7.7 A company manufactures products $A$ to $G$ using two types of machines $P_1, P_2$; and three raw materials $R_1, R_2, R_3$. Relevant data is given below.

<table>
<thead>
<tr>
<th>Item</th>
<th>Item input (in units) to make one unit of $A$</th>
<th>Item input (in units) to make one unit of $B$</th>
<th>Item input (in units) to make one unit of $C$</th>
<th>Item input (in units) to make one unit of $D$</th>
<th>Item input (in units) to make one unit of $E$</th>
<th>Item input (in units) to make one unit of $F$</th>
<th>Item input (in units) to make one unit of $G$</th>
<th>Max. available per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$ (in units)</td>
<td>0.1 0.3 0.2 0.1 0.2 0.1 0.2</td>
<td>$R_2$</td>
<td>0.2 0.1 0.4 0.2 0.2 0.3 0.4</td>
<td>$R_3$</td>
<td>0.2 0.1 0.1 0.2 0.1 0.2 0.3</td>
<td>$P_1$ time (mc. hrs.)</td>
<td>0.02 0.03 0.01 0.04 0.01 0.02 0.04</td>
<td>$P_2$ time</td>
</tr>
<tr>
<td>Profit ($/unit)</td>
<td>10 12 8 15 18 10 19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Formulate the product mix problem to maximize total daily profit as an LP, and solve it using one of the available software packages to obtain primal and dual optimum solutions. Also, answer each of the following questions about this original problem.

(i) Are the marginal values of the various items well defined in this problem? If so, what are they?

(ii) Is it worth increasing the supply of $R_1$ beyond the present 500 units/day? The current supplier for $R_1$ is unable to supply any more than the current amount. The procurement manager has identified a new supplier for $R_1$, but that supplier's price is $15/unit higher than the current suppliers'. Should additional supplies of $R_1$ be ordered from this new supplier?

(iii) The production manager has identified an arrangement by which 20 hours/day of either $P_1$- or $P_2$-time can be made available at a cost of $150/day. Is it worth accepting this arrangement?

(iv) The sales manager would like to know the relative contributions of the various products in the company's total profit. What are they?

(v) The sales manager believes that product $C$ is priced too low for a good image. This manager claims that if the selling price of $C$ were increased by $2/unit, the demand for it would be 600 units/day. What
is the effect of this change? The production manager claims that the manufacturing process for $G$ can be changed so that its need for $P_1$-time goes down by 50% without affecting quality, demand or selling price. What will be the effect of this change on the optimum product mix and total profit?

(vi) The production manager believes that by changing specifications, it should be possible to make product $B$ with 33.3% less of $R_1$, this would have no effect on the saleability of this product. What will be the effect of this change on the optimum product mix and total profit?

(vii) The company's research division has formulated a new product, $H$, which they believe can yield a profit of $8-10/unit made. The input requirements to make one unit of this product will be

<table>
<thead>
<tr>
<th>Item</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$P_1$-time</th>
<th>$P_2$-time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Is this product worth further consideration?

(viii) The sales manager feels that the selling price/unit of product $F$ can be increased by $2 without affecting the demand for it. Would this lead to any changes in the optimum production plan? What is the effect of this change on the total profit?

(D. C. S. Shearn [1984])
AMPL Model file

set P;
set Item;

param IO {Item, P};
param C {j in P};
param RHS {Item};
param l {j in P};
param u {j in P};

var x {j in P} >= 0;
maximize profit: sum {j in P} C[j] * x[j];
subject to MBEQ {i in Item}: sum {j in P} IO[i,j] * x[j] <= RHS[i];
limit {j in P}: l[j] <= x[j] <= u[j];

AMPL Data file

set P := A B C D E F G;
set Item := R1 R2 R3 P1 P2;
param IO default 0 :=

[R1,A] 0.1 [R1,B] 0.2 [R1,C] 0.2 [R1,D] 0.1
[R₁,E] 0.2 [R₁,F] 0.1 [R₁,G] 0.2
[R₂,A] 0.2 [R₂,B] 0.1 [R₂,C] 0.4 [R₂,D] 0.2
[R₂,E] 0.2 [R₂,F] 0.3 [R₂,G] 0.4
[R₃,A] 0.2 [R₃,B] 0.1 [R₃,C] 0.1 [R₃,D] 0.2
[R₃,E] 0.1 [R₃,F] 0.2 [R₃,G] 0.3
[P₁,A] 0.02 [P₁,B] 0.03 [P₁,C] 0.01 [P₁,D] 0.04
[P₁,E] 0.01 [P₁,F] 0.02 [P₁,G] 0.04
[P₂,A] 0.04 [P₂,C] 0.02 [P₂,D] 0.02 [P₂,E] 0.06
[P₂,F] 0.03 [P₂,G] 0.05;

param l default 0 :=
A 200;

param u default Infinity :=
B 800 E 400;

param RHS :=
R₁ 500 R₂ 750 R₃ 350 P₁ 60 P₂ 80;

param C :=
A 10 B 12 C 8 D 15 E 18 F 10 G 19;