Online Appendix for: Opting out of Incentive Mechanisms: A Study of Security as a Non-Excludable Public Good

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APPENDIX A

THE PIVOTAL MECHANISM: SOCIAL OPTIMALITY AND VOLUNTARY PARTICIPATION

We present two propositions to illustrate the main properties of the Pivotal mechanism, namely social optimality and voluntary participation. The proofs follows directly from the classical literature on VCG mechanisms and are included for completeness.

Proposition 1: In the Pivotal mechanism with taxes given by (6), reporting the true type, i.e., the true utility function \( u_i(\cdot) \), is a dominant strategy for all users \( i \). Therefore, the socially optimal solution is implemented.

Proof: The total utility of user \( i \) when reporting \( \hat{u}_i(\cdot) \), while others report \( \hat{u}_j(\cdot) \), \( j \neq i \), is given by:

\[
v_i(\hat{x}, t_i) = u_i(\hat{x}) + \sum_{j \neq i} \hat{u}_j(\hat{x}) - \sum_{j \neq i} \hat{u}_j(\hat{x}) ,
\]

where \( \hat{x} = \arg \max_{x \geq 0} \sum_{k=1}^{N} \hat{u}_k(x) \) is the allocation that is optimal given the reported types \( \hat{u}_k(\cdot), \forall k \). We first note that the last term is independent of user \( i \)'s report. Then, as the allocation \( \hat{x} \) is chosen according to the optimization problem \( \arg \max_{x \geq 0} \sum_{k=1}^{N} \hat{u}_k(x) \) over the reported types, the sum of the first and second terms is maximized at \( \hat{u}_i(\cdot) = u_i(\cdot) \). Therefore, users will reveal their true preferences, irrespective of other users’ reports. Consequently, the socially optimal investment profile will be prescribed by the mechanism designer.

Proposition 2: The Pivotal mechanism with taxes given by (6) satisfies voluntary participation.

Proof: The change in the utility of a user \( i \) when staying in vs. opting out of the mechanism is given by:

\[
v_i(x^*, t_i) - u_i(\hat{x}) = u_i(x^*) + \sum_{j \neq i} u_j(x^*) - \sum_{j \neq i} u_j(\hat{x}) - u_i(\hat{x}) = \sum_{j} u_j(x^*) - \sum_{j} u_j(\hat{x}) \geq 0 .
\]

The inequality is due to the fact that \( x^* \) is the socially optimal solution given by the maximizer of the sum of all users’ utilities. We conclude that it is in the best interest of users to participate in the Pivotal mechanism with the given taxes.

APPENDIX B

THE EXTERNAILITY MECHANISM: SOCIAL OPTIMALITY AND BUDGET BALANCE

We present two propositions to illustrate the main properties of the Externality mechanism, namely social optimality and budget balance.

Proposition 3: In the Externality mechanism with taxes given by (7), investing the socially optimal security effort \( x^* \) is a Nash equilibrium.

Proof: First, note that the taxes assigned to users at a vector of investments \( x \) (possibly off equilibrium) is given by:

\[
t^E_i(x) = - \sum_{j=1}^{N} x_j L_i \frac{\partial f_i}{\partial x_j}(x^*) - x_i \frac{\partial h_i}{\partial x_i}(x^*) .
\]

Now, assume all users other than \( i \) are investing at the socially optimal level \( x^*_i \). Then, user \( i \)'s total utility is:

\[
v_i(x_i, x^*_{-i}, t_i) = W_i - L_i f_i(x_i, x^*_{-i}) - h_i(x_i) + x_i L_i \frac{\partial f_i}{\partial x_i}(x^*) + \sum_{j \neq i} x^*_j L_i \frac{\partial f_i}{\partial x_j}(x^*) + x_i \frac{\partial h_i}{\partial x_i}(x^*) .
\]

The first derivative of \( i \)'s utility with respect to \( x_i \) is given by:

\[
\frac{\partial v_i(x_i, x^*_{-i}, t_i)}{\partial x_i} = -L_i \frac{\partial f_i}{\partial x_i}(x_i, x^*_{-i}) - \frac{\partial h_i}{\partial x_i}(x_i) + L_i \frac{\partial f_i}{\partial x_i}(x^*) + \frac{\partial h_i}{\partial x_i}(x^*) .
\]

We conclude that \( x^*_i \) is a best response for user \( i \), and hence the socially optimal effort profile \( x^* \) is a Nash equilibrium given the Externality taxes (7).

Proposition 4: The Externality mechanism with taxes given by (7) has strong budget balance.

Proof: The sum of taxes in (7) is given by:

\[
\sum_{i=1}^{N} t_i^E(x^*) = - \sum_{i=1}^{N} \sum_{j=1}^{N} x^*_j L_i \frac{\partial f_i}{\partial x_j}(x^*) - \sum_{i=1}^{N} x_i \frac{\partial h_i}{\partial x_i}(x^*) = \sum_{i=1}^{N} x_i \left( - \sum_{j=1}^{N} L_j \frac{\partial f_j}{\partial x_i}(x^*) - \frac{\partial h_i}{\partial x_i}(x^*) \right) = 0 .
\]

The last line follows from the observation that, using (2), at the socially optimal solution, either \( x^*_i \) or the term inside the parentheses in (1) is zero. Hence, the Externality mechanism guarantees (strong) budget balance.
APPENDIX C
THE WEIGHTED EFFORT MODEL - EFFECTS OF SELF-DEPENDENCE

In this appendix, we consider the security game where users' utilities are given by (8) and interdependence matrix (9). We solve for the socially optimal investment profile, and identify the possible exit equilibria, and parameter conditions under which each equilibrium is possible.

The socially optimal investment profile in this game will be given by:

\[ x_i^* = \frac{1}{a + N - 1} \frac{a + N - 1}{c}, \forall i. \]

To find the exit equilibrium for user \( i \), \( \hat{x}_i \), we write the first order conditions on (4). To simplify notation, denote \( x := \hat{x}_i \) and \( y := \hat{x}_j, \forall j \neq i \). The system of equation determining \( x \) and \( y \) is given by:

\[-a \exp(-ax - (N - 1)y) + c \geq 0\]
\[= 0\]
\[-(a + N - 2) \exp(-x - (a + N - 2)y) + c \geq 0. \]

(2)

There are four possible exit equilibria, depending on whether \( x \) and/or \( y \) are non-zero. We look at each case separately.

a) Exit equilibria with \( x > 0, y > 0 \): Intuitively, when user \( i \) steps out, both sides continue to invest in security, perhaps at reduced levels, but no user is fully free-riding. We would need the following to hold simultaneously:

\[-a \exp(-ax - (N - 1)y) + c \geq 0\]
\[-(a + N - 2) \exp(-x - (a + N - 2)y) + c \geq 0.\]

Solving for \( x, y \) leads to:

\[ x = \frac{1}{(a - 1)(a + N - 1)} \ln \left( \frac{a}{1 + \frac{N - 2}{a}} \right)^{a-1}(1 + \frac{N - 2}{a})^{-(N - 1)} \]
\[ y = \frac{1}{(a - 1)(a + N - 1)} \ln \left( \frac{a}{1 + \frac{N - 2}{a}} \right)^{a-1}(1 + \frac{N - 2}{a})^{a}. \]

To find the range of parameters for which the above holds, we need to ensure that \( x, y \) are indeed positive.

- If \( a > 1 \), then \( y > 0 \). For \( x > 0 \), we need:
  \[ \left( \frac{a}{c} \right)^{a-1} > \left( 1 + \frac{N - 2}{a} \right)^{N-1} \]

- If \( a < 1 \), then \( x > 0 \). For \( y > 0 \), we need:
  \[ \left( 1 + \frac{N - 2}{a} \right)^{a} < \left( \frac{a}{c} \right)^{1-a} \]

b) Exit equilibria with \( x > 0, y = 0 \): In this case, the participating users revert to investing zero, so that the outlier is forced to increase her investment:

\[-a \exp(-ax) + c \geq 0\]
\[-(a + N - 2) \exp(-x) + c \geq 0 .\]

As a result, we get \( x = \frac{1}{a} \ln \frac{a}{c} \). For this to be consistent with the second condition, we require:

\[ \left( 1 + \frac{N - 2}{a} \right)^{a} < \left( \frac{a}{c} \right)^{1-a} \]

The above always fails to hold for \( a > 1 \), as the LHS is always more than 1, while the RHS is surely less than 1 by the assumption \( a > c \). Intuitively, when self-dependence is higher than co-dependence on the outlier, the remaining users will not rely solely on externalities, and continue investing even when user \( i \) steps out.

For \( a < 1 \) on the other hand, for a small enough \( c \) (which leads to higher investment \( x \) be the outlier), the equation can hold.

\[ -a \exp(-(N - 1)y) + c > 0\]
\[-(a + N - 2) \exp(-(a + N - 2)y) + c = 0. \]

As a result, we get \( y = \frac{1}{a + N - 2} \ln \frac{a + N - 2}{c} \). For this to be consistent with the first condition, we need:

\[ (1 + \frac{N - 2}{a})^{N-1} > \left( \frac{a}{c} \right)^{a-1} \]

Note that this always hold for \( a < 1 \), but not necessarily for \( a > 1 \).

d) Exit equilibria with \( x = 0, y > 0 \): We would need the following to hold simultaneously:

\[-a + c > 0\]
\[-(a + N - 2) + c > 0 ,\]

which will never hold, as we initially required that \( c < a \).

A. Weak budget balance and voluntary participation constraints

We now separately analyze each of the possible cases identified in the previous section, summarized in Table I. Specifically, we are interested in the weak budget balance condition under the Pivotal mechanism, and users’ participation incentives in the Externality mechanism.

1) Case \( a \): In this case, the underlying parameters satisfy \( a > 1 \) and \( (1 + \frac{N - 2}{a})^{N-1} > \left( \frac{a}{c} \right)^{a-1} \). As a result, the exit equilibrium (EE) is such that \( x = 0 \), and \( y = \frac{1}{a + N - 2} \ln \frac{a + N - 2}{c} \).

Therefore, the utilities of users at the SO and EE are given by:

\[ u_j(x^*) = W - \frac{c}{a + N - 1}(1 + \frac{a + N - 1}{c}), \forall j \]
\[ u_j(\hat{x}_i) = W - \frac{c}{a + N - 2}(1 + \frac{a + N - 2}{c}), \forall j \neq i \]
\[ u_i(\hat{x}_i) = W - \frac{c}{a + N - 2}^{N-1} \]

a) Weak Budget Balance in the Pivotal mechanism: Note that \( \frac{1}{1 + \ln z} \) is a decreasing function of \( z \). Thus, \( u_j(\hat{x}_i) > u_j(x^*) \) for all \( j \), resulting in \( t_{ij}^p < 0 \), indicating rewards to all users \( i \), and thus a budget deficit in all scenarios. Intuitively, although when a user \( i \) steps out, other users have to invest less in security (thus decreasing their direct investment costs), still their overall security costs go up as a result of the increased risks. Consequently, each user \( i \) should be payed a reward to be kept in the mechanism, resulting in a budget deficit.
b) Voluntary Participation in the Externality mechanism: Voluntary participation will hold if and only if $u_i(\hat{x}^i) \leq v_i(x^*, t_i^E)$, that is:

$$\frac{c}{a + N - 2} \frac{N - 1}{a + N - 1} \geq \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 1}{c}\right)$$

$$\iff \frac{c}{a + N - 2} \frac{N - 1}{a + N - 1} \geq \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 1}{c}\right)$$

Based on the last inequality, define the function $g(z) := \kappa_1 z^{a-1} - (1 + \ln z)^a N^{-2}$. This function is increasing in $z$. As a result, it obtains its maximum when $z$ reaches its maximum value, which is the initial condition is given by $\frac{z}{c} = (1 + \frac{N - 1}{a})^\frac{a-1}{a}$. Thus,

$$g_{\text{max}} = \left(\frac{a + N - 1}{a + N - 2} \left(1 + \frac{N - 1}{a}\right) \frac{N - 1}{a + N - 1} \frac{a - 1}{a + N - 2} (1 + \ln(1 + \frac{N - 1}{a})) \right)^a N^{-2}$$

$$\iff (1 + \ln(1 + \frac{N - 1}{a})) \left(1 + \frac{N - 1}{a}\right) \frac{N - 1}{a + N - 2} (1 + \ln(1 + \frac{N - 1}{a}))$$

Let $z := \frac{N - 1}{a}$, and define $f(z) := (1 + z)^z \ln(1 + z - \frac{1}{z}) - z$ (i.e., we are assuming a fixed $a$). The derivative of this function wrt $z$ is given by:

$$\frac{1}{1 + z} + \ln(1 + z) + \frac{1}{a} + \frac{1}{1 + z - \frac{1}{z}} =$$

$$\ln(1 + z) + \frac{1}{a} + \frac{1}{1 + z - \frac{1}{z} - 1}\cdot$$

As the above is positive for all $a > 1$, we conclude that $f(z)$ is an increasing function in $z$. Furthermore, $\lim_{z \to 0} f(z) = 0$, which in turn means that $f(z) \geq 0, \forall z \geq 0$, and therefore, $g_{\text{max}}$ is always non-positive. This in turn means that the voluntary participation condition can never be satisfied.

2) Case $\beta$: For this case, the underlying parameters satisfy $a > 1$ and $(1 + \frac{N - 1}{a})^{a-1} < (\frac{a}{a+1})^{a-1}$. As a result, the exit equilibrium (EE) is such that $x > 0, y > 0$, and are given by $x = \frac{1}{a-1} a^{-1} a^{-1} \ln(\frac{a}{a+1})^{-1} a^{-1} a^{-1}$ and $y = \frac{a-1}{a} a^{-1} a^{-1} \ln(\frac{a}{a+1})^{-1} a^{-1} a^{-1}$. Therefore, the utilities of users at the SO and EE are given by:

$$u_j(x^*) = W - \frac{c}{a + N - 1} \left(1 + \ln \frac{a + N - 1}{c}\right), \forall j$$

$$u_i(\hat{x}^i) = W - \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 2}{c}\right), \forall j \neq i$$

$$u_i(\hat{x}^i) = W - \frac{c}{a + N - 1} \left(1 + \ln \frac{a + N - 1}{c}\right), \forall j \neq i$$

3) Case $\gamma$: Here, we only require that $a < 1$, and all other values of $N$ or $c$ will guarantee the existence of an equilibrium $x = 0$ and $y = \frac{1}{a N - 1} a^{N - 2}$. This is thus parallel with Case $\alpha$. Users’ utilities in the SO and EE are similarly given by:

$$u_j(x^*) = W - \frac{c}{a + N - 1} \left(1 + \ln \frac{a + N - 1}{c}\right), \forall j$$

$$u_j(\hat{x}^i) = W - \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 2}{c}\right), \forall j \neq i$$

$$u_i(\hat{x}^i) = W - \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 2}{c}\right), \forall j \neq i$$

a) Weak Budget Balance in the Pivotal mechanism: If $u_j(x^*) \leq u_j(x^*)$, the mechanism would always have a budget deficit. This holds if and only if:

$$\frac{c}{a + N - 1} \left(1 + \ln \frac{a + N - 1}{c}\right) \leq$$

$$\frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 1}{c}\right)$$

Based on the last inequality, define the function $g(z) := \kappa_1 z^{a-1} - (1 + \ln z)^a N^{-2}$. This function is increasing in $z$. As a result, it obtains its maximum when $z$ reaches its maximum value, which is the initial condition is given by $\frac{z}{c} = (1 + \frac{N - 1}{a})^\frac{a-1}{a}$. Thus,

$$g_{\text{max}} = \left(\frac{a + N - 1}{a + N - 2} \left(1 + \frac{N - 1}{a}\right) \frac{N - 1}{a + N - 1} \frac{a - 1}{a + N - 2} (1 + \ln(1 + \frac{N - 1}{a})) \right)^a N^{-2}$$

$$\iff (1 + \ln(1 + \frac{N - 1}{a})) \left(1 + \frac{N - 1}{a}\right) \frac{N - 1}{a + N - 2} (1 + \ln(1 + \frac{N - 1}{a}))$$

Let $z := \frac{N - 1}{a}$, and define $f(z) := (1 + z)^z \ln(1 + z - \frac{1}{z}) - z$ (i.e., we are assuming a fixed $a$). The derivative of this function wrt $z$ is given by:

$$\frac{1}{1 + z} + \ln(1 + z) + \frac{1}{a} + \frac{1}{1 + z - \frac{1}{z}} =$$

$$\ln(1 + z) + \frac{1}{a} + \frac{1}{1 + z - \frac{1}{z} - 1}\cdot$$

As the above is positive for all $a > 1$, we conclude that $f(z)$ is an increasing function in $z$. Furthermore, $\lim_{z \to 0} f(z) = 0$, which in turn means that $f(z) \geq 0, \forall z \geq 0$, and therefore, that the voluntary participation condition always fails to hold under these parameter settings.

3) Case $\gamma$: Here, we only require that $a < 1$, and all other values of $N$ or $c$ will guarantee the existence of an equilibrium $x = 0$ and $y = \frac{1}{a N - 1} a^{N - 2}$. This is thus parallel with Case $\alpha$. Users’ utilities in the SO and EE are similarly given by:

$$u_j(x^*) = W - \frac{c}{a + N - 1} \left(1 + \ln \frac{a + N - 1}{c}\right), \forall j$$

$$u_j(\hat{x}^i) = W - \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 2}{c}\right), \forall j \neq i$$

$$u_i(\hat{x}^i) = W - \frac{c}{a + N - 2} \left(1 + \ln \frac{a + N - 2}{c}\right), \forall j \neq i$$

a) Weak Budget Balance in the Pivotal mechanism: Note that $\frac{1 + \ln z}{z}$ is a decreasing function of $z$. Thus, $u_j(x^*) < u_j(x^*)$ for all $j$, resulting in $t_i^P < 0$, indicating rewards to all users $i$, and thus a budget deficit in all scenarios (exactly similar to case $\alpha$).
Voluntary participation will fail to hold if and only if \( u_i(x^*) \geq v_i(x^*, E^i) \), that is:
\[
\frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c}) \geq \frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c})
\]
\[
\Leftrightarrow \frac{a + N - 2}{N - 1} \geq 1 + \ln \frac{a + N - 1}{c}
\]
\[
\Leftrightarrow \left(\frac{\beta}{\tilde{\beta}}\right)_{N-1} \geq (1 + \ln \frac{a + N - 1}{c})^{a-1}
\]

First, we note that the RHS is always greater than 1, as \( x \geq 1 + \ln x \leq x \). On the other hand, since \( a < 1 \), \( a > 1 \) holds for all \( N \geq 3 \), so that the LHS will be less than 1. Therefore, the voluntary participation condition always fails.

4) Case \( \Upsilon \): This case has equilibrium investments similar to case \( \beta \), but under parameter conditions \( a < 1 \), and \( (1 + \frac{N - 1}{a})^a < \left(\frac{\beta}{\tilde{\beta}}\right) \).

\[
u_i(x^*) = W - \frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c}), \forall j
\]
\[
u_j(x^*) = W - \frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c}), \forall j \neq i
\]

5) Case \( \omega \): The last case emerges under parameter settings \( a < 1 \) and \( (1 + \frac{N - 2}{a})^a < (\frac{\omega}{\tilde{\omega}})^{a-1} \), and \( x = \ln \frac{a}{c} \) and \( y = 0 \) is the possible exit equilibrium. The users’ utilities in the SO and EE here are given by:
\[
u_j(x^*) = W - \frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c}), \forall j
\]
\[
u_i(x^*) = W - \frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c})
\]

A) Weak Budget Balance in the Pivotal mechanism: First we use \( (1 + \frac{N - 2}{a})^a < (\frac{\omega}{\tilde{\omega}})^{a-1} \) to conclude that \( (\frac{\omega}{\tilde{\omega}})^{a-1} < \frac{c}{a + N - 2} \). Now, for the mechanism to have weak budget balance, it would be enough to have \( u_i(x^*) \geq u_j(x^*) \), which holds if and only if:
\[
\frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c}) \geq \left(\frac{\omega}{\tilde{\omega}}\right)^{a-1}
\]

\[
\Leftrightarrow \ln(1 + \frac{N - 1}{a}) \geq 1 + \ln(1 + \frac{N - 1}{a})
\]

The last line follows from the previous because \( \frac{c}{a} > 1 \), and it is true because its LHS is \( \geq \ln N \) and its RHS is \( \leq 1/(N - 1) \). Therefore, the mechanism always has weak budget balance in this scenario.

B) Voluntary Participation in the Externality mechanism: The mechanism has voluntary participation if and only if:
\[
\frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c}) \leq \frac{c}{a + N - 1}(1 + \ln \frac{a + N - 1}{c})
\]
\[
\Leftrightarrow \frac{a + N - 2}{N - 1} \leq 1 + \ln \frac{a + N - 1}{c}
\]
\[
\Leftrightarrow \left(\frac{\beta}{\tilde{\beta}}\right)_{N-1} \leq (1 + \ln \frac{a + N - 1}{c})^{a-1}
\]

The last statement holds because the second element in the Inarith is always less than 1, due to \( a < 1 \), and the result follows as \( \ln(1 + z) \leq z \), for all \( z > 0 \).
A. Voluntary participation in the Externality mechanism

We now analyze the performance of the Pivotal and Externality mechanisms, under the different exit equilibria identified in the previous section, and summarized in Table II.

In the Externality mechanism, users’ taxes are given by:

\[
t^E_i(x^*) = cx^*_i \left( \frac{1}{N} - 1 \right)
\]

\[
t^E_j(x^*) = \frac{c}{N} x^*_i, \quad \forall j = 2, \ldots, N.
\]

For non-dominant users \(i \in \{2, \ldots, N\}\) to voluntarily participate in the mechanism, we require \(u_i(\hat{x}_i) \leq v_i(x^*, t^E_i(x^*))\):

\[
\frac{c}{a(N-1)} \geq \frac{c}{a} + \frac{c}{aN} \ln \left( \frac{aN}{c} \right) \Leftrightarrow \frac{1}{N-1} \geq \ln N + \ln \frac{a}{c}.
\]

However, \(\ln N \geq \frac{1}{N-1}, \forall N \geq 3\), and \(a > c\). Therefore, the voluntary participation constraints will always fail to hold for free-riders in the Externality mechanism.

A perhaps more interesting aspect is that the voluntary participation of user 1, i.e., the main investor who is receiving a reward, may also fail to hold. Specifically, when \(a < N - 1\), user 1 will participate voluntarily if and only if \(u_1(\hat{x}^1) \leq v_1(x^*, t^E_1(x^*))\), which reduces to:

\[
\frac{c}{N-1} \geq \frac{c}{aN} + \frac{c}{aN} \ln \left( \frac{aN}{c} \right).
\]

However, the above inequality does not necessarily hold. For example, with \(N = 10\), \(c = 0.45\), and \(a < 5\), the above will fail to hold, indicating that the main investor will also prefer to opt out. It is also interesting to mention that when \(a > N - 1\), the voluntary participation of the main investor always holds.

B. Weak budget balance in the Pivotal mechanism

Finally, we analyze the total budget in the Pivotal mechanism. The taxes for the non-dominant users \(i \neq 1\) will be given by:

\[
t^P_i = \frac{c}{a} \left( \ln \frac{N}{N-1} - 1 \right).
\]

The taxes for user 1 will depend on the realized exit equilibrium. If \(a < N - 1\), this tax is given by:

\[
t^P_1 = (N - 1) \frac{c}{aN} - c(1 + \ln \frac{N-1}{c})
\]

The sum of the Pivotal taxes under this parameter conditions will then be given by:

\[
\sum t^P_i = c \left( \frac{N-1}{a} \left( \ln \frac{N}{N-1} - 1 + \frac{1}{N} \right) - (1 + \ln \frac{N-1}{c}) \right)
\]

Note that \(\ln z - \frac{1}{z} < 0, \forall z < \frac{3}{2}\), and therefore, with \(N \geq 3\), the above sum is always negative. We conclude that the Pivotal mechanism will always carry a deficit.

On the other hand, when \(a > N - 1\), the tax for the dominant user is given by:

\[
t^P_1 = (N - 1) \frac{c}{aN} - (N - 1) \frac{c}{a} = (N - 1) \frac{c}{a} \left( \frac{1}{N} - 1 \right).
\]

The sum of the Pivotal taxes will then be given by:

\[
\sum t^P_i = \frac{c(N-1)}{a} \left( -1 + \ln \frac{N}{N-1} - 1 + \frac{1}{N} \right)
\]

By the same argument as before, the above sum will always be negative, indicating a budget deficit in the Pivotal mechanism under this scenario as well.