IOE-510 HW5 Winter 2002 Due 13 Feb 2002.

Formulate 8.5 as a Goal Program.

8.5 Wood pulp is produced by cooking hardwood and bamboo chips mixed in certain proportion in a Kamyr digestor with a cooking liquor consisting of chemicals. Three important quality characteristics of pulp are **k-number, burst factor,** and **breaking length.**

Overcooking results in excess disintegration of chips and low k-number, and consequently rejection of the material. Undercooking leads to high k-number, this will cause increased consumption of the bleaching agent when the pulp is processed into paper. Thus the k-number is the most important pulp characteristic that should be strictly maintained within set limits.

The burst factor and the breaking length of pulp are also important because they determine the strength of the finished paper into which pulp is processed.

The important raw material variables and process variables in the cooking process are

x_1 = Hardwood % (should be between 20 - 40).
x_2 = Upper cooking zone temperature in ^0C (should be between 140 - 175).
x_3 = Lower cooking zone temperature in ^0C (should be between 140 - 173).
x_4 = LP steam pressure in kg/cm^2 (should be between 2 to 4.4).
x_5 = HP steam pressure in kg/cm^2 (should be between 8 to 20.5).
x_6 = Active alkali as Na OH % (should be between 20 - 35).
x_7 = Sulphidity of white liquor in % (should be between 13 - 25).
x_8 = Alkali index number (should be between 12.5 - 18.7).

The output characteristics are

y_1 = k-number, which should be $\geqq$ 16, and desired to be $\leqq$ 18
y_2 = Burst factor, which is desired to be equal to 35 as far as possible
y_3 = Break length, which is desired to be equal to 5000 m as far as possible.

From data collected at the plant over a period of two weeks, the following relationship has been shown to exist between the output characteristics and the raw material variables and process variables.

$$y_1 = 22.84 + 0.06x_1 - 0.05x_2 + 0.004x_3 - 0.67x_4 + 0.24x_5 - 0.13x_6 + 0.19x_7 - 0.18x_8$$
$$y_2 = 38.94 + 0.05x_1 - 0.02x_2 + 0.002x_3 + 1.67x_4 + 0.21x_5 + 0.06x_6 + 0.02x_7 - 0.69x_8$$
$$y_3 = 3273.40 - 24.37x_1 + 9.997x_2 + 8.48x_3 - 268.68x_4 + 120.92x_5 + 67.27x_6 + 27.89x_7 - 138.46x_8.$$

Formulate the problem of determining the best values for the controllable raw material and process variables x_1 to x_8 within their permissible limits, that yield the most desirable values for the output characteristics, as a goal programming problem. Take the penalty coefficients for unit deviation of y_1, y_2, y_3 from their desired ranges to be 10, 7, 4, respectively, in your formulation. Find an optimum solution by solving it using an LP software package and discuss the results. ([S. Sengupta, March 1981]).

2.11 A company manufactures two types of cake mixes A and B using two raw materials $R_1 and R_2$. The following table gives the necessary data.

Raw material	Units needed to make 1 unit of A	Units needed to make 1 unit of B	Units available
R_1	1	2	6000
R_2	2	1	8000
Net profit per unit made	7	5	
Maximum demand	3500	2500	

Formulate the problem of determining how many units of A and B to make, as an LP.

Solve the problem geometrically. Determine the marginal values associated with all the RHS constants in the model. Interpret them.

At this stage, how much extra profit can the company make if the supply of R_1, R_2 is increased by one unit?

A new cake mix developed by the company's kitchen needs 2 units of R_1 two units of R_2 as input per unit. What is the minimum net profit that a uni this new cake mix should make, if it were to be competitive with A, B?

2.19 Solve each of the following LPs geometrically and identify all the possible optimum solutions for each problem.

$$\begin{aligned} \text{Minimize} \quad & -10x_1 - 10x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 4 \\ & x_1 + 2x_2 \geq 2 \\ & 2x_1 + x_2 \geq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} \text{Maximize} \quad & -x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \geq 2 \\ & 2x_1 + x_2 \geq 2 \\ & -x_1 + x_2 \geq 0 \\ & x_1, x_2 \geq 0 \end{aligned}$$