Models for Optimum Decision Making in Crude Oil Production and Refining

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Abstract

We discuss mathematical models for several important applications of optimum decision making techniques in crude oil production, and in the operations of a crude oil refinery

Key words: Optimum decision making in crude oil production and refining operations.

1 History of Crude Oil Refining

"Crude oil" or "petroleum" (from Greek: "petra" ("rock") and "oleum" ("oil")) is a naturally occurring yellow to black liquid found in geological rock formations beneath the earth's surface. It is a "fossil fuel" formed in very ancient times when large quantities of dead organisms were buried underneath sedimentary rock and subjected to both intense pressure and heat underground. For a very long period, people in many different parts of the world have been noticing natural seepage of subsurface crude oil collecting in oil pits. In [1, 6] it was reported that oil collected by hand from such pits was used more than 4000 years ago in Babylon in the construction of walls and towers, asphalt used to seal water tanks at Mohenjo-Daro [8], in Ancient Persia for medicinal and lighting uses, and also in some parts of the Roman Empire. The earliest known oil wells were reported to have been drilled using bamboo poles in China by the 4th century CE. By the 7th century CE, the Japanese statesman Shen Kuo of the Song Dynasty coined the word "rock oil" for crude oil.

Crude oil is flammable, so people tried to use it for lighting. But the burning crude produced an awful smell and a great deal of smoke, so it was hard to sell as an illuminant. So, they realized that it needs to be refined by distillation to get the best lamp oil from it. Crude oil is a mixture of a vast number of hydrocarbon compounds ranging from the smallest fractions such as methane or natural gas to large asphaltenes and paraffin chains; these can be separated because they have different boiling points. The lightest molecules have the lowest boiling points and the heaviest ones the highest.

Every oil field produces crude oil with a different mixture of hydrocarbons. Crude oil is classified by density into 3 categories "light, medium, heavy". It is "light" ("heavy") if it has a higher proportion of "lighter" ("heavier") compounds. Compounds used in gasoline blends are lighter hydrocarbons (these are hydrocarbons with molecular weight less than that of heptane (C_7H_{16})), where as those used for making lubricating oils, wax, asphalt are heavier hydrocarbons.

Crude oil is classified as "sweet" if it is low in sulfur (less than 1% sulfur content), "sour" if it is high in sulfur (may contain 1-5% sulfur).

The terms "low (heavy)ends" refer to "lower (higher)" boiling point components in a mixture of hydrocarbons. At 90^0F, the lightest fractions of crude oil vaporize, these are gases such as methane, ethane, and butane. Naphtha, which is a compound in the gasoline blend is vaporized between 90 - 220^0F, Kerosene is vaporized between 315 - 450^0F, and fuel oil between 450 - 800^0F. "Light oils" refers to products distilled from crude oil up to but not including the first lubricating oil distillate. Heavy fuel oil is fuel oil having a high density and viscosity. When the temperature of crude oil raises above 1000^0F, heavy hydrocarbons in it begin to "crack" (i.e., break up into lighter compounds). This "cracking" feature was not part of "straight-run distillation" used in earlier times, it was unknown till modern times.

Crude oil accessible from natural fields was being distilled in oil refineries for the production of chemicals like tar, and flammable substances like kerosene in Persia, Arab countries, Europe and Russia by the 13th century CE.

These first oil refineries used “straight-run distillation” (the first form of “atmospheric distillation” used), the simplest distillation method, to extract kerosene, the most valuable product until 1915; and other light fractions like naphtha and gasoline. Compared to oil refineries today, these early refineries were tiny, they distilled crude oil in batches in horizontal cylindrical iron stills that only hold 5 to 6 barrels at a time, built over a furnace. They would raise the temperature of the oil very slowly. The vapors would be captured in a condenser and allowed to condense back into liquid, After each fraction of crude oil is boiled off, the still was heated further until the next fraction boils. The operator would control the product coming out of the condenser by controlling the temperature of the still. Each fraction would be collected in a separate container, and possibly redistilled to produce a purer fraction (see [2, 3]).

The left over residue, approximately half the volume of the input crude oil, was too heavy to vaporize in the process used, it was stored in tanks until it was shipped out to be processed into lubricating oils and greases, bitumen, or sold as fuel oil. There was no market for gasoline or naphtha at that time, and their explosiveness made storage a problem, so they were simply poured into open pits and burned, or disposed off in a nearby river. The very lightest fractions like methane, butane, propane were either flared off or escaped into the atmosphere.

In those days, refiners used to sell their product to “jobbers” who would run retail operations. The jobbers would deliver kerosene from a tank on the back of a wagon to homes and businesses. Hardware and grocery stores stored barrels of kerosene and other petroleum products in the back of the store; customers would bring their own containers to be filled from the barrel.

By the 13th century crude oil was being distilled in this way in the Arab world, Persia, Europe, Russia, China and other parts of Asia to make flammable products and other chemicals like lubricants etc. Early British explorers to Myanmar documented a flourishing oil extraction industry based in Yenangyaung that in 1795 CE had many hand-dug wells under production.

In mid-18th century under the Empress Elisabeth of Russia, a refinery was built in Russia. Through this process of distillation of the rock oil they produced a kerosene-like substance which was used in oil lamps in Russian churches and monasteries, even though households still relied on candles. By 1825 Russia was producing and processing 3500 barrels/day of oil this way.

Modern oil drilling began in Azerbaijan in 1848, and two large pipelines were built in the

Russian empire: the 833 km long Baku-Batumi pipeline to transport oil from the Caspian to the Black Sea Port of Batumi completed in 1906, and the 162 km long pipeline to carry oil from Chechnya to the Caspian. At the turn of the 20th century Russia's output of crude oil accounted for half of the world's production and dominated international markets.

The first modern oil refinery based on continuous distillation was built by Ignancy Lukaciewicz near Jasha in Poland in 1854-56. Compared to refineries of today, it was a small refinery, but built according to modern designs based on fractional distillation. The refined products used were kerosene, asphalt, fuel oil, and lubricants. As Lukaciewicz's Kerosene lamps gained popularity, the refining industry grew in the area.

The name "kerosene" for the product was coined by the Canadian geologist Abraham Pineo Gesner, he developed the process of making it from coal, bitumen and oil shale. In 1850 he created the "Kerosene Gas Light Company", and began installing lighting in the streets of Halifax and other cities in Canada.

The first commercial oil well in North America was drilled in Canada by James Miller Williams, and it started producing oil in 1858. He discovered a rich reserve of oil four meters below ground. In 1862 Canada's first major oil gusher came into production, shooting crude oil into the air at a recorded rate 3000 barrels/day [1].

In 1859 two separate events occurred that would jumpstart both the petroleum and the auto industries. In that year, Edwin L. Drake in the US drilled the first working oil well in the US in Titusville, Pennsylvania (a 70 foot deep well) at a cost of $15000 (Nelson, 1958 [11]); and the French engineer J. J. Etienne Lenoir made the first dependable internal combustion engine powered by gasoline. Drake's oil well kicked off the petroleum industry, and Lenoire's work lead to the development of the automobile of today (see [4]). Between 1860-1900 CE many technological innovations occurred in the developing oil and auto industries. By 1908, the year that Henry Ford's Model T made its market debut, the US had roughly 125000 passenger cars. By 1911 gasoline dethroned kerosene as the top selling product for Standard Oil of New Jersey, the largest refiner in the US at that time. Kerosene's slide was hastened by the 1910 invention of the tungsten filament for electric light bulbs by William David Coolidge (the tungsten filament outlasted all other types of filaments, and Coolidge made the costs practical). Drake's first refinery, and others around that time, used batch distillation to separate kerosene and heating oil from other crude fractions. However with increasing demand for petroleum products, continuous refining became a necessity, and the first continuous refinery plants emerged around 1912 (Nelson, 1958 [11]).

In 1878 Ludwig Nobel and his Branobel company revolutionized oil transport by commissioning the first oil tanker and launching it on the Caspian sea ([1]). It was built in Sweden and operated between Baku and Astrakhan

In the first quarter of the 20th century, the US overtook Russia as the World's largest oil producer. By the end of this quarter, oil fields had been established in many countries including Canada, Poland, Sweden, Ukraine, US, Peru, and Venezuela.

In 1913 refiners developed thermal cracking which enabled production from a barrel of crude oil, of more gasoline, diesel fuel, and kerosene which has become a primary component in aviation fuel for modern jet engines, and soon several other process innovations followed allowing refiners to meet market needs.

The products refined from the liquid fractions of crude oil today can be placed into 10 major categories. These are (see [3]): asphalt (used for paving roads), diesel (produced in fractional distillation between 392- 662^0F, higher density than gasoline, commonly used in transportation), fuel oil (heaviest commercial fuel produced in refineries, burned in furnaces to generate heat, part of it is used as bunker fuel for ocean-going vessels), gasoline (almost 50% of output from refineries, it is a blend of paraffins, naphthenes, olefins etc., mainly used as fuel for internal combustion engines), kerosene (collected in fractional distillation between 302 - 527^0F, used as jet fuel and heating fuel; it is thin, clear, and burns without any smell as it contains no sulfur compounds), LPG or Liquified Petroleum Gas (includes propane, butane; these are a mixture of gases used in heating appliances, refrigerants, aerosol propellants; since these are gases at normal atmospheric pressures they are marketed in pressurized steel bottles), lubricating oils (used between two surfaces to reduce friction and wear, an example is motor oil), paraffin wax (excellent electrical insulator, also used in drywall to insulate buildings, and in candles), bitumen (also known as tar, it is from the bottom fraction in fractional distillation, used in paving roads, waterproofing roofs, also used as thin plates to soundproof dishwashers), and other petrochemicals (includes ethylene used to make anesthetics and plastics like polyethylene, antifreeze, and detergents, propylene, benzene, toluene, xylene used in making many different chemical products for our daily use).

Crude oil and liquid products from oil processing are usually measured in barrels. A barrel is 42 gallons. In oil refining, from a 42-gallon barrel of input crude oil, the output is roughly 45 gallons of petroleum products. This is due to the overall density changes that occur in refining process.

Today crude oil refineries are operating all over the world. The Jamnagar Refinery of Reliance Industries in India refining 1240000 barrels/day, SK Energy in Ulsan, South Korea processing 1120000 barrels/day, and the Paraguana Refinery complex in Venezuela processing 940000 barrels/day are the world's largest. The top 11 of the World's largest refineries are located in India, South Korea, Venezuela, Singapore, Texas in the US, Saudi Arabia and Greece.

For clean performance when they are in use, for each product stringent specifications have been developed on several important performance characteristics. As an example, we will explain the function of the specifications on the characteristic "octane rating" for gasoline (see [5]). Gasoline must meet three primary requirements- it must have an even combustion pattern, start easily in cold weather, and meet prevailing environmental requirements. Gasoline must burn smoothly in the automobile engine, otherwise "knocking", "severe knocking" can occur jolting passengers and damaging car engine. Experiments determined that the most severe knocking occurs with fuel of pure normal heptane, while least knocking was produced by pure isooctane. This led to the development of the "octane scale" for defining gasoline quality. When a motor gasoline gives same knocking performance as a mixture of 90% isooctane and 10% normal heptane, it is given the octane rating of 90. Adding oxygenated compounds such as ethyl alcohol to the gasoline blend also helps to increase its octane rating and also to reduce emissions of carbon monoxide and nitrogen oxides.

Octane rating or Octane number is not an indicator of the energy content of the fuel. It is only a measure of the fuel's tendency to burn in a controlled manner, rather than exploding in an uncontrolled manner. It is possible for a fuel to have Research Octane Number (RON, most commonly used octane rating worldwide) more than 100 because isooctane is not the most knock-resistant fuel available. For example racing fuels have octane ratings of 110 or greater.

Most gas stations offer 3 octane grades: regular (usually 87 octane), mid-grade (usually 89 octane), and premium (usually 92 or 93). Regular octane is recommended for most cars. Some cars with high compression engines (like sports cars, luxury cars) need mid-grade or premium-grade to prevent knocking.

In 1901 one of the largest and most significant oil strikes in history occurred near Beaumont, Texas on a mound called Spindletop. Drillers there brought in the greatest gusher ever seen within the US. By 1910 crude oil production within the US was more than equalled that of the rest of the world combined. In the 20th century Oil companies prospered along with automakers. At the beginning of this century, ocean-going steamships converted from using coal as their primary fuel, to using heavy fuel oil (under the name bunker fuel), which gave the big advantage of being pumped into the engine rather than being shoveled. Refiners built more refineries and expanded existing facilities. Researchers improved thermal cracking techniques and developed other catalytic processes to produce high-grade products. High-octane gasoline emerged in the 1930's from these efforts. These developments played a major role in WWII(World War II), in which US Oil Refiners supplied more than 80% of the fuel used by Allied Forces. Running on high-octane fuel that optimized engine performance, the equipment of Allied Forces in WWII outmaneuvered German equipment using inferior fuels. Joseph Stalin of the Soviet Union said in a toast during a banquet towards the end of the war: "This is a war of engines and octanes. I drink to the American Auto and oil industries". See ([4]). Another important development was the building of long-distance pipelines, which had overtaken rail tank cars as the main method of transporting hydrocarbons. One example is "The Big Inch" spanning 1254 miles for moving crude from Texas oil fields to East Coast refineries built in 1942, 1943 (world's longest crude oil pipeline at that time), followed by the "Little Inch" which transported refined products.

After the end of WWII the countries in the Middle East took the lead in crude oil production from the US. There were several important developments after WWII, including Ocean and deep-water drilling, growth of the global shipping network for crude oil relying on huge oil tankers and pipelines, international organizations like OPEC (Oil Producing and Exporting Countries) playing a major role in setting petroleum prices and policy. With consumption over 30 billion barrels/year, crude oil refining is one of the major industries all over the world today. Today crude oil has become an essential commodity for the sustainability of our modern way of life; its importance is due to all the machines based on internal combustion engines that we use in our daily living, raise of commercial aviation, and to industrial organic chemistry in manufacturing fertilizers, pesticides, plastics, solvents, adhesives, and so many other chemicals that we use daily. Today crude oil refining is the most prominent and complex industry in the World offering many challenging applications of optimization techniques. A typical refinery has many different crude oils and other feedstocks to choose from, dozens of different products to manufacture to maximize profit based on consumer demands in the global marketplace; and hundreds of important decisions to be made in daily operations. But the crude oil industry faces serious environmental concerns due to GHG (green house gas) emissions into the atmosphere from the burning of fossil fuels, and the consequent global warming; and oil spills and their cleanup.

In 2009 the total world crude oil reserves have been estimated to be 1342 billion barrels; with Venezuela as the oil kingpin in OPEC with 301 billion barrels of proved oil reserves (24.8% of OPEC's total), and Saudi Arabia as the country with the 2nd highest reserves of 267 billion barrels (22% of OPEC's reserves).

2 Operations inside a Crude Oil Refinery

Crude oil is either locally produced, or imported. Local crude comes from production sites which may be either offshore or onshore. Offshore crude oil production wells are drilled from either floating (discussed in Chapter 13 of [10]) or fixed platforms. that are placed on the ocean floor. Offshore production of crude oil is transported either by pipelines or tankers to maritime terminals. From there it is either exported or shipped to refineries for processing.

Fixed platforms on the ocean floor contain storage tanks for gas and oil, as well as pumps to transport the resource by pipeline, or to fill tanker ships. They may contain bedrooms and lounges for up to 200 workers. The power supply for the many items of equipment, and for the living quarters is produced by generators on the fixed platform.

Typical production fluids coming from the oil wells are a mixture of crude oil, gas and produced water. Many permanent offshore platforms have full oil production facilities on board. Smaller platforms and subsea wells export raw production fluid to the nearest production facility which may be an onshore terminal or a nearby offshore processing platform.

The processing consists of a separator to separate the three components. The separated oil is usually routed to a coalescer before being metered and pumped to an onshore terminal. The produced water goes through treatment to remove entrained oil and solids and then either re-injected into the reservoir or dumped overboard. The produced gas (dubbed "wet gas") is treated to remove the liquids , and the resulting "dry gas" is either exported by pipeline, or re-injected into the reservoir, or used as fuel for installation's power generation, or flared.

Modern Petroleum Refineries are typically large sprawling industrial complexes with tubes of various diameters running throughout. These tubes carry different streams of fluids between various processing units in the refinery. Other distinct signs of an oil refinery are processing towers of various heights (like the Fractionation Tower for fractional distillation described in Chapter 11 of [10]) and farms of many storage tanks for storing crude oil until the refinery is ready to process it, and storing various liquid products from the refinery. Unless maintenance is needed refineries operate round the clock every day.

Crude oil refineries are the main part of the downstream side of the petroleum industry, they are typically large scale plants processing hundred thousand to several hundred thousand barrels of crude oil/day. Because of the high capacity of these plants, making decisions in the daily operations in them optimally, can lead to substantial savings.

Oil refineries use a lot of steam and cooling water, so they have to be located at a site with an abundant source of water, like near a navigable river, or seashore near a port. And their major outputs are transported by river, or sea, or by pipeline. Small volume outputs are transported by railcars, or road tankers. So the site of an oil refinery should have infrastructure for receiving supply of raw materials, and for shipment of products to markets.

Also, since the refining process releases several chemicals into the air and polluting wastewater on the ground, and poses risks of fire, explosions, and harmful release of chemicals; residential communities vigorously oppose the siting of a new oil refinery close to their neighborhood. So, typically oil refineries have to be located some distance away from populated areas.

The Crude distillation unit (CDU) described in Chapter 11 of [10] , also referred to as as the *atmospheric distillation unit* since it operates at slightly above the atmospheric pressure; is the first processing unit in all oil refineries; it separates the crude oil into various fractions in it of different boiling ranges, each of which undergoes additional processing in other processing units in the refinery.

Based on the way the crude oil is distilled and separated into fractions, the products from a refinery are usually grouped into these categories:

Light distillates: The gaseous and more volatile liquid hydrocarbons produced in an refinery are collectively known as light hydrocarbons or lightends. This group includes LPG, gasoline, naphtha, these fractions have the lowest densities and boiling points

Middle distillates: Kerosene, diesel; these are fractions with boiling points and densities between those of kerosene and lubricating oil fractions

Heavy distillates: Heavy fuel oil, lubricants, wax, asphalt, and tar, these fractions have the highest boiling points and densities.

Other output products are hydrogen, and light hydrocarbons which are steam-cracked in an ethylene plant, and the produced ethylene is polymerized to produce polyethylene.

The refining process actually begins at the oil well itself. The pumped oil comes mixed with brine (salt water) and mud as it comes out of the well. In the settling tanks, the oil, brine and mud, settle into layers. The crude oil is pumped from the top of the tank into a pipeline or truck for delivery to a refinery. The brine and mud are injected right back into the oil-bearing geologic formation to repressurize the formation to make the oil easier to pump.

Here are various processing units in a typical oil refinery [7] . In modern oil refineries these process units are combined into functional categories: crude supply and blending (includes receiving facilities, tank farm for receiving and storage, and either blending, or sent directly to the processing system), separation (basically distillation to separate various fractions)), upgrade, treatment, other processing and blending according to product specifications. This portion also includes utilities that provide the refinery with power, fuel, steam, cooling water, compressed air etc., all necessary things for operations in the refinery. And, finally dispatch of finished products.

1. **Desalter unit:** This unit is designed to wash salt out from crude oil before it enters the CDU. It mixes crude oil with water, the salt will dissolve in water, and as the water and oil are separated, the salt is washed away.

2. **Atmospheric Distillation Unit:** Also called CDU, this unit separates the crude oil into its various fractions.

3. **Vacuum Distillation Unit:** This unit further distills residual bottoms from the CDU. Then **Solvent Dewaxing Units** remove the heavy waxy constituent petrolatum from Vacuum Distillation production.

4. **Naphtha Hydrotreater Unit:** In this unit, hydrogen is used to desulfurize naphtha from the CDU , before it is sent to the catalytic reformer unit.

5. **Catalytic Reformer Unit:** This unit converts the naphtha evaporating range products (see Table 11.1 in Section 1.2.2 in Chapter 11 of [10]) into higher octane reformate which contains higher proportion of aromatics and cyclic hydrocarbons. This unit also produces hydrogen as a byproduct, this hydrogen is used in hydrotreaters (described in no. 4 in this list), or the hydrocrackers (described in no. 8 in this list).

6. **Distillate Hydrotreater:** This unit desulfurizes distillates like diesel from the CDU.

7. **Fluid Catalytic Cracker (FCC) Unit:** This unit upgrades heavier fractions from the CDU, into more income producing lighter products.

8. **Hydrocracker Unit:** This unit uses the hydrogen from no. 5 in this list to also upgrade heavier fractions from the CDU into lighter, more profitable products.

9. **Visbreaking Unit:** This unit is used to convert heavy residual oils from the CDU by thermally cracking them into lighter more profitable reduced viscosity and pour point products.

10. **Merox Unit:** This unit is used to treat LPG, kerosene, jet fuel, to oxidize mercaptans in them into organic disulfides. Other units (**Doctor sweetening process, Caustic washing**) are also used for this purpose.

11. **Coking Units:** There are 3 types of coking units, Delayed Coking, Fluid Coker, Flexicoker. These units process heavy residual oils into gasoline and diesel fuel, leaving Petroleum Coke as a byproduct.

12. **Dimerization Unit:** This unit converts olefins into higher-octane gasoline blending components. Butenes can be dimerized into isooctene which may subsequently be hydrogenated to form isooctane.

Alkylation Unit uses sulfuric acid or hydrofluoric acid to produce high-octane components for gasoline blending.

Isomerization Unit converts linear molecules into higher-octane branched molecules for blending into gasoline or feed to Alkylation units.

13: Steam Reforming Unit: This unit produces hydrogen for the Hydrotreaters, Hydrocrackers (discussed in nos. 4, 8 in this list).

14: Amine Gas Treater, Clause Unit, and Tail Gas Treatment: These convert hydrogen sulfide from hydrodesulfurization in no. 6 in this list , into elemental sulfur.

15. Solvent Reforming Units: These units use solvent (cresol, or furfural) to remove unwanted aromatics from lubricating oil stock or diesel stock.

Solvent Dewaxing Units remove the heavy waxy constituents petrolatum from vacuum distillation products from no. 3 in this list.

The refinery also has **Utility units**, these are **cooling towers** circulating cooling water, **Boiler plants** generating steam, and an **Electrical substation** generating electricity for the operations in the plant, **Wastewater collecting and treating** systems to make used water suitable for reuse or disposal.

The output from these processes are classified into three main classes of hydrocarbons based on their molecular structure.

Saturated hydrocarbons: These include two groups: paraffins or alkanes (this group includes methane, ethane, propane, butane, pentane, hexane, heptane, octane, nonane, decane etc.), and naphthanes or cycloalkanes.

Unsaturated hydrocarbons or Olefins: These include two groups: alkenes, and alkynes.

Aromatic hydrocarbons: These compounds are important petrochemical feedstocks, and in gasoline they help increase the octane number.

Gaseous fuel outputs like propane are stored and shipped in specialized vessels at high pressure to maintain them in liquid form. Lubricants (light machine oils, motor oils, greases etc.) are usually shipped in bulk to an offsite packing unit. Paraffine wax is usually shipped in bulk to another site for preparing it in the form of packaged blocks. Sulfur, sulfuric acid, byproducts of sulfur removal from petroleum, are sold as industrial materials. Bulk tar is sent to an offsite unit for packaging it for use in tar-and-gravel roofing and other uses. Asphalt is shipped in bulk to customers. Petroleum coke is either sold as solid fuel, or to makers of speciality carbon products. Other liquid fuel finished products are stored in tanks allocated for their storage , and then used in blending them into gasoline, diesel, kerosene, aviation fuel etc.

3 How Much Crude Oil Should an Oil Exporting Country Produce Annually for Export?

3.1 Historical Background

At the beginning of the year 2015 international press had many headlines with news about the resurgence of the US Oil & Natural Gas industry using the newly developed horizontal drilling and hydraulic fracturing (fracking) techniques. These techniques of course require huge investments. News reports indicated that with these investments US natural gas production will go up so much that the country won't have to import natural gas; and even the US crude oil production will go up to such an extent that pretty soon the country will move from an oil importing country to an oil exporting country. Crude oil produced using these new techniques is usually referred to as *shale oil*. For a background, we remind the reader that for many years, the US has been a net importer of crude oil with 2014 imports of 7 million barrels/day.

Saudi Arabia (SA), the leader of OPEC (Oil Producing and Exporting Countries), is the world's largest exporter of crude oil with exports of 7.4 million barrels/day at the beginning of 2015. Russia which is not a member of OPEC, is the 2nd largest crude oil exporter. Both SA and Russia mainly depend on the revenue from crude oil exports for their sustenance. So, these countries, as well as all other oil exporting countries were mainly interested in raising the crude oil price to maximize their revenue from its sale at that time.

At that time crude oil price was ranging between \$100-110/barrel (in this book the symbol " \$ " denotes "US dollar"). Due to huge public expenditures in the country, SA's budget break-even crude oil price was estimated to be \$88/barrel.

3.2 The Two Players Involved in the Decision Making Problem

One of the players with great interest in the outcome of the decision making problem that we will discuss is the US Government. For several years before 2015, the value of the US total annual imports was running higher than that of its total annual exports, resulting in a net annual trade deficit of around \$500 billion. This has a negative effect on the growth of the US economy. US is the world's largest importer of crude oil until 2014, also crude oil imports were a large part of US imports. So, the US is very much interested in keeping the crude oil price as low as possible.

Also, there was another major event that happened in Europe around this time. There was a civil war in the Crimean Peninsula (or Crimea) that was part of Ukraine at that time. The vast majority of inhabitants of Crimea are Russian people speaking the Russian language, and they never liked being a part of Ukraine. At that time the Russian Army marched into Crimea, and annexed it as a part of Russia. Supporting the Ukraine Government, the US Government imposed some sanctions on Russia. When these sanctions had no effect on Russia, the US Government was looking for some other actions they could take for punishing Russia. Since Russia depends heavily on their revenues from the sale of crude oil, reducing crude oil price will result in a severe punishment to Russia.

The 2nd player is the SA Government represented by their Oil Minister. The income from their exports of crude oil is the major source of revenue for the country, another source for them is the revenue from the Haj pilgrimage. SA imports most of their groceries and several other commodities using this income, also SA's population is growing at the annual rate of about 2%. Given these facts, clearly SA is interested in keeping the crude oil price at the then prevailing rate of between \$100-110/barrel, and perhaps growing it on par with the growth rate in the prices of other commodities. In fact for a very long time, SA has been very modest in their desire for higher crude oil prices.

Other crude oil exporting countries have been trying vigorously to push up crude oil prices;

as the World's largest producer of crude oil, SA has been playing the role of a moderator of these ambitions by increasing their crude oil production rate to keep price raises in check. The world really appreciates this moderating role that SA has been playing regarding crude oil prices.

3.3 History of Saudi Oil Industry

The Kingdom of Saudi Arabia was formed in 1932 with Abdulaziz Al-Saud as its King. A major source of income for the country at that time was the taxes paid by Haj Pilgrims to visit the holy sites of Islam. But the great depression of the 1930's resulted in the number of pilgrims/year decreasing from 100,000 to below 40,000. This hurt the Saudi economy greatly, so they started looking for an alternate source of income. Oil seepages have been observed at Quatif on the eastern seaboard, so the King invited mining engineers to explore the eastern regions of the country for oil. They identified a promising site near Dammam and over the following 3 years they started drilling for oil in that region. They stuck oil on 3 March 1938. This discovery turned out to be the first of many, eventually revealing the largest source of crude oil in the world. For the kingdom, oil revenues became a crucial source of wealth, and these discoveries altered Middle Eastern political relations forever. Soon the name of the company dealing with these oil producing operations was changed to Arabian American Oil Company (ARAMCO). By 1988, ARAMCO was officially bought by the SA Government and became known as the Saudi ARAMCO.

Now SA is the world's largest producer and exporter of crude oil and has 25% of world's known oil reserves (over 264 billion barrels). As the world's largest producer and exporter of crude oil, SA plays the leading role in the global energy industry, its policies have a major impact on the energy market and the global economy. Mindful of this responsibility, the country is committed to ensuring stability of supplies and prices; so far they have been covering any drop in oil supplies by increasing their output.

With domestic demand and population raising rapidly, a very important objective for the SA Oil Minister to consider in reaching important decisions about oil production levels, is to make sure that his country's domestic oil needs, and needs for revenues from oil exports, will be covered for *as long a planning horizon in future as possible.* This important objective function is measured by:

Objective 1: Lifetime of the country's crude oil reserves resource = (country's estimated crude oil reserves)/(country's annual crude oil production rate) in years.

For Saudi Arabia the estimated oil reserves in 2015 were 264 billion barrels, and the country was producing 9.6 million barrels of crude oil daily at that time, so Objective 1 at that time was (264 billion)/(9.6 $\times$ 365 million) $\approx$ 75 years. After all, the oil reserves are a fixed finite resource that is being depleted by oil production. The SA Oil Minister is aware that about 40 years ago, the countries in Northern Europe were plush with oil from the reserves in the North sea; now these reserves are nearing depletion, and many of these countries are importing crude oil from Russia. One of his prime responsibilities is to make sure that this does not happen to his country SA, for the largest possible time interval in future, i.e. maximize Objective 1.

At that time they were exporting 7.4 million barrels/day of crude oil. At \$100/barrel this was providing oil revenues of \$100 $\times 7.4 \times$ 365 million $\approx$ 270 billion/year. This revenue of \$270 billion was meeting their needs for oil revenues at the beginning of 2015. This is the 2nd important objective function to consider in our decision making problem to decide how much crude oil the country should export annually, it is:

Objective 2: Amount in US \$ the country's exports of crude oil generate annually.

For Saudi Arabia at the beginning of 2015 it was \$270 billion.

The 3rd important objective function to consider in our decision making problem to decide how much crude oil the country should export annually is:

Objective 3: Market share of the country in the oil export market = (the country's annual export of crude oil in barrels)/(Worldwide annual crude oil exports in barrels).

Saudi Arabia at the beginning of 2015 was exporting 7.4 million barrels/day, and the Worldwide crude oil exports was 44 million barrels/day. So this Objective 3 had a value of 7.4/44 =

17% of market share in the crude oil exports market for Saudi Arabia at that time.

3.4 The Decision Making Problem, Background in Early 2015

In early 2015, when International News headlines were predicting that with shale oil production US may change from an oil importing country into an oil exporting country, US Secretary of State had discussions with the Saudi Oil Minister and other top level persons in the **Saudi Arabian (SA)** Government. He briefed them about these developments.

One of the countries in the Middle East with which SA does not have good relations is another OPEC member, Iran. At that time, the permanent members of the UN Security Council had imposed sanctions on Iran under the suspicion that the country is engaged in secret research to produce nuclear weapons, even though Iran has always denied these allegations. For many years now, negotiations have been going on between Iran and these big 6 powers on conditions that Iran has to meet, to lift the sanctions placed on it. In discussions with Saudi authorities, US Secretary of State has pointed out that these Iran nuclear negotiations may at last be successful, and if so the sanctions on Iran will be lifted at that time. If that happens, Iran will begin oil production and enter the oil export market. He pointed out that these recent developments may affect SA's *market share in the crude oil export market*; and that they have to do something to protect their market share.

News reports in Western Media at that time indicated that the SA Oil Minister considered the emerging US Shale oil industry a big threat to the Saudi market share in the crude oil export market. So, in order to discourage additional investment in the US Shale oil industry, he decided to do everything in his power to make these investments unattractive. Knowing that this can be achieved by reducing the price of crude oil in the oil market, which can be achieved by flooding the market with crude oil; he took the decision of increasing the value of the Country's crude oil production to the maximum extent that the Saudi oil industry can pump.

This recent Saudi action is contrary to the actions of all the OPEC members including SA until recently. Previously all the OPEC countries had the United policy of maintaining capacity limits for oil production by each country, to either increase the crude oil price or at least keep it from falling. That was to make sure that they are rewarded adequately for making their precious natural resource available to the World.

This action by SA was not popular with other OPEC members, they kept their crude oil production levels more or less stable. These actions raised the Saudi crude oil production to 10.6 million barrels/day, a record high. They resulted in the overall OPEC production increasing from 32 million barrels/day in 2014 to 33.1 million barrels/day in early 2015. The net result was that within a few days crude oil price dropped to 52$/barrel, and kept falling still further, a 50% drop due to the this action.

We will analyze this action taken by the Saudi Government in the next section.

3.5 Analysis of the Decision Taken

First let us examine the consequences of the decisions implemented. In a few days after the decisions mentioned were implemented, the price of crude oil dropped by about 50%. So as a result of this action by SA, all the oil exporting countries experienced a similar decline in their returns/barrel exported. Prices of all other goods and commodities remained stable, so the net result was that SA is now having to export twice the amount of their precious natural resource to import the same quantity of other goods and commodities for their national needs, or use up some of their dollar reserves.

It is quite possible that maintaining market share is a matter of national pride, but is it worth having to pay twice the amount of the country's fixed natural resource for importing the same quantity of other goods and commodities? Also at the current production rate of 10.6 million barrels/day, the estimated lifetime of the current known reserves of SA will be (264 billion)/(10.6×365 million) ≈ 68.2 years, which is a significant drop of the other objective function Objective 1 from its previous value of 75 years, as a result of the decision implemented.

3.6 The Procedure That OPEC IS Using Currently to Decide The Oil Production Quota Of Each Of Its Members

Each OPEC country wants to maximize the amount of money in US $ that they receive by exporting their crude oil, while making sure that their crude oil export volume is the same or higher than that in the recent past.

3.7 Lessons to be Learnt from the Events Over the Last Two Years (2015 - 2017)

Crude oil prices have remained below US $50/barrel all this time, and even now with OPEC and other major crude oil producers like Russia limiting their crude oil production, they are struggling to reach 50 US $ /barrel. Here briefly are the lessons we can learn from the events over this period.

1. To maintain the desired price level, unity of OPEC countries in abiding by agreed upon production limits is essential. Even if one country produces above their limit, the results may slip.

2. Keeping Objective 3, Country's Market Share at a high level may be a matter of National pride for the time being; but Ojective 1, Lifetime of the country's crude oil resource is very important for the well being of the next generation in the country. So, Objective 1 is very important and should not be ingnored in decision making.

3. Just increasing country's crude oil production is not enough to make more money in US $, it is also necesssary to make sure that the increased production does not result in lowering crude oil prices.

4. For every country, the country's crude oil reserves is a fixed finite resource, and crude oil production by any technique including fracking, depletes that resource.

5. When OPEC countries and others like Russia stick to their agreed upon crude oil production limits; if the US tries to keep crude oil price artificially low , by flooding the market with crude oil produced through fracking, they will soon realize that this effort is depleting their crude oil reserves resource faster than necessary.

6. To keep crude prices at desired levels, once this level is reached, it is necesssary to make sure that the agreed upon production limits leads to available crude oil in the market equal to the demand for that oil.

3.8 Simple Approach to Calculate Crude Oil Export Quota for Each OPEC Country

Under these conditions, we get the following simple approach for calculating the crude oil export quota for each OPEC country:

1. Each Country Selects its Desired Value for the Lifetime of its Crude Oil Resource: For each country, calculate (its crude oil reserves in barrels)/(its current annual crude oil production level in barrels) = its current value for "Lifetime of its crude oil crude oil resource". Then let the country modify that value to their "desired lifetime for its crude oil resources".

This implies that the country's "desired annual crude oil production rate" = (its crude oil reserves in barrels)/(its desired lifetime of its crude oil resource). Denote this quantity by the symbol y_i for OPEC Country i.

2. Determine x = the Amount of Crude Oil in Barrels that OPEC as a Group Should Export for Coming Year: = (total estimated world crude oil demand in barrels) - (expected amount that non-OPEC countries are likely to export if they are an oil exporting country, next year).

3. Distribute the Quantity x Among OPEC Countries in Proportion to Their Desired Annual Crude Oil production Rate y_i: There are two cases to consider here:

Case 1: If $x \geq (\sum_p (y_p$: over all OPEC countries p)), make the crude oil export quota for the year as y_i for OPEC country i ; and leave $x - \sum_p (y_p$: over all OPEC countries p) as portion of the crude oil demand for the year, x , for other non-OPEC countries to fill.

Case 2: $x < (\sum_p (y_p$: over all OPEC countries p)): In this case, for OPEC country i, make its crude oil export quota = $x \times (y_i / (\sum_p y_p$:over all OPEC countrys p)).

4 Clustering problems in offshore drilling of crude oil wells.

First a brief introduction to offshore oil exploration. With worldwide consumption of crude oil over 30 billion barrels/year, and natural gas over 3500 billion cubic meters/year; crude oil and natural gas production and processing is the major industry all over the world today.

In 2009 the total world crude oil reserves have been estimated to be 1342 billion barrels, with Saudi Arabia as the country with the second highest reserves of 267 billion barrels. Its crude oil reserves are Saudi Arabia's most valuable asset, and Saudi Aramco handling these reserves is possibly the World's most valuable company.

Now crude oil and natural gas have become the most valuable commodities for international trade, and for countries exporting these commodities, the quantity of their reserves of these commodities has become a hallmark of their economic status. For Governments of countries among these who have some sea shore, this has provided a great incentive to explore for crude oil and natural gas reserves beyond their land border in their offshore areas. This has led to the development of *offshore drilling*, which has now become a thriving multi-billion dollar industry in several areas of the world. However offshore drilling is much more expensive compared to conventional drilling on land.

Developing an area for hydrocarbons production begins with a complex process of geological surveys and analysis, seismic exploration, sedimentary basin analysis and reservoir characterization (mainly in terms of the porosity and permeability of geologic reservoir structures) to identify underground rocks enriched with hydrocarbons indicating the presence of a reservoir of these hydrocarbons underground. Once such an area is identified, step-out wells drilled from mobile drilling rigs are used to determine the size and other characteristics of the field, and establish a reservoir model and determine the locations of *sweet spots* or suitable sites for production wells. By means of reservoir simulations, estimates of the total reserves and production as a function of time for a well at each of those sites, are obtained. A reservoir simulation is a computer model of underground rock and fluid conditions, which helps predict oil and gas recovery profile from a well at a target location. Ofcourse this prediction depends on many factors like reservoir characteristics, flow properties, size, pressure near the well site, and the number of other wells already producing from the reservoir.

Th information obtained from this activity is used to decide the locations for drilling of production wells, or *targets*. A typical 3×3 mile area of an offshore field will have between 25 to 300 production wells.

In offshore crude oil production, drilling of production wells is carried out by *drilling rigs* from either *fixed platforms* positioned on the seabed, or *sub-sea templates* placed on the seabed with flowline to a processing platform (for descriptions of these see Chapter 13 in reference [10]), or *bottom jackup rigs*, or *combined drilling and production facilities* with either bottom grounded or floating platforms, or *deep-water mobile offshore units* including semi-submersibles and drill ships. The choice of technology depends on parameters like the size of the field, water depth, and bottom conditions. In relatively shallow waters like the Gulf of Arabia (water depth upto 390 feet) a jackup rig (a self-elevating drilling platform with attached legs that can be lowered into the ocean floor and the hull raised above the sea level well above ocean waves and offering a relatively steady and motion-free platform for drilling) is most suitable.

A jackup rig can also be moved from one location to another, however moving takes time and money, as all the equipment on top of the platform has to be taken down for this mobilization.

Drilling an oil/gas well is carried out in phases. At the end of each phase, a steel tube called *casing* is placed and cemented inside the drilled hole, and the next phase begins. As a result, each well drilled will be completed with several cemented tubulars inside each other all the way to the targeted depth. Once drilling the well is completed, a *wellhead* is installed in it on top of the sea floor where it will be connected to the production pipeline.

Improvements in drilling technology have led to several improvements in drilling practices. Today we are not limited to drilling a well vertically all the way to the bottom. With *directional drilling* it is now possible to drill deviated up to a horizontal direction. Directional drilling allows several oil producer wellheads to be grouped together and drill all of them with the jackup rig fixed in one location; and it allows increased exposure to the reservoir rock.

In directional drilling, drilling of a well begins in the vertical direction first, then at a point of specific depth known as *kickoff point* the well inclination begins to increase. With this directional drilling practice, it is not necessary to move the rig to each well location to drill it; it is quite possible to group several sweet spots where wells have to be drilled into a group and drill all of them one after the other with the rig fixed in a central location with all of them around that location.

4.1 Application Involving Clustering problems in offshore drilling

The input for a clustering problem is a set of objects, with relavent data about some characteristics on these objects. The desired output is a partition of the set of objects into disjoint clusters (also called classes or groups) satisfying certain constraints on their cardinality, that minimizes a specified objective function [9].

In this Chapter we discuss an important application of clustering models in offshore oil field development. This section is based on work carried out in developing an offshore oil field located 60 miles off the Eastern coast of Saudi Arabia.

The cost of drilling a production well depends on the distance of the well from the position of the rig and several other factors. Getting this cost exactly is very hard, it is estimated approximately using information from past drilling jobs.

Given the number and locations of production well targets in an offshore oil field to be developed, the probem is to to partition the set of production well targets into clusters or groups, each group to be drilled by a single rig fixed in a single location; and also determine the best location to position the rig, to minimize the total cost of developing the field.This is a clustering problem involving huge sums of money. We discuss the mathematical model used to solve the problem. One parameter that is used for this decision making is:

$D = 10,500$ feet, maximum value of $y =$ distance between the target location to be drilled, and the current position of the rig, for the rig to drill this target without moving.

If $y > D$, then drilling the long lateral position in the horizontal direction is itself more expensive than that of moving the rig to the target location, so this rig in its present position will not be considered for drilling that well. Also a large value of D may be undesirable for other reasons besides its cost. So, the value chosen for D may vary from country to country, hence in an application, find out the preferred value for it; here we illustrate with $D = 10,500$ feet.

When a rig fixed in a position has to drill several target locations around it, it typically drills only one vertical hole, and from that reaches all the target locations it has to drill by drilling horizontal sections. The crude oil being produced from all these target locations has to come up to the ground from the casing installed in that single vertical hole. So, the number of target locations that the rig in its present position can drill is limited by the capacity of that casing to accommodate all the crude oil being produced from these target locations.

Several different cost components contribute to the cost of drilling a distance of length ℓ feet (either horizontal or vertical) using a rig. These various components can be approximated by linear functions of ℓ, obtained by linear regressions based on past drilling data from a field in the Gulf of Arabia, but may vary from field to field.

(i): Daily working rates paid to the drilling contractor: In crude oil production, the daily rate is the amount a drilling contractor gets paid by the oil company, for operating his rig per day. In offshore drilling, the daily rates of drilling rigs vary by their capability, and the market availability at the time the contract for the rig is signed.

The number of days required to drill a well of length ℓ feet has been estimated to be: $0.0147\ell + 10.081$

(ii): Cementing: Cement is used to hold casing in place and to prevent fluid migration between subsurface formations. The price of cementing jobs vary based on additives used in mixing, well depth, well geometry, and rock characteristics in the field. From previous wells drilled and completed in this field, cementing cost has been estimated to be : $ 16850 / foot of length.

(iii): Drilling Fluid: Drilling fluids, also referred to as drilling mud, are added to the wellbore to facilitate the drilling process by controling pressure, stabilizing exposed rock, providing buoyancy, and cooling and lubricating while drilling. This also depends on type of fluid used and rock characteristics. From previous wells completed in this field, this cost has been estimated to be: $94671 / foot of length.

(iv): Transportation, overhead, and other services: These expenses refer to transportation of equipment, tools and personnel to and off the rig, spare parts and iron tubes used to run the completion; drilling companies overhead such as office, personnel, fuel, water, logistics, third party vendor services, etc. We will subdivide these costs into three main categories, these are:

Cost of overhead: estimated to be $ 80, 080 per foot of length of the well
Cost of transportation: estimated to be $ 118, 170 per foot of length of the well
Cost of services: estimated to be $ 280, 280 per foot of length of the well.

So, all these costs sum to 590051ℓ + (daily working rates paid to the drilling contractor for $(0.0147\ell + 10.081)$ days).

The total length of the vertical portions to be drilled depends on the number of vertical holes drilled (which depends on the choice of the parameter D defined above), and the depth of the holes to be drilled; which are not factors that we can choose; so in a sense they are fixed. But the total length of the horizontal portions to be drilled depends on the order in which the rigs drill the wells, the optimum order minimizing the sum of the horizontal portions to be drilled by all the rigs can be found using the following procedure.

Let n be the total number of target locations to be drilled, and m the total number of rigs available. Also, let r_k be the drilling rate of the kth rig in feet drilled per hour, $k = 1$ to m.

Let $\mathcal{P}$ be the shortest Hamiltonian path covering all the n target locations to be drilled , with the distance between each pair of these target locations as the cost coefficient corresponding to that pair; i.e., the path of minimum total length containing all these target locations.

Divide $\mathcal{P}$ into m segments with the kth segment $\mathcal{P}_k$ containing the nearest integer to $n(r_k/(r_1 + r_2 + ... + r_m))$ of the n target locations. Then allocate the target locations on the kth segment, $\mathcal{P}_k$ as those to be drilled by the kth rig, $k = 1$ to m; in the order in which they appear on $\mathcal{P}_k$ starting from the first target location on this segment. When the kth rig moves to one of these target locations, it will drill a vertical hole there first, and then it drills the horizontal section in $\mathcal{P}_k$ connecting all the remaining target locations on this segment in that order within distance D of that target location. When the drilling bit of rig k has just reached target location i in this set, it will try to continue drilling the horizontal section to the next target location $i+1$ along the path $\mathcal{P}_k$ if that is feasible. If it is unable to do that due to a bend in the path or for whatever reason, it will return to rig k. From the current position of rig k, it will then drill the horizontal section to that next target location $i + 1$, and continues the same way. After it reaches the last target location in this set, rig k then moves to the next target location remaining in this segment, and continues the same way until all the target locations in this segment are covered.

Software for solving the Shortest Hamiltonian Problem (SHP) is available for download. For example consider an SHP covering s nodes with the matrix $c = (c_{ij})$ of order $s \times s$, where c_{ij} = cost of traveling from node i to node j, for $i, j = 1$ to s. This is equivalent to the Traveling Salesman Problem (TSP) with C of order $(s+1) \times (s+1)$ as the cost matrix, where nodes 1 to s are the same as the nodes in the above SHP, and node $s+1$ is an artificial node with both the costs of traveling to and from node $s+1$ to and from every other node is 0. Given an optimum tour for this TSP, just deleting this artificial node and arcs incident at it, from this tour, leaves the Shortest Hamiltonian Path for our SHP problem. One software package for solving the TSP is the " Concorde TSP Code", the source code for which can be downloaded at:

http://www.math.uwaterloo.ca/tsp/concorde/downloads/downloads.htm

Information on this Concorde TSP solver is available for example in Mathematica. My thanks to William J. Cook, University of Waterloo, for sharing this information with me.

5 A Product mix optimization application at the Saudi ARAMCO company, Sadara Chemicals

Saudi ARAMCO has been the World's largest exporter of crude oil for a long time. In early 2000's they had the idea that instead of exporting crude oil directly, they can make more money by using that crude oil to manufacture and market high value-added chemical products and performance plastics used as chemicals and additives in the oil & gas industry; in making chemicals for water treatment and numerous consumer products like soaps, detergents, cosmetics and other personal care products, fibers, paints, bedsheets, automobile furnishings, adhesives and sealants, break fluids, packaging materials, etc.; and in the energy, transportation, electronics, and furniture industries. In addition to higher revenues, this will also generate over 20,000 quality direct and indirect jobs for Saudi nationals, and also attract several downstream investments.

Of course the manufacture of all these new products will require lots of fresh water, of which Saudi Arabia does not have any natural supply. So if plants are set up to make these new products, their fresh water requirements have to be met using expensive desalinated sea water.

The feeling is that in spite of this, these ventures will be profitable.

The chemical products and performance plastics considered for manufacture from crude oil can be grouped into the following groups: G_1 (ethylene), G_2 (propylene), G_3 (amines), G_4 (propylene glycol), G_5 (polyurethanes), G_6 (isocyanates) , G_7 (glycol ethers), G_8 (polyether polyols), G_9 (polyolefin elastomers), G_{10} (propylene oxide), G_{11} (glycol ethers), and G_{12} (low density polyethylenes).

SEVERAL GROUPS. PLEASE

MAKE LIST COMPLETE AND CORRECT.

The main feedstock for this new company will be naphtha obtained in fractional distillation of crude oil; and ethane obtained both from fractional distillation, and produced as associated gas in crude oil production from the wells. The feedstock will be supplied from the Saudi ARAMCO and its Total Refining and Petrochemical Company (SATROP)'s refinery and other local refineries. Other feedstocks for the propylene oxide unit will be supplied by a new hydrogen peroxide plant being constructed. CO, H_2, ammonia (NH_3) gases for the chemical complex used in the production of aromatics and amines will be supplied by another facility being constructed.

OTHER FEEDSTOCKS NEED TO BE LISTED TO MAKE LIST COMPLETE AND CORRECT.

The process begins by cracking naptha into propylene and ethylene at the cracker units.

Estimated input data for analyzing the profitability of this new venture is shown below. Here the index i represents feedstocks; $i = 1$ to 6 correspond to feedstocks naphtha, ethane, hydrogen peroxide, CO, H_2, and ammonia respectively. The index j represents chemical product group G_j for $j = 1$ to 12. Let

a_{ij} = estimated average tons of input i needed to make one ton of products in group G_j, for $i = 1$ to 6, $j = 1$ to 12.

b_i = estimated tons of input product i expected to be available per month for $i = 1$ to 6.

d_j = maximum sales potential of chemical products in group G_j, i.e., estimate of the maximum expected sales of products in this group per month, $j = 1$ to 12.

c_i^F = estimated average sale price per ton of feedstock i, for $i = 1$ to 6.

c_j^S = estimated average sale price per ton of a chemical product in group G_j, for $j = 1$ to 12.

This data is shown in the following table.

GIVE TABLE OF ABOVE DATA.

Define "Total income" as the income from the monthly sales of all products in all the groups, as a function of x_j = amount in tons of products in group G_j that we sell/month. Then the model for finding $x = (x_j : j = 1 \text{ to } 12)$ to maximize this "Total income" subject to the above data is:

maximize Total income $= z = \sum_{j=1}^{12} c_j^S x_j - \sum_{i=1}^{6} c_i^F (\sum_{j=1}^{12} a_{ij} x_j)$
subject to $\sum_{j=1}^{12} a_{ij} x_j \leq b_i \quad i = 1$ to 6.
and $\quad 0 \leq x_j \leq d_j \quad j = 1$ to 12.

OTHER CONSTRAINTS NEED TO BE INCLUDED TO MAKE MODEL COMPLETE AND CORRECT.

The optimum solution of this model is ..... corresponding to a Total income value of ...... Of course one has to subtract the estimated costs of running these plants from this, to get an idea of the expected profits.

Based on this preliminary analysis, ARAMCO has decided to go ahead with this venture and awarded the contract to Dow Chemical Co. to build these plants in Jubail , Saudi Arabia, and construction has commenced in 2007. By 2016, 26 manufacturing units have been set up with over 3 million tons capacity per year, and the construction is expected to be completed in 2017. When completed the venture is expected to generate revenues of over \$ 10 billion per year. Exports will go to Asia (45 %), Europe (10 %), Middle east (25 %).

Also, encouraged by these results, Saudi Arabia, the World's largest crude oil exporter, made plans to invest in Asian refineries in China, India, Indonesia, Malaysia, and Vietnam; and build new refineries there as part of a plan to almost double its global refining capacity from around 5.4 million barrels a day in 2014, to around 10 million barrels per day globally in 10 years. It already exports crude oil to these countries, and by investing in refining crude oil there, they can ensure that they have plenty of buyers in the fastest growing region for fuel demand; and it will give the Kingdom a key outlet for its crude oil in these countries, and gives it influence in determining pricing of oil products worth billions of dollars each day.

Exercises

1. Find the new optimum solution $x = (x_j : j = 1 \text{ to} 12)$ to maximize the monthly " total income" with the following changes in the data:

(a) : all the data remains the same except the sales prices vectors $c^F = (c_i^F : i = 1 \text{ to } 6)$, $(c^S = (c_j^S : j = 1 \text{ to } 12)$ which changed to $c^F = (.......)$, $c^S = (.......)$.

(b) : all the data remains the same except the vector $b = (b_i : i = 1 \text{ to } 6)$ which has changed to $(.......)$.

(c) : all the data remains the same except the vector of maximum sales potential of chemical products $d = (d_j : j = 1 \text{ to } 12)$, which has changed to $(......)$.

IF POSSIBLE GIVE OTHER EXERCISES SUCH AS FINDING NEW OPTIMUM SOL. WHEN OTHER CHANGES OCCUR SUCH AS WHEN AN EXISTING GROUP IS SPLIT INTO TWO GROUP, WHEN NEW GROUP IS INTRODUCED, etc.; and possibly other exercises. HESHAM'S STUDENT MENTIONED SEVERAL OTHER POSSIBLE INTERESTING EXERCISES TO INCLUDE.

6 Blending Operations in Crude Oil Refineries

The outputs from the CDU (Crude Distillation Unit), and other treatment units in an oil refinery are generally not commercially usable directly. They are only semi-finished products, which can be blended into products meeting various specifications that the market place demands.

There are two classes of blending operations carried out in most refineries. The first is mixing different crude oils to prepare a crude blend with desired properties to feed into the CDU.

Reasons for Blending Crude Oils in Oil Refineries

The processing units in each crude oil refinery are designed to process certain slate of crudes based on their properties, a decision made depending on the availability and cost of crudes at the time the refinery was built. The more consistent the supply of crude oil to the specific refinery , the more that refinery can tailor its operation to that specific crude supply.

PUT A FIGURE HERE WITH CRUDES OF DIFFERENT COLORS, VISCOSITIES.

In its natural, unrefined state crude oil ranges in density and consistency, from very thin, light weight, volatile fluidity; to a very think semi-solid heavy oil. Its color ranges all the way from a light golden yellow to a deep dark black. Oil from different geographical locations has its own very unique properties like viscosity, volatility, color, density, etc. Crude oil is a complex liquid mixture made up of a vast number of hydrocarbon compounds, and small amounts of organic compounds containing sulpher, oxygen, nitrogen and metals such as vanadium, nickel, iron, copper, etc. The commercial price of a crude oil is determined by its property called *gravity*, generally expressed as $API\ gravity =$ (141.5)/(its specific gravity) - 131.5, where specific gravity of a crude oil is (its density)/(density of water) at 60^{0F}. The API gravity of crude oils ranges from < 8.5 for very heavy crudes, 8.5 to 29 for heavy crudes, 29 to 38 for medium crudes, and > 38 for light crudes. There are over 160 different crude oil types traded in the World market these days. The three primary ones among them are:

WTI (West Texas Intermediate): is a premium quality crude oil, greatly valued because more and better gasoline can be made from a single barrel of this, than most other types available on the market. It is a light, sweet (with only 0.24% sulfer) crude oil. Its price in the market is often $ 5 - 7 higher per barrel than OPEC basket price.

Brent Blend: is a combination of different crude oils from 15 fields in the North Sea, it is light and sweet (0.37% sulfer), excellent for making gasoline and the middle distillates, typically priced $ 4 per barrel higher than OPEC basket price.

OPEC Basket: is a group of 7 different crude oils from Algeria, Saudi Arabia, Indonesia, Nigeria, Dubai, Venezuela, and the Mexican Isthmus, not as sweet or as light as the above two groups.

Refineries can accommodate a different crude slate in one of two ways. If the properties of the crude vary only slightly, then process cut points, charge rate, and operating set points of existing units can accommodate the change through modifications in operating conditions. However, when there are more than minor changes in crude properties, refineries require finding a blend of several available crude oils that comes quite close in properties as the current slate.

INCLUDE HERE CRUDE BLENDING EXAMPLES WITH DATA. GIVE EXAMPLES AND EXCERCISES WITH DATA FOR STUDENT PROJECT PROBLEMS. ALSO DISCUSS CLEARLY HOW PROCESS CUT POINTS , CHANGE RATES etc. ARE DETERMINED USING CRUDE PROPERTIES.

The other is mixing different output products from the CDU and other treatment processes in the crude oil refinery; and other additives like octane rating enhancers, metal deactivators, antioxidants, anti-knock agents, rust-inhibitors, detergents, etc. and other components to produce finished products with specified properties and other desired characteristics to sell in the markets.

Products can be blended in-line by injecting proportionate amounts of each component in the main stream where turbulence promotes thorough mixing; or blended by batches in tanks and vessels.

In product blending for example, gasoline is produced by blending a number of products including alkylate (output from alkylation units), reformate (upgraded naphtha resulting from catalytic or thermal reforming), fluid catalytic cracking gasoline, oxygenated additives such as MTBE (Methyl Tertiary Butyl Ether to increase the octane rating), etc.

In this chapter we will discuss the two classes of blending problems, and the mathematical models used for finding optimum blending recipes for each of them; and how optimum solutions are computed for them, and implemented in practice.

Final quality of the outputs from blending are always checked by lab tests before dispatch to the markets. Here are brief descriptions of these properties from [12].

Pour point (PP) of an oil product is defined as the lowest temperature at which a sample of this oil product will flow. It indicates how difficult it is to pump this oil product in cold weather. Lower pour point means that the paraffin content in it is low.

Viscosity (VIS) of an oil product determines the resistence of this oil product to flow. More viscous oils create a greater prefure drop when they flow in pipes.

Refractory index of an oil product is the ratio (velocity of light in a vacuum)/(velocity of light in the oil product), used as a characterization parameter for the products composition.

Freezing point of an oil product is the temperature at which the hydrocarbon mixture solidifies at atmospheric pressure, an important property for kerosene and jet fuels due to the very low temperatures encountered at high altitudes by jet planes.

Aniline point (AP) of an oil product is the lowest temperature at which an equal volume mixture of the oil product and aniline are miscible. Since aniline is an aromatic compound, a petroleum product with high aromatic content will be miscible at ambient conditions. Oil products with more paraffins will have higher aniline points. This is an important specification for diesel fuels.

Flash point of a liquid hydrocarbon mixture is the lowest temperature at which sufficient vapors are produced above the liquid to cause a spontaneous ignition in the presence of a spark. An important specification for gasoline and naptha related to safety in storage and transport in high temperature environments, and an indicator of fire and explosion potential of a liquid fuel; a lower flash point fuel is a higher fire hazard.

Octane number (ON) is a measure of knocking tendency of gasoline fuels in spark ignition engines, is measured under two environments. MON (Motor Octane Number) indicates engine performance at highway conditions with high speeds, RON (Research Octane Number) is indicative of performance at low speed city driving. The PON (Posted Octane Number) is the average of MON and RON.

Cetane number (CN) of a diesel fuel is the percentage of pure cetane (n-hexadecane) in a blend of cetane and alpha methyl naphthalene which matches the ignition quality of the diesel fuel sample.

Smoke point is the maximum height in mm, of a smokeless flame of fuel, it measures the burning qualities of kerosene and jet fuel.

Reid vapor pressure (RVP) of a liquid oil product is the vapor pressure determined in a volume of air four times the liquid volume at 37.8^{0c} ($= 100^{0F}$). It measures the vapor-lock tendency of a motor gasoline in which excessive vapors in the fuel line causing interruption of the supply of liquid fuel to the engine. It indicates the evaporation and explosion hazards of the fuel.

Blending of output products from the CDU and other treatment processes into finished products for the marketplace

Output products from treatment processes are blended into finished products like gasoline, kerosenes, diesel oil, etc. for the marketplace. Here we discuss the mathematical models used to determine the optimum blending recipes for making these finished products to meet the specifications on various properties on them.

Gasolines are checked for octane rating, Reid Vapor Pressure (RVP), and volatility. Kerosenes are tested for flash point, volatility. Gas oils (middle distillate petroleum fractions with a boiling point in the range 350-750^{oF} including diesel fuel, kerosene, heating oil, light fuel oil, etc.) are tested for pour point, viscosity, etc.

Properties (such as octane number, specific gravity, RVP, viscosity, flash point, pour point, cloud point, aniline point, etc.) of a blended product can be determined as functions of the corresponding property of the components in the blend, and the quantities of the components in the blend. Consider a blend with the following data corresponding to volumes of the components blended:

n = number of components in the blend.

$i = 1$ to n is the index representing the components in the blend.

x_i: $i = 1$ to n is the proportion (volume fraction) of compound i in the blend. So $\sum_{i=1}^{n} x_i$ $= 1$.

p_i: $i = 1$ to n is the value of a property for component i in the blend.

If the property in linear (i.e., additive) under blending, then the value of the property for the blend will be $\sum_{i=1}^{n} x_i p_i$. Additive properties include specific gravity, boiling point, sulfer content. However, properties like viscosity, flash temperature, pour point, aniline point, RVP, cloud point and octane number are not additive. Petroleum industry has developed techniques for getting estimates of the property values of nonadditive properties of petroleum component blends. One such empirical method developed by Chevron oil trading co. is based on "*blending indices*" , see [12]. These blending indices are specific to the property, for each component in the blend, whether the blending calculations are being performed in terms of proportions by volume, or weight; and for each nonadditive property there are separate blending indices. We will now discuss these blending indices and how to use them for estimating each property of the blend, see [12]. We will show the procedures when blending calculations are performed in terms of proportions by volume. Corresponding procedures exist if the preference is to do these blending calculations using proportions by weight. We will discuss these blending indices (BI) for each property.

Reid Vapor Pressure (RVP) of the blend: RVP of an oil product is the vapor pressure in psi at 100^{0F} determined in a volume of air 4 times the liquid volume, it is not an additive property. The blending index used for RVP, $BI_{RVP,i} = (RVP_i)^{1.25}$ is the blending index for component i in the blend in psi, where RVP_i is the RVP in psi for component i in the blend. And the estimate $BI_{RVP,Blend}$ of the blend is $\sum_{i=1}^{n} x_i BI_{RVP,i}$, and hence the estimate of the RVP of the blend is $(BI_{RVP,Blend})^{1/1.25}$.

Example 1: Consider a blend consisting of components: LSR gasoline, HSR gasoline, Reformate, FCC gasoline with following data. Quantities are measured in BPD (barrels per day).

Quantities and RVP values of components blended

Component $i =$	Quantity (BPD)	x_i	RVP_i in psi	$BI_{RVP,i}$
1. LSR gasoline	5000	0.227	11.1	20.26
2. HSR gasoline	4000	0.182	1.0	1.0
3. Reformate 94 RON	6000	0.273	2.8	3.62
4. FCC gasoline	7000	0.318	13.9	26.84

So here $n = 4$, $p_i = RVP_i$ and $BI_{RVP,i}$ in psi are given in the table. Hence $\sum_{i=1}^{4} x_i BI_{RVP,i}$

= 14.305. Consequently the RVP of the blend is $(14.305)^{1/1.25} = 8.4$ psi.

Example 2: Additives such as i-butane, n-butane can be added to the blend in Example 1 to increase its RVP. Consider adding n-butane with RVP of 52 psi to the blend prepared in Example 1 to get a mixture with RVP = 10 psi. How much n-butane should be added? Let λ denote the BPD of n-butane added to the blend prepared in Example 1 to achieve this goal. Then we have the following data for the new mixture.

Quantities and RVP values of components blended

Component $i =$	Quantity (BPD)	x_i	RVP_i in psi	$BI_{RVP,i}$
1. LSR gasoline	4000	$5000/(22000 + \lambda)$	11.1	20.26
2. HSR gasoline	4000	$4000/(22000 + \lambda)$	1.0	1.0
3. Reformate 94 RON	6000	$6000/(22000 + \lambda)$	2.8	3.62
4. FCC gasoline	7000	$7000/(22000 + \lambda)$	13.9	26.84
5. n-butane	λ	$\lambda/(22000 + \lambda)$	52	139.64

So, the requirement that the RVP of the mixture should be 10 leads to the equation

$(10)^{1.25} = (5000 \times 20.26 + 4000 \times 1.0 + 6000 \times 3.62 + 7000 \times 26.84 + \lambda \times 139.64)/(22000 + \lambda)$

i.e., $10^{1.25}(22000 + \lambda) = 314912 + 139.64 \times \lambda$

This leads to $\lambda = 625.7$ BPD which is the amount of n-butane to add to the blend in Example 1 to bring the RVP of the mixture to 10 psi.

INCLUDE HERE RVP COMPUTATION of PRODUCT BLEND EXAMPLES WITH DATA. GIVE EXAMPLES AND EXCERSISES WITH DATA FOR STUDENT PROJECT PROBLEMS.

Pour Point (PP) of the Blend: The Pour Point is the lowest temperature at which an oil product can be stored and it is still capable of flowing or or pouring without stirring, when it is cooled. It is not an additive property , and so PP blending indices are used which blend linearly when quantities of components in the blend are measured on a volume basis. Let PP_i be the pour point in ^{0C} of the ith component in the blend. First convert all these PP temperatures into the Kelvin scale and measure them in ^{0R}. Then the blending index $BI_{PP,i}$ of component i in the blend in ^{0R} is

$BI_{PP,i} = 3262000(PP_i/1000)^{12.5}$

The estimate of the blending index, $BI_{PP,Blend} = \sum_{i=1}^{n} x_i BI_{PP,i}$ and hence the estimate of the PP_{Blend} is $1000[(BI_{PP,Blend})/3262000]^{1/12.5}$ in Kelvin ^{0R}. Convert it back into ^{0C} yielding the estimate of the PP of the blend in ^{0C}.

Example 3: Consider a blend consisting of 4 components with data given in the following table.

Quantities and PP values of components blended

Component $i =$	Quantity (BPD)	x_i	PP_i in ^{0C}	PP in ^{0R}	$BI_{PP,i}$
1. Catalytic cracked gas oil	2000	0.1818	-15		227.3
2. Straight run gas oil	3000	0.2727	-3		461.0
3. Light vacuum gas oil	5000	0.4545	42		2748
4. Heavy vacuum gas oil	1000	0.0909	45		3093

So, $\sum_{i=1}^{4} x_i BI_{PP,i} = 1697.2$. So, $PP_{Blend} = [(1697.2)/(3262000)]^{1/12.5} \times 1000 = 546^{0R}$. Converting back to ^{oC} we get $PP_{Blend} = 30^{oC} = 86^{oF}$.

Note: An alternate formula for $BI_{PP,i}$ is also being used in the industry in recent times, it is $BT_{PP,i} = (PP_i)^{1/(0.08)}$. As an exercise compute the PP_{Blend} in this example using this formula.

THIS IS FROM PAGES 242-243 IN [12]. WHAT IS ^{oR}. PLEASE CHECK AND CHECK WHETHER CALCULATIONS IN THIS EXAMPLE 3 ARE CORRECT. ALSO PLEASE FILL EMPTY COLUMN IN EXAMPLE 3.

Cloud Point (CP) of the Blend: Cloud Point (CP) of an oil product is the lowest temperature at which the oil becomes cloudy and the first particles of wax crystals are observed as the oil is cooled gradually under normal conditions. IT is not an additive property so CP blending indices are used, which blend linearly when quantities of components included in the blend are measured on a volume basis, to estimate the CP of the blend.

We are given the CP of component i included in the blend , for $i = 1$ to n in oC. Convert it into the Kelvin scale, and let CP_i be the cloud point of component i in the blend in ^{oK}. Cloud point is not an additive property, so we use cloud point blending indices, $BI_{CP,i}$, which blend linearly when quantities of components included in the blend are given on a volume basis. The $BI_{CP,i} = (CP_i)^{1/0.05}$. So $BT_{CP,Blend} = \sum_{i=1}^{n} x_i BI_{CP,i}$.

From this we can calculate $CP_{Blend} = (BI_{CP,i})^{0.05}$, in ^{oK}, and we convert it into ^{oC}.

Example 4: Consider a blend with 3 components with the following data.

Quantities and CP values of components blended

Component $i =$	Quantity (BPD)	x_i	CP_i in ^{oF}	CP_i in ^{oK}	$BI_{CP,i}$
1. A	1000	0.167	-5	258.0	1.101×10^{48}
2.B	2000	0.333	5	258.0	1.70110^{48}
3.C	3000	0.5	10	260.8	2.119×10^{48}

So $BI_{CP,Blend} = \sum_{i=1}^{3} x_i BI_{CP,i} = 1.8098 \times 10^{48}$. So, $BI_{Blend} = (1.8098 \times 10^{48})^{0.05}$ in $^{oK} = 258.756^{oK} = -14.25^{oC} = 6.35^{oF}$.

Aniline Point (AP) of the Blend: AP of an oil is the minimum temperature at which equal volumes of that oil and pure aniline are completely miscible, and indicates the degree of aromaticity of the oil. It is not an additive property, and so blending indices which blend linearly when proportions of components in the blend are given on a volume basis.

Let AP_i be the AP of component i in the blend in ^{oC} . Then the AP of the i-th component in the blend , $BI_{AP,i} = 1.124[exp(0.00657AP_i)]$. Then $BI_{AP,Blend}$, the blending index of the blend $= \sum_{i=1}^{n} x_i BI_{AP,i}$, where x_i is the proportion by volume of the ith component in the blend. And $AP_{Blend} = log_n(BI_{AP,BLend}/1.124)/0.00657$ in ^{oC}.

Example 5: Consider a blend of 3 components with the following data:

Quantities and AP values of components blended

Component $i =$	Quantity (BPD)	x_i	AP_i in ^{oC}	$BI_{AP,i}$
1. Light Diesel	4000	0.4	71.0	1.792
2.Kerosene	3000	0.3	60.7	1.675
3.Light cycle gas oil	3000	0.3	36.8	1.432

Then $BI_{AP,Blend} = \sum_{i=1}^{3} x_i BI_{AP,i} = 1.6468$. So, $AP_{Blend} = log_n(\sum_{i=1}^{3} x_i BI_{AP,i}/1.124)/0.00657$ $= 58.17^{oC}$.

Smoke Point (SP) of the Blend: SP of an oil is the maximum flame height in mm at which the oil burns without smoking when tested at specified standard conditions.

Consider the blend in terms of volume fractions discussed earlier under Aniline point of the blend in Example 5. The SP of that blend in mm is SP_{Blend} determined using the following formula:

$$SP_{Blend} = -255.26 + 2.04AP_{Blend} - 240.8log_n(SG_{Blend}) + 7727(SG_{Blend}/AP_{Blend})$$

where the AP_{Blend}, SG_{Blend} are the aniline point and specific gravity respectively of that blend. The specific gravity is an additive property, hence the specific gravity of a given blend with component fractions by volume in the blend x_i, and specific gravities of components as SG_i, is $SG_{Blend} = \sum_{i=1}^{n} x_i SG_i$.

Example 6: The Specific Gravities of components in the blend discussed in Example 5, SG_i, are 0.75, 0.8, 0.8 respectively. So the Specific gravity of that blend is $SG_{Blend} = 0.4(0.75) + 0.3(0.8) + 0.3(0.85) = 0.805$, and the smoke point of that blend $SP_{Blend} = -255.26 + 2.04(56.54) - 240.8log_n(0.805) + 7727(0.805/56.54) = 22.33$ mm.

Viscosity of the blend: Viscosity is not an additive property, so Viscosity blending indices $BI_{Vis,i}$ are used for determining the viscosity of a blend, V_{Blend}. Here the blending index of component i in the blend in terms of its proportion by volume measured in cSt, v_i , is $BI_{Vis,i} = (log_{10}v_i)/(3 + log_{10}v_i)$. Then

$BI_{Vis,Blend} = \sum_{i=1}^{n} x_i BI_{Vis,i}$

and the viscosity of the blend is $v_{Blend} = 10^{3(BI_{Vis,Blend})/(1-BI_{Vis,Blend}))}$ in cSt.

Example 7: Consider a blend of 3 components with the following data:

Quantities and Viscosity values of components blended

Component $i =$	Quantity (BPD)	x_i	V_i in cSt	$BI_{Vis,i}$
1 Fraction 1l	2000	0.222	75	1.385
2. Fraction 2	3000	0.333	100	0.400
3. Fraction 3	4000	0.445	200	0.434

Here blending index $BI_{Blend} = \sum_{i=1}^{3} x_i BI_{Vis,i} = 0.4117$. So, the viscosity of the blend is $V_{Blend} = 10^{3(0.4117)/(1-0.4117)} = 125.73$ cSt.

Flash Point (FP) of the blend: The flash point of an oil is the lowest temperature at which the vapors coming from the oil ignite. It indicates the maximum temperature at which the oil can be stored safely without causing a serious hazard. If the flash point of a petroleum product is considered to be high, it can be blended with other fractions to lower it. FP is not an additive property, and so flash point blending indices are used which blend linearly when volume fractions of components in the blend are used. Let x_i, FP_i be the volume fraction, flash point temperature of the ith component in the blend in ^{oK}. Then the blending index $BI_{FP,i} = FP_i^{1/(-0.06)}$.

Example 8: Consider a blend of 3 components A, B, C with the following data:

Quantities and Flash point values of components blended

Component $i =$	Quantity (BPD)	x_i	FP_i in ^{oK}	$BI_{FP,i}$
1 A	2500		120	
2 B	3750		100	
3 C	5000		150	

COMPLETE THS EXAMPLE.

Octane Number (ON) of a Gasoline Blend: The Octane Number (ON) is an important characteristic of fuels used for spark ignition engines. ON is a measure of the fuel's tendency to "knock" in an engine test. Post octane number (PON) of a fuel, also known as the road octane number, is the average of the fuel's RON and MON.

There are several additives such as oxygenated ethers or alcohols that can be added to gasoline to enhance its octane number. Some like tetra-ethyl lead used in the past have now been phased out, but other oxygenates like methanol, ethanol, MTBE (Methyl-tertiary-butyl ether), ETBE (Ethyl-tertiary-butyl-ether), TBA (Tertiary-butyl alcohol), TAME (Tertiary-amyl-methyl ether) are still being used in practice for this purpose.

The octane number discussed below, ON, may be either RON, MON, or PON, the procedure described below for estimating the octane number of a blend with the given data applies to all of them.

Let ON_i, x_i be respectively the octane number, the fraction by volume of the i-th component in the blend. If the octane number is an additive property under blending in terms of volume fraction, the octane number of the blend ON_{Blend} will be $= \sum_{i=1}^{n} x_i ON_i$. But in practice it has been observed that this estimate needs minor corrections to get the true value. Blending indices for components in the blend are used to get a closer estimate for the octane number , ON_{Blend} , of the blend.

The blending index of a component in the blend, is a function BI_{ON} of its octane number ON. This blending index function BI_{ON} is a piecewise nonlinear function of ON. The range of values of ON is divided into 3 intervals and in each interval BI_{ON} is defined as given below:

$11 \leq ON \leq 76, \quad BI_{ON} = f_1(ON) = 36.01 + 38.33(ON/100) - 99.8(ON/100)^2 + 341.3(ON/100)^3 - 507.02(ON/100)^4 + 268.64(ON/100)^5$

$76 \leq ON \leq 103, \quad BI_{ON} = f_2(ON) = -299.5 + 1272(ON/100) - 1552.9(ON/100)^2 + 651(ON/100)^3$

$103 \leq ON \leq 106, BI_{ON} = f_3(ON) = 2206.3 - 4313.64(ON/100) + 2178.57(ON/100)^2$

Let R_1, R_2, R_3 be the interval of values of BI_{ON} corresponding to the three intervals of ON above.

Then the blending index for the blend, $BI_{ON,Blend} = \sum_{i=1}^{n} x_i BI_{ON_i}$. To get the estimate ON_{Blend} of the octane number of the blend , we find which of the three intervals R_1, R_2, R_3

contains the value of $BI_{ON,Blend}$ and if it is R_j , then ON_{Blend} is the value of ON that satisfies $f_j(ON) = BI_{ON,Blend}$.

Example 9: Consider a blend of 3 components A, B, C with the following data:

Quantities and RON values of components blended

Component i	Quantity (BPD)	x_i	RON_i	$BI_{RON,i}$
1 Alkylate $c_4^=$	6000	0.4	97.3	67.66
2 Coker gasoline	4000	0.267	67.2	53.65
3 FCC gasoline	5000	0.333	83.2	58.78

If the octane number is an additive property under blending in terms of volume fraction, the octane number of the blend ON_{Blend} will be $= \sum_{i=1}^{n} x_i ON_i = 84.57$

Using the blending index method, $BI_{RON,Blend} = 0.4(67.66) + 0.267(53.65) + 0.333(58.78) = 60.98$. 60.98 is contained in the interval R_2 in this example. So, RON_{Blend} is the solution of $f_2(RON) = 60.98$, which leads to the estimate $RON_{Blend} = 88.09$.

Example 10: Consider the blending problem discussed in Example 9. Suppose it is required to produce a gasoline with a RON of 95 by adding the oxygenate MTBE with RON of 115 to the blend in Example 9. Find out how much MTBE should be added to that blend.

Let V_{MTBE} denote the BPD of MTBE added to the blend discussed in Example 9 to meet this requirement; and let BLEND denote this new blend. So V_{BLEND}, the total volume of this BLEND is $15000 + V_{MTBE}$ BPD.

We want the RON value of BLEND to be 95; i.e., using the blending index method we want $BI_{RON,BLEND}$ to be $f_2(95) = 65.56$. Since MTBE has a RON of 115, we get $BI_{RON,MTBE} = f_3(115) = 126.77$. So, the requirement $BI_{RON,BLEND}$= 65.56 becomes the equation

$$67.76(6000/V_{BLEND})+53.65(4000/V_{BLEND})+58.78(5000/V_{BLEND})+126.77(V_{MTBE}/V_{BLEND}) = 65.56$$

or $65.56(15000 + V_{MTBE}) = 6000(67.66) + 4000(53.65) + 5000(58.78) + 126.77V_{MTBE}$

leading to $V_{MTBE} = 1126$ BPD.

Method to find an optimum blending recipe using blending indices

Here we will consider the problem of finding an optimum blending recipe to blend components from a given set of petroleum components, to prepare a blend satisfying specified constraints on various properties like octane number etc., using blending indices. A solution called a *blend* is a vector $x = (x_i)$ satisfying $\sum_{i=1}^{n} x_i = 1$, where x_i is the proportion by volume of component i in the blend; and these are the decision variables.

Given an initial blend $\bar{x}$, may be heuristically generated, compute blending indices corresponding to it while computing for $\bar{x}$ the values of various properties on which we have constraints specified. Use those blending indices values to express the bound constraints on those properties as linear constraints on input fractions x_i of various components in a general blend x. Also include other constraints on availabilities of components, etc. This way we get a linear programming model for finding the optimum blend with minimum cost with the given data.

Using the optimum solution of that LP as an initial blend, repeat the above work. This can be repeated a few times until changes in the objective value come within an accepted to lerance. The final solution is the blending recipe to implement.

PLEASE CHECK HOW WELL THIS METHOD WORKS ON REAL DATA. ALSO ADD EXAMPLES WITH GOOD DATA, AND PROJECT EXERCISES FOR STUDENTS WITH DATA.

7 Production planning in a crude oil refinery

Production planning in a crude oil refinery deals with determining how much crude oil to process in the CDU, and the quantities of various products to produce to meet the expected demand for them in each planning period. Oil refineries deal with a lot of products, which can be classified into 3 classes:

Class 1 products are the products, fractions in crude oil, which are the direct outputs from fractional distillation of crude oils.

Class 2 products include all Class 1 products which are marketed as they are; and products obtained from further processing (through one or more processes like cracking, vacuum distillation, solvent dewaxing, catalytic reforming, distillate hydrotreating, coking, dimerization, alkylation, etc.). Some Class 1 products like diesel oil, undergo some treatment ; diesel oil from CDU undergoes desulfurization in distillate hydrotreater; but continues to be called by the same name diesel oil, even after the treatment. For such products, we will label the product in Classes 1, 2 as diesel oil_1, diesel oil_2.

Class 3 products include all Class 2 products which are marketed as they are, and all other products that are produced by blending two are more Class 2 products and then sold in the market.

Here is the notation that we will use in this section.

k = number of different types of crude oils processed during the planning period

ℓ, m, n = numbers of different Classes 1, 2, 3 products produced respectively; during the planning period.

g, h, i, j = indices used to denote different crude oils used, Classes 1, 2, 3 products respectively; during the planning period.

$a_1^0, ..., a_k^0$ = amount in barrels of different types crude oil available for refining during the planning period.

p_{gh}^{01} = units (barrels or tons) of Class 1 product h produced in CDU per barrel of crude oil g.

p_{hi}^{12} = units (barrels or tons) of Class 2 product i produced when a unit of Class 1 product h goes through treatments post-CDU.

J_i^{23} = set of Class 3 products whose optimal blend contains the i-th Class 2 product.

I_j^{23} = set of Class 2 products in the optimal blend for the j-th Class 3 product.

b_i^{j2} = proportion of the ith Class 2 product in the optimal blend for the j-th Class 3 product, found using the procedure discussed in the previous chapter. So, One unit of the jth Class 3 product = $\sum_{i \in I_j^{23}} [b_i^{j2}(\text{one unit of the } i\text{-th Class 2 product})]$.

Then

a^1_h = units of the hth Class 1 product available in this period is $\sum_{g=1}^{k} [\, a^0_g p^{01}_{gh}]$ for $h =$ 1 to ℓ.

a^2_i = units of the ith Class 2 product available in this period is $\sum_{h=1}^{\ell} [\, p^{12}_{hi} a^1_h]$ for $i = 1$ to m.

Let

$\bar{a}^3_j$ be the maximum quantity in units of the jth Class 3 product that can be sold in the market during this period, for $j = 1$ to n.

$\bar{c}^3_j$ be the price in $ per unit of the jth Class 3 product sold, for $j = 1$ to n.

Then the problem of determining the optimum values y^3_j = units of the jth Class 3 product to produce in this period for $j = 1$ to n is:

$$\begin{array}{lll} \text{Maximize} & \sum_{j=1}^{n} \bar{c}^3_j y^3_j & \\ \quad \text{subject to} & \sum_{j \in J^{23}_i} b^{j2}_i y^3_j \leq a^2_i & \text{for each } i = 1 \text{ to } m \\ \quad \text{and} & y^3_j \geq 0 & \text{for all } j = 1 \text{ to } n. \end{array}$$

Let ($\hat{y}^3_j$: $j = 1$ to n) be an optimum solution for this LP model. Then the quantity of the ith product in Class 2 needed for this is $\sum_{j \in J^{23}_i} b^{j2}_i \hat{y}^3_j = \hat{a}^2_i$ for $i = 1$ to m.

If $\hat{a}^2_i < a^2_i$ for any $i = 1$ to m, then $a^2_i - \hat{a}^2_i$ of this product can be kept in storage for meeting the demand in a future period, or the amount of this product produced can be reduced by this amount, by reducing the Class 1 product h such that $p^{12}_{hi} > 0$ put through post-CDU treatments.

MAY BE GIVE A LIST OF MAJOR CLASS 1, 2, 3 PRODUCTS.

GIVE SOME EXAMPLES USING A MONTH AS THE PLANNING PERIOD. SOME AS PROJECT EXERCISES, ONE WORKED OUT.

STILL NEED TO WORK ON THIS SECTION. MY FEELING IS THAT WE SHOULD

DISCUSS REAL TYPICAL CASES WITH DATA AND DEVELOP THIS SUBJECT THROUGH DISCUSSING THE SOLUTION OF THOSE CASES.

8 Product Sequencing on a Machine Processing Multiple Products in Oil Refining

When a machine processes multiple products during a planning period, the problem of finding the optimum sequence for producing these products on this machine can be formulated as a Minimum cost Hamiltonian Path problem.

WE NEED TO DEVELOP THIS SECTION 9 TO DISCUSS APPLICATIONS OF SEQUENCING AND SCHEDULING MODELS IN OIL REFINING. WE NEED TO FIND OUT WHAT GOOD HEURISTICS ARE USED FOR THESE PROBLEMS IN INDUSTRY, AND COMPARE THESE WITH THE ABOVE FORMULATION.

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