correspondingly increased demands for support from a large number of distant relations. [Lewis, 1955:114]}
\]

Laboratory and field experiments also provide evidence on redistributive norms and social sanctions. Bernhard et al. (2006), Fehr and Fischbacher (2004) and Falk et al. (2005), among others, show that agents engage in costly punishments against unfair income distributions. Henrich et al. (2001, 2005) conduct experiments in 15 small-scale societies and find large cultural differences in sharing norms and social sanctions.

While there are lots of examples of sharing norms, there is extremely limited evidence on whether such norms affect public goods provision. Endogeneity and measurement problems make it hard to distinguish the impact of these norms on voluntary contributions. This paper increases our understanding of the relationship between norms of redistribution and public goods provision. We see that social norms that impose redistribution may help to overcome free-riding problems. Therefore, this may have important consequences for societies to preserve and enhance their social norms and values.\(^{24}\)

However, this interpretation has to be taken with caution. As explained in Section 3.1, there may be an interaction between ex-ante and ex-post equality. In a society with very unequal earnings (which may be because luck is an important determinant of wealth), egalitarian norms may develop to be very strong, i.e., as in hunter-gatherer societies.\(^{25}\) So, ex-ante inequality and the level of redistribution may be positively correlated. However,\(^{24}\)

\(^{24}\)Note that this result holds for societies where contributions are observable. If contributions are not taken into account, then egalitarianism could imply opposite results.

\(^{25}\)A society may have a very unequal wealth distribution but sharing may still exist. Another example is provided by Lenski (1966) that African kings, chiefs and other high officials are expected to be generous.
it may also be the case that in societies with strong egalitarian norms where luck is not a main determinant of income, agents have equal opportunities, and therefore, initial income inequalities are lower. This implies a negative correlation.

Societies with higher norms of redistribution may therefore have higher or lower ex-ante inequality compared to societies with lower norms of redistribution. Public good provision is higher in a more egalitarian society if initial inequality is also higher. Otherwise, the two effects go in opposite direction and the net effect of egalitarianism and initial inequality on total provision is ambiguous.

The effects of norms of redistribution on public good provision when total income depends endogenously on the sharing rule is also ambiguous. Lewis (1955) presents the possibility that rich individuals may decrease their effort levels and, therefore, productivities due to the threat of social obligations to share: “There are many reports from Asia and Africa of able men who have refused promotion because the material benefit would accrue mostly to relatives whose moral claims they do not recognize.” In that case, Section 3.2 implies that redistribution may not improve welfare. On the other hand, norms of redistribution may increase the social welfare if redistribution increases the total productivity of the individuals in the community (Bardhan and Udry, 1999; Dasgupta and Ray (1986)). Higher redistribution would increase the nutritional intake of poor individuals, and hence it would increase work efficiency. Higher degrees of egalitarianism could also diminish credit market imperfections, reduce crime and decrease political instability. Poverty traps can be eliminated if, for example, the poor have enough resources to educate their children, to start their own businesses or to undertake productive investments that enlarge their scales of production.

5 Conclusion

This paper studies the relationship between redistributive taxation and public goods provision. In particular, I examine the impact of the tax rate on charitable giving. If everybody is a contributor in the equilibrium, voluntary giving is monotonically increasing in the degree of redistribution because the price of charitable giving is now lower. When there are non-
contributors in the equilibrium, redistribution has two opposite effects on voluntary giving: substitution and income effects. However, I show that, for a large class of utility functions, the substitution effect always dominates the income effect. Hence, voluntary contributions are increasing in the degree of redistribution. While this is the broad finding of the paper, it is important to note that there are subtle qualifications. First, higher redistribution may be associated with higher pre-tax income equality. Since pre-tax income equality has a negative effect on contributions, the net effect of a lower level of pre-tax income inequality and a higher degree of redistribution is ambiguous. Surprisingly, purely egalitarian societies, which fully iron out all ex-post differences in wealth, have efficient levels of public good provision and the highest total welfare, independent of the initial income distribution. Second, when effort is endogeneously determined, welfare may strictly decrease with the tax rate. I show that when the agents have Cobb-Douglas preferences, in the interior equilibrium, the result that welfare is maximized with $\gamma = 1$ can be generalized to the endogenous effort case as the number of people increases.

A Proofs

Proof of Result 1. It is enough to see that the equilibrium level of provision, $G$, is unique. Suppose that there are two equilibria $G$ and $G'$. Without loss of generality let $G' > G$. Therefore, there exists an individual $i$ such that $g'_i > g_i$,

$$u'(w_i - g_i' - t(w_i, g_i', g_{-i}'))[1 - \frac{n - 1}{n}\frac{1}{\gamma}] = v'(G'),$$

and

$$u'(w_i - g_i - t(w_i, g_i, g_{-i}))[1 - \frac{n - 1}{n}\frac{1}{\gamma}] \geq v'(G).$$

This implies:

$$w_i - g_i - t(w_i, g_i, g_{-i}) < w_i - g_i' - t(w_i, g_i', g_{-i}'),$$
or, equivalently

$$(1 - \gamma)(w_i - g_i) + \gamma\left(\frac{W - G}{n}\right) < (1 - \gamma)(w_i' - g_i') + \gamma\left(\frac{W - G'}{n}\right).$$

However, this contradicts $G' > G$. \hfill \Box

**Proof of Result 5.** It is enough to prove the case of transferring income from non-contributors to contributors. To see that this increases public goods provision, let’s consider a transfer from a non-contributor to a contributor $i$. Suppose after the transfer the total amount of public good (weakly) decreases: $G' \leq G$.

Claim: Contributor $i$ increases his contribution after the transfer ($g_i' > g_i$).

Proof: Suppose $0 \leq g_i' \leq g_i$. Then the following two equations have to hold simultaneously:

$$u'(1 - \gamma)(w_i - g_i) + \frac{W - G}{n}[1 - \frac{n - 1}{n} \gamma] = v'(G)$$

and

$$u'(1 - \gamma)(w_i' - g_i) + \frac{W - G'}{n}[1 - \frac{n - 1}{n} \gamma] \geq v'(G').$$

where $w_i' > w_i$. Since $v'(G') \geq v'(G)$,

$$[(1 - \gamma)(w_i' - g_i') + \frac{W - G'}{n}] \leq [(1 - \gamma)(w_i - g_i) + \frac{W - G}{n}].$$

However, this contradicts $w_i' > w_i$, $g_i' \leq g_i$ and $G' \leq G$. Q.E.D.

Since agent $i$ increases his contributions after the transfer, there exists at least one agent $j$ who decreases his contributions after the transfer: $0 \leq g_j' < g_j$. The following two equations have to hold simultaneously:

$$u'(1 - \gamma)(w_j - g_j) + \frac{W - G}{n}[1 - \frac{n - 1}{n} \gamma] = v'(G)$$

and

$$u'(1 - \gamma)(w_j - g_j') + \frac{W - G'}{n}[1 - \frac{n - 1}{n} \gamma] \geq v'(G').$$
Similarly, this contradicts \( g'_i < g_i \) and \( G'_i \leq G \). Therefore, \( G' > G \). □

**Proof of Theorem.** When everybody is a contributor, public good provision increases with \( \gamma \) (see Result 2). Now suppose there are non-contributors at \( \gamma \). Let \( I(\gamma) \) be the set of agents that are indifferent between giving and not giving to the public good at \( \gamma \). Suppose \( w_1 \leq w_2 \leq ... \leq w_n \). I would like to show that charitable giving increases with \( \gamma \).

case i) Let \( I(\gamma) = \emptyset \). Then, the set of contributors do not change in a neighborhood of \( \gamma \) and contributors give strictly positive amounts to the public good. Continuity of contributions guarantee that this neighborhood exists. Let \( k \) be the number of contributors. Contributors must satisfy the first order condition with equality; so there are \( k \) equations with \( k \) unknowns:

\[
u'((1 - \gamma)(w_i - g_i) + \frac{W - G}{n})(1 - \frac{n - 1}{n} \gamma) - v'(G) = 0 \tag{15}\]

with \( i \in \{n - k + 1, ..., n\} \), since only the richest \( k \) people contribute.

I would like to see how a change in \( \gamma \) changes the contributions. So I apply the Implicit Function Theorem.

\[
\begin{bmatrix}
-\alpha_{n-k+1} A - v''(G) & -\alpha_{n-k+1} \frac{2}{n} - v''(G) & \cdots & -\alpha_{n-k+1} \frac{2}{n} - v''(G) \\
-\alpha_{n-k+2} \frac{2}{n} - v''(G) & -\alpha_{n-k+2} A - v''(G) & \cdots & -\alpha_{n-k+2} \frac{2}{n} - v''(G) \\
\vdots & \vdots & \ddots & \vdots \\
-\alpha_{n} \frac{2}{n} - v''(G) & -\alpha_{n} \frac{2}{n} - v''(G) & \cdots & -\alpha_{n} A - v''(G)
\end{bmatrix}
\begin{bmatrix}
d\frac{g_{n-k+1}}{d\gamma} \\
d\frac{g_{n-k+2}}{d\gamma} \\
\vdots \\
d\frac{g_{n}}{d\gamma}
\end{bmatrix}
= -
\begin{bmatrix}
-\alpha_{n-k+1} \lambda_{n-k+1} - \frac{n-1}{n} u' \alpha_{n-k+1} \\
-\alpha_{n-k+2} \lambda_{n-k+2} - \frac{n-1}{n} u' \alpha_{n-k+2} \\
\vdots \\
-\alpha_{n} \lambda_{n} - \frac{n-1}{n} u' \alpha_{n}
\end{bmatrix}
\]

\(^{26}\)One can show that uniqueness of the equilibrium and the Maximum Theorem grantee the continuity of \( g(\gamma) \). Proof is available from the author upon request.

24
where \( A = (1 - \frac{n-1}{n} \gamma) \), \( \alpha_i = u''((1 - \gamma)(w_i - g_i) + \gamma \frac{W-G}{n})A \), \( \beta_i = (1 - \gamma)(w_i - g_i) + \gamma \frac{W-G}{n} \) and \( \lambda_i = A((w_i - g_i) - \frac{W-G}{n}) \). It is easy to see that \( \alpha_i = \alpha_n = \alpha \), \( \lambda_i = \lambda_n = \lambda \), \( \beta_i = \beta_n = \beta \) for all contributors. Since the Jacobian is not singular, the implicit function theorem implies that \( g(\gamma) \) is a differentiable function. Note that \( \frac{dg_i}{d\gamma} = \frac{dg_n}{d\gamma} \) for all contributors. Therefore, the above equation implies the following:

\[
[\alpha A + v''(G) + (k - 1)(\alpha \frac{\gamma}{n} + v''(G))\frac{dg_n}{d\gamma} = -u''(\beta)\lambda - \frac{n-1}{n}u'() \quad (16)
\]

Since \( v'' < 0 \) and \( \alpha < 0 \), \( [\alpha A + v''(G) + (k - 1)(\alpha \frac{\gamma}{n} + v''(G))] \) is not zero, and I can divide both sides by the same argument. Since the denominator will always be negative, the numerator will determine the sign of the change.

\[
-u''(\beta)\lambda - \left(\frac{n-1}{n}\right)u'(\beta) \quad (17)
\]

It is not hard to see that \( \lambda < \beta(\frac{n-1}{n}) \). Therefore, since \( -u''(\beta) \) is nonnegative, we have

\[
-u''(\beta)\lambda - \left(\frac{n-1}{n}\right)u'(\beta) < -u''(\beta)\beta\frac{n-1}{n} - \left(\frac{n-1}{n}\right)u'(\beta) \quad (18)
\]

\[
< \left(\frac{n-1}{n}\right)(-u''(\beta)\beta - u'(\beta)). \quad (19)
\]

But we know that \(-\frac{u''(\beta)x}{u'(x)} \leq 1 \) for all \( x \). Hence \( -u''(\beta)\lambda - \left(\frac{n-1}{n}\right)u'(\beta) < -u''(\beta)\beta - u'(\beta) \leq 0 \). This implies that \( \frac{dg_n}{d\gamma} > 0 \). Hence, \( G(\gamma) \) is strictly increasing in \( \gamma \) as long as the set of contributors stays the same. If \( I(\gamma) = \emptyset \) for all \( \gamma \), then we know that each contributor increases their contribution.

---

27Nonsingularity of the Jacobian is not hard to see since the Jacobian becomes a symmetric matrix.

28The last claim can be seen in two different ways. First, since \( w_i - g_i = w_j - g_j \) for all contributors, for small changes in \( \gamma \), contributions will change in the same way. Another way to see this is direct inspection of the equations above. Since there will be \( n \) symmetric equations, the only way for all of them to hold is for all the derivatives to be equal.

29I want to show: \((w_i - g_i) - \frac{W-G}{n}\)(1 - \(\frac{n-1}{n} \gamma)) < ((1 - \gamma)(w_i - g_i) + \gamma \frac{W-G}{n})(\frac{n-1}{n})\)

Let \( a = (w_i - g_i) \) and \( b = \frac{W-G}{n} \) and \( c = n-1 \). So I want the following to hold:

\((a - b)(1 - c\gamma) < c((1 - \gamma)a + \gamma b)\). This simplifies to \((1 - c)a < b, \) or \( \frac{(w_i - g_i)}{n} < \frac{W-G}{n} \), which is obviously true since \( w_i - g_i < W - G \).
case ii) Let \( I(\gamma') \neq \emptyset \) for some \( \gamma' \). Define \( A(\gamma) \) as the set of agents with strictly positive contributions and \( B(\gamma) \) as the set of agents who strictly prefer not to contribute at given \( \gamma \). The continuity of \( g_i \)'s implies that there exists a neighborhood, \( O(\gamma') \), of \( \gamma' \) such that \( A(\gamma') \subset A(\gamma) \) and \( B(\gamma') \subset B(\gamma) \) for all \( \gamma \in O(\gamma') \).

Note that all agents in \( I(\gamma') \) have the same income level, say \( w^* \), hence, they behave in the same way. Note that \( w^* = w_k - g_k(\gamma') \) where \( k \in A(\gamma') \). Assume that \( A(\gamma) = A(\gamma') \) for all \( \gamma' \leq \gamma \in O(\gamma') \) then we already know that \( g_i(\gamma) \) is strictly increasing in \( \gamma \geq \gamma' \) for all \( i \in A(\gamma') \). Therefore net income of individuals in the set \( A(\gamma') \) decreases as \( \gamma \) increases, i.e., \( w_i - g_i(\gamma) < w_i - g_i(\gamma') = w^* \). This implies agents with income level \( w^* \) should contribute as well. This contradicts \( A(\gamma) = A(\gamma') \).

Therefore \( A(\gamma) = A(\gamma') \cup I(\gamma') \), for all \( \gamma' \leq \gamma \in O(\gamma') \). Therefore \( w_i - g_i(\gamma) = w^* - g_k(\gamma) < w_i - g_i(\gamma') = w^* \) for all \( i \in A(\gamma') \) and for all \( k \in I(\gamma') \). Hence, \( G(\gamma) \) strictly increases.

**Proof of Result 7.** To show that \( G \) increases with \( \gamma \), take the derivative of equation (14) with respect to \( \gamma \):

\[
\frac{\partial G}{\partial \gamma} = \frac{\partial w}{\partial \gamma} \left[ (n + 1) - (n - 1)\gamma \right] + (n - 1)W n \left[ (n + 1) - (n - 1)\gamma \right].
\]

Since \( [ (n + 1) - (n - 1)\gamma ] > 0 \), we need to check the sign of the numerator.

First denote \( \lambda = n - (n - 1)\gamma \). It is routine to check that the sign of the numerator is equal to the sign of the following:

\[
A = \left( \frac{2}{3} - \frac{1}{3\lambda} \right) \left( \frac{\lambda}{4n} \right)^{1/3} + \left( \frac{1}{3} + \frac{4}{3\lambda} \right) \left( \frac{\lambda}{4n} \right)^{4/3}
\]

\( A \) is positive since \( \lambda \geq 1 \).

**Proof of Result 8.** This can be proven by using the Implicit Function Theorem:

\[
\gamma'(n) = -\frac{(\gamma - 1)(5\gamma - 9)}{10(n - 1)\gamma - 11 - 14n}
\]

It is easy to see that \( \gamma'(n) > 0 \), since \( 0 < \gamma < 1 \).
B Figures

For the following figures, assume $w_1 = 5$ and $w_2 = 45$.

Case i) $u(.) = v(.) = 2\sqrt{x}$

Figure 3: Total provision (strictly increasing)

Case ii) $u(.) = v(.) = -\frac{1}{x}$

Figure 4: Total provision (strictly decreasing)
Case iii) \( u(.) = v(.) = -\frac{x^{0.4}}{0.4} \)

Figure 5: Total provision (non-monotonic)

References


