

## Preface

The earliest instance of a network model for a routing problem seems to have been constructed by L. Euler in his solution of the Konigsberg bridges problem in the early 18<sup>th</sup> century.

Initially, theoretical aspects of networks were developed by mathematicians under the name of graph theory. It was not until the 1940's that network models were used to study transportation and distribution problems by F. L. Hitchcock, L. V. Kantorovitch, and T.C. Koopmans. The study of network models got a big boost with the development of linear programming, and particularly after G. B. Dantzig showed that the computational version of the simplex method developed by him in the late 1940s simplifies considerably for the special case of the transportation problem. But the publication in 1962 of the book *Flows in Networks* by L. R. Ford and D. R. Fulkerson is the landmark event after which the study of network flows really took off.

Since then, network optimization has become an established branch of operations research that attracted a lot of talented researchers. It has grown rapidly in its sheer theoretical elegance, in its scope, and in its range of application. Beginning with the 1960s, work on data structures developed by computer scientists has been adopted for improving the performance of implementations of network algorithms. For several network optimization problems, the best implementations available today can solve very large-scale problems with reasonable running times and memory requirements. Because of this, network models and algorithms for solving them, have come into widespread use in many areas.

Flows (of goods, materials, vehicles, people, services, messages, etc.) are an essential component of modern society. In fact, as the Telugu poet V. Rao Dhulipala states, life itself is a manifestation of the flow of blood and of nerve signals.

నెట్వర్కింగ్ కోసం ఘోషం లైన్లు, కంప్యూటర్లు చూడాలా; పుస్తకాలు తిరగేయాలా?

గుండెల్లోనుండి రక్త నాళాల ద్వారా నిరంతరం ప్రవహించే రుధిరం

మెదడు నుండి నాడీ మండలం ద్వారా అనుక్షణం నడిచే వార్తావాహనం

ప్రతిప్రాణి ఒక నెట్వర్క్ సుమా, ఇలలో ఇంతకుమించి వేరొకటి లేదు సుమా!

(For flows and networks, does one have to look at modern innovations like phone lines and computer systems?, nay, these things are as old as life itself; for, the essence of life is nothing but the flow of the red fluid through blood vessels, and of nerve signals through the nervous system).

Network models find many applications in controlling the various flows in our society. The application of network algorithms to solve these models benefits humanity

enormously by optimizing the distribution, communication, construction, and transportation costs.

Thus, network flow models are a special class of mathematical programs for which very efficient algorithms exist. This special network methodology constitutes a fundamental pillar of optimization. New developments are occurring at a fast pace due to the dramatic increase in research efforts devoted to network problems. This makes the study of network flows very interesting and exciting.

The primary aim of this book is to cover the significant advances in network flow methods ranging across modeling, applications, algorithms, their implementations and computational complexity. While almost all real world problems are essentially nonlinear, the importance of linear models stems from their ability to provide a reasonable approximation to many problems. This point is brought home by the well-known engineering proverb that exhorts researchers to **be wise, linearize**. This book deals with problems on network structures that can be handled by linear programming techniques or their adaptations.

Some familiarity with matrix algebra (which can be gained from an undergraduate course), especially the concept of linear independence and the pivotal methods for solving systems of linear equations is necessary. Also some knowledge of linear programming (LP) is needed. The basics of LP are reviewed in Chapter 1.

Chapter 1 presents definitions of all the important network concepts which form the basis for the material developed in the rest of the book, and several formulation examples of various types of network models. At the end of this chapter, we discuss a formulation of the Chinese postman problem (an important routing problem with many applications) as a minimum cost perfect matching problem. We also discuss algorithms for finding an Euler route in an Eulerian network, which are needed to solve the Chinese postman problem, once the solution of the corresponding minimum cost perfect matching problem is given. This chapter concludes with several formulation exercises from a variety of areas of application of network models.

Chapter 2 presents the maximum value flow problem in pure single commodity flow networks. Various algorithms for this problem are discussed, and their computational complexity is analyzed. Methods for doing sensitivity analysis in this problem are also discussed.

Chapter 3 covers the Hungarian and signature methods for assignment problems, and the classical primal-dual method for the transportation problem together with the polynomially bounded version of it obtained by scaling.

Chapter 4 discusses various algorithms for several types of shortest chain problems and their applications.

Chapter 5 presents algorithms for minimum cost flow or circulation problems in pure networks, based on a variety of approaches. We also present a recently developed strongly polynomial algorithm for minimum cost circulation problems.

Chapter 6 covers an efficient structured programming implementation of the primal simplex algorithm applied to a pure single commodity network flow problem subject to additional linear constraints.

Chapter 7 deals with applications of network algorithms in scheduling the various jobs in a project according to the precedence constraints. Discussed here are critical path methods (CPM), and methods for finding the project shortening cost curve.

In Chapter 8 we discuss efficient implementations of the primal simplex method for solving generalized network flow problems using rooted loop labelings.

Chapter 9 presents various algorithms for finding minimum cost spanning trees in undirected networks.

And, in Chapter 10, we present blossom algorithms for matching, and matching/edge covering problems.

Most chapters contain many exercises including formulation problems from a variety of application areas.

This book is ideally suited for a second semester course in the first year of graduate programs dealing with mathematical programming, following an introductory course on linear programming. Such courses are offered in disciplines such as operations research, industrial engineering, other branches of engineering, or business schools. It could also serve as a text for a course on algorithmic graph theory in mathematics or computer science programs or as a supplemental book in the study of computational complexity and algorithms. Portions of this book can be used to supplement the material in other courses (such as Chapters 1 and 7 in a course on CPM in civil engineering).

I am deeply indebted to several friends and colleagues for their help in preparing this book. The one person whose explanations and insights helped me enormously is Santosh Kabadi. He reviewed several revisions of this manuscript, pointed out errors and ways of correcting them, and suggested many improvements. I also learnt a great deal about network flows and matchings through many discussions with R. Chandrasekaran and Clovis Perin, and from the subtle questions raised by enquiring students in my classes. The comments of many people including Abdo Alfakih, Mohammed Partovi, Horst Hamacher, Tamas Solymosi, Ghanta Subbarao, Vasant Tikekar, and Paul Tseng have been very helpful. The intellectually stimulating environment in our department at the University of Michigan in Ann Arbor has motivated me to think and work harder. I am grateful to Bala Guthy, Teresa Lam, Jolene Glaspie, and Ruby

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Other books by Katta G. Murty:

*Linear and Combinatorial Programming* first published in 1976, available from R. E. Krieger Publishing Co., Inc., P. O. Box 9542, Melbourne, FL 32901.

*Linear Programming*, published in 1983, available from John Wiley & Sons, Inc. Publishers, 605 Third Avenue, New York, NY 10158.

*Linear Complementarity, Linear and Nonlinear Programming*, published in 1988, available from Heldermann Verlag, Nassauische Str.26, D-1000 Berlin 31, Germany. Now can be accessed at the public website:

[http://ioe.engin.umich.edu/people/fac/books/murty/linear\\_complementarity\\_webbook/](http://ioe.engin.umich.edu/people/fac/books/murty/linear_complementarity_webbook/)

Other books prepared recently in self-learning style can be accessed on the web through his webpage, whose address is given at the bottom of this preface.

## Preface for the Internet Edition

This is one book where the blossom algorithms for 1-edge matching/covering problems are discussed in clear detail. To make the book widely accessible, I have decided to make this available at this public website on the occasion of my 70th birthday later this year.

Several people have expressed an interest in having the book available also in hard copy form in the market. So, Prentice Hall continues to publish the hard copy form of the book, but have permitted to make the PDF files of the book available at this website. My thanks to them.

In preparing this book for the internet, all the figures (over three hundred) had to be resurrected from the old files, and several had to be redrawn. Julia Angstrom, Shital Thekdi, and Professor Fengtien Yu helped me with this effort, and I am grateful to them. For the convenience of those who prefer reading the book on the screen, when the book was typeset for the internet edition, slightly smaller page length than in the print edition was used, with the result that the number of pages in the book has gone up. So the pages in this web edition do not correspond to the same number pages in the print edition. Since the PDF file of each chapter is put on the web as a separate file, each chapter has its own index instead of a single index at the end of the book. Some errors in the print edition have been corrected.

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