IOE 510 HW #8 Winter 2002 Due. 20. Mar.

#1. Write the duals of the following problems and all the complementary pairs

(a) Maximize $4x_1 - 3x_2 + 8x_3$

Subject to
$$-2 \leqq x_1 \leqq 6$$
$$4 \leqq x_2 \leqq 14$$
$$-12 \leqq x_3 \leqq -8$$

(f) Maximize $z'(x) = -17x_2 + 83x_4 - 8x_5$

Subject to
$$-x_1 - 13x_2 + 45x_3 + 16x_5 - 7x_6 \geqq 107$$
$$3x_3 - 18x_4 + 30x_7 \leqq 81$$
$$4x_1 - 5x_3 + x_6 = -13$$
$$-10 \leqq x_1 \leqq -2$$
$$-3 \leqq x_2 \leqq 17$$
$$16 \leqq x_3 \qquad x_4 \leqq 0$$
$$x_5 \text{ unrestricted} \qquad x_6, x_7 \geqq 0$$

(g) Minimize $z(x) = 3x_1 - 7x_2 + 6x_4 + 5x_5 - x_6$

Subject to
$$5x_1 + 8x_2 + 3x_3 + 3x_4 + 2x_5 + 11x_6 = 200$$
$$5 \leqq x_j \leqq 20 \quad \text{for all } j$$

#2

4.24 Consider the following LP, where A is an $m \times n$ matrix. Write the dual of this problem. Using duality theorem prove that if the problem is feasible, then the minimum value of $z(x)$ in this problem is finite iff c can be expressed as a linear combination of the row vectors of A. Using this prove that either $z(x)$ is a constant or unbounded below on the set of solutions of this problem.

Minimize $z(x) = cx$

Subject to $Ax = b$

#3 Write the dual and the complementary slackness conditions for optimality.

Minimize $z(x) = -2x_1 + 13x_2 + 3x_3 - 2x_4 + 5x_5 + 5x_6 + 10x_7$

Subject to
$$x_1 - x_2 + 4x_4 - x_5 + x_6 - 4x_7 = 5$$
$$x_1 + 7x_4 - 2x_5 + 3x_6 - 3x_7 \geqq -1$$
$$5x_2 + x_3 - x_4 + 2x_5 - x_6 - 2x_7 \leqq 5$$
$$3x_2 + x_3 + x_4 + x_5 + x_6 - x_7 = 2$$
$$x_j \geqq 0 \quad \text{for all } j$$