Additional Descent Steps in the Sphere Method

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Abstract

In this short note we propose two new descent steps to be used in each iteration of the sphere method discussed in [1], that have the prospect of speeding up that method.

Key words: Sphere method for Linear programming (LP), two new descent steps.

Murty and Oskoorouchi [1] discuss the implementation details of the sphere method for LP, each iteration of which consists of a centering step, and some descent steps. Among these two, the centering step involving some matrix inversion operations, is computationally the more expensive one. Each of the descent steps involves only computing a minimum ratio, which is far cheaper in comparison. So, any strategy that helps to reduce the number of times the centering step has to be carried out, at the expense of carrying out some more descent moves, is very worth while. In this note we suggest two such descent steps for every iteration of this method.

The Additional Descent Steps For Each Iteration of the Sphere Method

The computational results reported in [1] with the Sphere Method indicate that Descent Steps 5.1 yielded the best results among all the five descent steps discussed there. This observation has led us to the following two additional descent steps that seem to hold the promise of yielding even much better results with minimal effort (that of carrying out a few more minimum ratio steps). We describe these new descent steps using the same notation as in [1], which please refer.

D5.2, Descent Steps 5.2: In the General iteration $r+1$, the method comes to this descent step after completing Descent Step 5.1. $\tilde{x}^{r1}$, the best point obtained in Descent Step 5.1 is the initial interior feasible solution for this step.

By the way the descent steps are carried out, it is clear that $\tilde{x}^{r1}$ is close to the boundary of K, and $\delta(\tilde{x}^{r1}) \leq \epsilon_1$. Find the touching set $T(\tilde{x}^{r1})$ = set of all i that tie for the minimum in $\{A_{i.}\tilde{x}^{r1} - b_i : i = 1 \text{ to } m\}$.

For each $i \in T(\tilde{x}^{r1})$, from $\tilde{x}^{r1}$ take a descent step in the GPTC direction $-c^i$ and include the resulting point along with its objective value in a new **List 5.2**.

At the end, let $\tilde{x}^{r2}$ denote the best point in List 5.2 by objective value. If $c\tilde{x}^{r1} - c\tilde{x}^{r2}$ is:

> $\leq$ some selected tolearance for objective value reduction, take $\tilde{x}^{r2}$ as the output of this Descent Step 5.2, put $\tilde{x}^{r2}$ along with its objective value in the List, and go to Descent Step 6.
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> $>$ the selected tolearance for objective value reduction, with $\tilde{x}^{r2}$ as the initial interior feasible solution repeat this Descent Step 5.2; and continue the same way.

D6, Descent Step 6: This step involves solving a 2-variable LP. We will first describe this step assuming that we came here at the end of Descent Step 5.1.

We denoted by $\tilde{x}^{r1}$, the best point obtained in Descent Step 5.1. Suppose this point came up from a descent step from the NTP $\hat{x}^{gr}$ in the descent direction $-c^g$ for some $g \in T(\bar{x}^r)$, where $\bar{x}^r$ is the center in this Iteration $r+1$.

The 2-dimensional plane containing the triangle $\bar{x}^r\ \hat{x}^{gr}\ \tilde{x}^{r1}$ can be represented by $\{x(\alpha, \beta) : \alpha, \beta \in R^1\}$ where $x(\alpha, \beta) = \hat{x}^{gr} + \alpha(A_{g.})^T + \beta c^g$, since $(\hat{x}^{gr} - \bar{x}^r)$, $(\tilde{x}^{r1} - \hat{x}^{gr})$ are multiples of $(A_{g.})^T$, c^g respectively.

The intersection of this 2-dimensional plane with K can be represented by the system of following constraints in variables α, β:

$$Ax(\alpha, \beta) \geq b.$$

Solve the LP in the two variables α, β of minimizing the objective function $cx(\alpha, \beta)$ subject to the above constraints in variables α, β beginning with the its BFS $(\alpha, \beta) = 0$, and let $(\bar{\alpha}, \bar{\beta})$ be an optimum solution of this 2-variable LP. Then the point $\epsilon \bar{x}^r + (1 - \epsilon) x(\bar{\alpha}, \bar{\beta})$ is the interior feasible solution obtained at the end of this descent step, put it in the List along with its objective value.

Suppose we came to this step from Descent Step 5.2. We denoted by $\tilde{x}^{r2}$, the best point obtained in Descent Step 5.2. Suppose this point came up from a descent step from the interior feasible solution $\tilde{\bar{x}}^r$ in the descent direction $-c^g$ for some $g \in T(\tilde{\bar{x}}^r)$. Then this step is the same as above, with the exception that $x(\alpha, \beta) = \tilde{\bar{x}}^r + \alpha (A_{g.})^T + \beta c^g$.

After all these descent steps are carried out, the sphere method goes to the next iteration with the best point in the List at that stage as the initial interior feasible solution.

The computational effectiveness of these new descent steps has not been evaluated yet. If they perform well, they can help reduce the number of centering steps needed. In fact if Descent Step 5.2 gives good results, then we may be able to solve the LP (1) effectively without any centering steps (and consequently no matrix inversion operations), by starting with any interior feasible solution, applying Descent Step 5.1 once, and then applying Descent Steps 5.2, 6 alternately.

[1] K. G. Murty, and M. Oskoorouchi, "Note on Implementing the New Sphere Method for LP Using Matrix Inversions Sparingly", to appear in *Optimization Letters*.