IOE 510 HW # 11 Winter 2002 Due 15. Apr.

#1

Exercises

6.3 Solve the following LPs by the dual simplex method.

(a)

x_1	x_2	x_3	x_4	x_5	x_6	x_7	$-z$	b
1	2	0	−3	0	2	−3	0	−4
0	−1	0	2	1	−1	2	0	1
0	2	1	−3	0	1	−2	0	1
0	4	0	5	0	9	9	1	0

$x_j \geqq 0$ for all j; minimize z.

#2

(b)

Minimize $3x_1 + 4x_2 + 2x_3 + x_4 + 5x_5$

Subject to

$$x_1 - 2x_2 - x_3 + x_4 + x_5 \leqq -3$$
$$-x_1 - x_2 - x_3 + x_4 + x_5 \leqq -2$$
$$x_1 + x_2 - 2x_3 + 2x_4 - 3x_5 \leqq 4$$
$$x_j \geqq 0 \quad \text{for all } j$$

#3

9.16 Consider the family's diet problem discussed in Chapter 4. The original tableau for it is given at the top of page 319. Here x_1 to x_6 are the kilograms of the primary foods 1 to 6 in the family's diet, and x_7, x_8 are the slack variables representing the excess amounts of the nutrients, vitamins A and C, in the diet over the minimum requirements. The basis B_1,

Original Tableau for the Family's Diet Problem

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	$-z$	b
1	0	2	2	1	2	−1	0	0	9
0	1	3	1	3	2	0	−1	0	19
35	30	60	50	27	22	0	0	1	0 (minimize z)

$x_j \geq 0$ for all j.

P.319

associated with the basic vector (x_5, x_6) is optimal to this problem. The optimum inverse tableau is:

Basic Variables	Inverse Tableau			Basic Values
x_5	$-\frac{1}{2}$	$\frac{1}{2}$	0	5
x_6	$\frac{3}{4}$	$-\frac{1}{4}$	0	2
$-z$	−3	−8	1	−179

Answer each of the following questions with respect to this original problem:

(a) Suppose a new primary food, food 7, is available in the market at 88 cents/kg. One kilogram of this food contains two units of vitamin A and four units of vitamin C. Should the family include this new food in its diet? If not, how much should the cost of this food decrease before the family can consider including it in its diet?

At this breakeven cost show that there is an optimum diet that includes food 7 and another that does not.

What is the optimum diet if food 7 is actually available at 32 cents/kg?

(d) What is the marginal effect of increasing the minimum requirement of vitamin C on the cost of the optimal diet in the original family's diet problem? How much can the minimal requirement of vitamin C increase before the basis B_1 becomes nonoptimal to the problem? What is an optimal diet when this requirement is 39 units?

(e) In the original family's diet problem, suppose that each additional unit of vitamin A in the diet is expected to bring an average savings of 10 cents in medical expenses. How does this alter the optimal diet?

(f) For what range of cost per kilogram of primary foods 4 and 5 does the basis B_1 remain optimal to the original family's diet problem?

(g) What happens to the optimal diet in the original family's diet problem if the vitamin C content of food 4 changes? For what range of values of this quantity does the basis B_1 remain optimal? Suppose a richer version of food 4 containing $1 + \alpha$ units of vitamin C per kilogram is available at a cost of $50 + 4\alpha$ cents/kg for any $\alpha \geq 0$. What is the minimum value of α at which it becomes attractive for the family to include it in its diet?