Practice problems from textbook (do not submit):
• 1,3,5,6,7,8,9 (p.41)
• 1–15 (p.49, constructions can be done with Huffman codes.)

Due on Tuesday January 29th: Work on the following problems. Each problem is worth 10 points.

I. Decide if each of the following codes is uniquely decipherable and explain why:
   a) \( C = \{0, 10, 110, 1110, 11110, 11111\} \).
   b) \( D = \{0, 10, 110, 1110, 1111, 1101\} \).

II. Prove that if \( C \) is not uniquely decipherable then there are some \( c_1, \ldots, c_n \in C \) and \( c'_1, \ldots, c'_n \in C \) with \( c_1 \ldots c_n = c'_1 \ldots c'_n \) and \( c_i \neq c'_i \) for at least one \( i \).

III. Let \( A \) be a alphabet with at least 2 characters. Let \( C \) be a maximal instantaneous code over \( A \) with codeword lengths \( \ell_1, \ldots, \ell_q \). If \( L = \max\{\ell_i\} \), show that \( C \) must contain at least two codewords of length \( L \).

IV. Let \( C \) be an instantaneous code over an alphabet \( A \). Prove that the following are equivalent:
   a) \( C \) is maximal instantaneous.
   b) Every string \( x_1 \ldots x_n \in A^* \) is a prefix of some string \( c_1 \ldots c_m \) with \( c_1, \ldots, c_m \in C \).

V. (Bonus) Design and prove an algorithm to check if a given code \( C \) is uniquely decipherable. How long is the running time? (Hint: one possible method is to reduce the problem to \textsc{Graph-Reachability}, see \url{https://en.wikipedia.org/wiki/Reachability}.)