MATH 567 – WINTER 2019 – HOMEWORK 9

Practice problems from textbook (do not submit):

• 1–7, 11–13 (p.244 textbook).

Due on Thursday April 23rd: Work on the following problems. Each problem is worth 10 points.

I. Let $\mathcal{R}(m)$ be the Reed–Muller code described in Chapter 6.2. For $m = 3$, decode:
   a) $x = 01111100$.
   b) $x = 01101001$.
   Show your steps.

II. Let $V = \mathbb{Z}_p^n$ viewed as an $n$-dimensional vector space. Denote by $a_p(n, k)$ the number of $k$-dimensional subspaces of $V$. Show that:
   a) $a_p(n, 1) = \frac{p^n - 1}{p - 1}$.
   b) $a_p(n, 2) = \frac{(p^n - 1)(p^n - p)}{(p^2 - 1)(p^2 - p)}$.

III. Same assumption as question II.
   a) Show that the total number of all bases for $\mathbb{Z}_p^n$ is
   $$(p^n - 1)(p^n - p) \ldots (p^n - p^{n-1}).$$
   Here different orderings of a basis are also counted. For example if $\{v_1, v_2, v_3\}$ is a basis then so is $\{v_2, v_1, v_3\}$.
   b) Show that for general $0 \leq k \leq n$:
   $$a_p(n, k) = \frac{(p^n - 1)(p^{n-1} - 1) \ldots (p^{n-k+1} - 1)}{(p^k - 1)(p^{k-1} - 1) \ldots (p - 1)}.$$