Economic Arguments For Comparative Negligence and The Reasonable Person Standard

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November 6, 2012

Abstract

Rules that apportion monetary damages between tort litigants affect the precautionary levels of individuals engaged in a risky activity. Concurrently, the expected outcome of litigation impacts individual choices on how frequently to engage in the activity. This paper investigates both the direct and indirect effects of cost apportionment rules when individuals face evidentiary uncertainty, arriving at two important results. For a general class of accidents, comparative negligence can induce individuals to take efficient levels of precaution despite evidentiary uncertainty. Drawing on this result, the paper further shows that the efficient level of caution can be induced from both parties with the introduction of uncertainty, via less rigid legal standards, even when it cannot be achieved under perfect information. This in turn leads to the second important result, that given the correct cost apportionment rule some uncertainty is welfare improving as it induces efficient precaution and provides proper incentives for individuals to internalize a portion of the social cost of repeated engagement in the activity.

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1 Introduction

The choice of a cost apportionment rule and standard of care in a tort liability rule determine not only how prudent individual actors are when they engage in an inherently risky activity but also how frequently they choose to engage. The relationship in the bilateral accident setting is simple. Given a liability rule, a would-be injurer and victim each choose a level of precaution, which in equilibrium determines how likely each is to be found liable for damages sustained by the victim were an accident to occur. This expected liability, i.e., each party's expected share of the social cost, in turn becomes part of the calculus as the potential tortfeasors weigh the benefits and cost of engagement in the activity.

The question of how potential injurers and victims will adjust their levels of precaution when facing different cost apportionment rules has received plenty of attention in both perfect and imperfect information settings[1]; and there is a copious amount of scholarly work on the externality issues inherent in the level of engagement problem. However, the questions are often modeled and addressed separately despite being intimately related. This paper considers the problems simultaneously, focusing first on the question of efficient care, then extending the results to the more general problem of care and activity level. Two important questions are considered: Can any mechanism induce efficient care under general conditions and what is the effect of uncertainty on activity level?

Before turning to the problem with uncertainty, a discussion of the problem with perfect information provides the necessary context. The question of efficient care has traditionally been modeled as the solution to a cost minimization problem, with the objective to minimize aggregate private and social cost of the agents engaged in the activity. The problem naturally consists of dueling incentives from the perspective of the individual actors; while precaution is costly, it reduces the likelihood of an accident. Economic analysis of the problem shows that when parties have perfect information about the legal standard and juries are able to perfectly observe their behavior, any of three cost apportionment rules, namely simple, contributory or comparative negligence, will induce socially efficient levels of care from both parties in equilibrium.[2] This result dates back to Shavell's seminal paper on the subject.

[1] Calfee and Craswell (1986); Haddock and Curran (1985); Ulen and Cooter (1984)

[2] This result of course assumes that the legal standard of care is correctly assigned.

Since the objective is not to do away altogether with accidents as complete prevention may be too costly, models of the problem assume some solution to the social planner's objective function where the optimal level of care results in a non-zero probability of an accident. This implies a positive social cost at the optimum. Therefore, a corollary to the finding is that at the equilibrium where efficient levels of cautiousness are exercised by both parties, repeated engagement increases total social cost. The effect of repeated engagement is considered in the more general model where individual agents derive utility from the activity. Assuming that courts cannot monitor the frequency of engagement (and accordingly establish legal standards or quotas), these decisions can be judicially regulated only to the extent that the tort liability rule indirectly affects them. This question has been addressed in the perfect information setting.

The Shavell Theorem[3] shows that there exists no legal mechanism that can provide incentives to both injurer and victim to choose both the efficient level of care and the efficient level of engagement in the activity. Assuming that each chooses the efficient level of care in equilibrium, injurer will always over-engage.[4] This result is the by-product of the perfect information setting. Under any of the three negligence rules, at the equilibrium injurers know with certainty that meeting the legal standard of care bars the victim from recovery.[5] Therefore, her only cost consideration is the reduction in the private cost of care, as her share of the social cost is zero.

While the perfect information setting provides important insights about the efficient design of tort law, in the real world individuals face some form of judicial uncertainty. Uncertainty in the bilateral accident setting can come in the form of imperfect observation of care by juries or alternatively uncertainty about the legal standard of care on the part of individuals engaged in the activity. This paper considers uncertainty as the by-product of the "reason-

[3] Shavell (1987)

[4] As I show later in the paper, in the perfect information setting, in any pure strategy equilibrium, injurers will choose the efficient level of care while victims will be overly cautious whenever social cost is increasing the the injurer's level of activity.

[5] Technically, if the standard of care is set above the efficient level of care with a contributory or comparative negligence rule there is no pure strategy equilibrium, therefore parties do not always face probability 1 or 0. However, Endres and Querner (1995) show that any mixed strategy equilibrium under contributory negligence must be socially inefficient.

able person" standard, the prevalent standard against which the negligence of individuals is measured in American courts.

One of the elements that must be established by a victim seeking recovery for damages in any tort suit is a showing that injurer failed to meet the standard of care owed to the victim. Whether the standard of care has been breached by injurer is typically decided on the level of prudence a reasonable person in the injurer's situation would have exercised. The reasonable person, when engaging in an activity, is to consider the forseeable risk of harm her actions create, the extent of this risk, the likelihood that the risk will cause harm to another person, and alternatives that reduce the risk. The subjective nature of the standard naturally leads to a degree of uncertainty from the perspective of the individual agents, and with it several economic problems arise.

Amongst others, the introduction of uncertainty raises the following key questions:

1. How will would-be injurers and victims adjust their level of care in response to uncertainty under various legal mechanisms?[6]

2. Which mechanism is most efficient in this setting?

3. Can any mechanism induce the efficient level of care under general conditions despite the presence of uncertainty?

4. How will the distortions in level of care affect agents' decisions on how frequently they engage in the risky activity?

5. Is the reasonable person standard, i.e., uncertainty, desirable when the general problem is considered?

The first two questions have been the focus of the literature on the cost minimization problem with imperfect information. Considered are simple negligence, which assigns liability to the injurer when he is negligent, negligence with a defense of contributory negligence, which assigns liability to the injurer when he is negligent and the victim is not, and comparative negligence. Comparative negligence assigns all liability to the injurer when he is negligent

[6] Throughout the paper I use the terms legal mechanism and cost apportionment rule interchangeably.

and the victim is not and some portion of the liability when both parties are negligent according to a "division of liability" rule.[7] Given the different incentives that each rules provides to injurer and victim we would certainly expect the rules to have different distortionary affects on equilibrium choices of care by injurer and victim.

Ulen and Cooter[8] addressed the possibility that one rule is generally more efficient, arguing essentially that since simple negligence causes the largest distortion in injurer's behavior and smallest distortion in victim's and contributory negligence does the opposite, comparative negligence must be most efficient since it causes average levels of distortions for both. This notion was shown to be incorrect by Bar-Gill and Ben-Shahar. Their 2003 paper showed that for a specific division of liability rule, each of the three cost apportionment rules can be "most" efficient depending on the accident prevention technology and the level and distribution of uncertainty. Specifically, the paper showed numerically that for a fixed standard of care and division of liability rule, contributory and comparative negligence outperform simple negligence at low levels of uncertainty while simple negligence is most efficient at high levels.

Two caveats are necessary. The objective of that paper was to show the absence of a general conclusion on the matter, so only one division of liability rule was considered, i.e., a 50/50 rule. Additionally, the numerical simulation fixed the level of care at the socially efficient level without uncertainty. As others have shown in the literature, the optimal level of care differs across liability rules with the introduction of uncertainty.[9]. The conclusion of the paper, however, is supported even when the standard of care and division of liability rule are allowed to vary. In Section 3 I show that even when the legal standard of care is set optimally, either contributory or comparative negligence may be the more efficient regime depending on the accident avoidance technology. I then prove that for a general class of accidents, comparative negligence is most efficient. When considering activities where the effort of either party reduces the likelihood of an accident independently of the other's, con-

[7] The division of liability rule varies amongst US states, with most states adopting a 50/50 division rule. Other versions of the rule include a division that depends on how negligent injurer and victim, with a larger proportion of liability assigned to the party who was more negligent.

[8] Cooter, Robert C., and Thomas S. Ulen. 1986. "An Economic Case for Comparative Negligence," 61 New York University Law Review 1067110.

[9] Bar-Gill and Ben-Shahar (2003); Edlin (1997); Shavell (1987)

tributory and simple negligence cannot induce the efficient equilibrium. Under comparative negligence however, there always exists some sufficiently small level of uncertainty to induce such an equilibrium.

The possibility of an efficient care equilibrium and the welfare implications of uncertainty:

The introduction of uncertainty to the more general problem, where both care and activity level are considered, generates a trade-off. Generally speaking, uncertainty generates deadweight loss as one or both parties choose an inefficient level of care. The magnitude of this additional social cost depends of course on the particulars of the problem, including the level of uncertainty. However, uncertainty has the secondary effect of leaving the injurer with some non-zero ex-ante probability of bearing the cost of the accident. This in turn, induces the injurer to internalize some fraction of the social cost and adjust her level of activity accordingly. The trade-off between the increase in social cost distortion in care and the increase in aggregate utility stemming from a more efficient levels of activity is ambiguous.[10]

What is unambiguously true however is that a cost allocation mechanism that induces efficient levels of caution from both potential injurer and victim yet never guarantees either party fully escape or bear liability should be preferred to one where injurer is guaranteed not to bear any portion of the social cost. No such rule exists in pure strategy equilibria in a perfect information setting.

The paper simultaneously contemplates the social planner's cost minimization problem and the more general welfare maximization problem in a setting with evidentiary uncertainty. Section 2 formally describes the cost minimization problem and its solutions under contributory and simple negligence where parties face no evidentiary uncertainty. For standards of care at or below the efficient level, as identified in Section 2, Section 3 characterizes the set of possible equilibria for sufficiently low levels of uncertainty under both contributory and simple negligence. Throughout Section 3, I assume that parties' actions affects the likelihood of an accident independently of their counterpart's, i.e., an additively separable

[10]For example, an individual who drives once a year but chooses to drive at 200 miles per hour may be preferred to one who drives every day but always at 55 miles per hour, or he may not. Clearly, the choice reduces to the increased risk of driving 200 miles per hour versus 55 and the additional utility drivers get from the excess speed.

accident avoidance technology. Section 4 contains the main contribution of the paper with relation to the cost minimization problem: for the class of accidents considered there exists some sufficiently small level of uncertainty to support an equilibrium where both parties choose the socially efficient level of care despite evidentiary uncertainty. Sections 3.2 to 4.2 contain the proof of this sufficiency result.

Using numerical simulations of the model, Section 5 demonstrates the results of the previous sections. Section 6 contemplates the general welfare maximization problem with no uncertainty, showing that the problem always results in over-engagement in the activity by potential injurers and under-engagement by the victim, and may result in over-cautiousness by the victim whenever the social cost of the activity is increasing in the injurer's level of activity. Section 6 also contains the contribution of the paper in relation to the welfare maximization problem, showing that given the findings in Section 4, uncertainty is unambiguously welfare improving as parties choose the efficient level of care and potential injurers internalize some non-zero portion of the social cost they impose, thereby adjusting their level of engagement in a welfare improving way. Section 7 concludes.

2 The Cost Minimization Problem With No Evidentiary Uncertainty

The problem involves a potential injurer and victim that engage in some inherently risky activity. Risk takes the form of the possibility of an accident, which in turn causes the victim to incur monetary damages. Each agent chooses some level of caution when engaging in the activity and while care is costly, additional care on the part of either injurer or victim reduces the risk of an accident according to some exogenous accident avoidance technology. The injurer and victim are identical in every respect until an accident occurs, at which point victim attempts to recover monetary damages from the injurer under tort law. Their choice of cautiousness has a symmetric effect on the likelihood of an accident and they face the same level of exogenous evidentiary uncertainty or absence thereof. In the absence of a cost apportionment rule, victim would always bear the cost of the accident as it is presumed throughout that injurer and victim can not form a contractual agreement for allocation of damages.

Throughout the paper I assume that the exogenous accident avoidance technology and the ratio of the cost of a unit of cost of care and the monetary damages sustained in an accident are such that the socially efficient outcome demands that each agent exercises some positive level of care.[11] The symmetry of the problem in the level of care of the two parties implies that each should choose the same non-zero level of care at the optimum.

Injurer and victim generally have three incentives to consider in choosing how cautious to be when engaging in the activity. Firstly, they consider their private cost of care. Each would prefer to expend as little effort as possible as care is costly. Secondly, given the other party's level of care, each has an incentive to reduce total social cost, to the extent that she will bear some portion or all of it, by exercising more care. Finally, given the other party's level of care, each party has an incentive to reduce her expected share of the social cost. In a world with perfect information, each agent knows with certainty her share of the social burden given the cost apportionment rule, the standard of care, and the level of caution exercised by both parties.

This is not the case when evidentiary uncertainty is introduced. Rather than knowing the standard of care with certainty, each agent takes the standard as the mean of the symmetric distribution of all possible standards that a jury will apply. This is an alternative way of thinking about the lack of perfect observation of precautionary effort by juries or the effects of the reasonable person standard. Agents are unaware of the exact standard of care (they have some idea that the legal standard is on average the standard applied by juries) as the standard is governed by the reasonable care criteria.

The social planner's objective is to design a liability rule, consisting of a cost apportionment rule and standard of care, that minimizes the sum of the agents' private and social costs. Upon announcement of the apportionment rule and standard of care, each party simultaneously chooses a level of care that minimizes the sum of her private cost and her share of

[11] The reader should note here that in the absence of a liability rule, injurer would exercise no care while engaging in the activity, resulting in social inefficiency. In the degenerate case where zero care by one of the parties is optimal, the questions addressed by this paper are moot.

damages. Since each party knows that the other has full information, in equilibrium the level of care of each party must minimizes those costs for the level of care of the other party. Formally, the problem can be written as

$$\min_{\bar{x},\Gamma} \qquad \hat{x}_n + \hat{x}_v + \lambda \cdot L(\hat{x}_n, \hat{x}_v) \;\; s.t.$$

$$\begin{aligned}
\hat{x}_n &= \operatorname*{argmin}_{x_n} \qquad x_n + \lambda \cdot \Gamma \cdot L \\
\hat{x}_v &= \operatorname*{argmin}_{x_v} \qquad x_v + \lambda \cdot (1 - \Gamma) \cdot L
\end{aligned}$$

where x_i is the level of care of party i, λ measures the ratio of monetary damages sustained by the victim in the event of an accident to the cost of a unit of care, $L \equiv L(x_n, x_v)$ is a continuous, twice differentiable function, that describes the accident avoidance technology. L is decreasing and convex in both arguments. *Throughout the remainder of the paper I consider only the class of accidents where precaution by one party is a substitute for precaution from her counterpart. This implies that $L_n \equiv L_n(x_n)$ and that $L_v \equiv L_v(x_v)$, where L_i is the effect of additional caution by agent i on the likelihood of an accident.* Finally, these conditions imply that $L_{ij} = 0$, where L_{ij} is the cross partial derivative of the accident avoidance function.

$\Gamma \equiv \Gamma(x_n, x_v; \bar{x})$ is an indicator function that assigns liability based on the level of care jointly exercised by the injurer and the victim and the standard of care $\bar{x}$. The indicator function differs across cost apportionment rules and will be defined individually for each of the rules considered.

The assumptions of linear cost and a convex accident avoidance technology imply that there must be some $(x_n, x_v) = (x^*, x^*)$ which is the interior solution to the social planner's problem. *For the remainder of the paper, this will be referred to as the "socially efficient level of care". References to the efficient equilibrium throughout the paper will be in reference to this equilibrium and behavioral distortions when uncertainty is introduced in Section 3 will be measured by deviations from this equilibrium.* At the optimum, each party chooses a level of care such that $L_i = -\frac{1}{\lambda}$. The literature shows that in the absence of uncertainty, when the standard of care $\bar{x}$ is set at the socially efficient level of care, each of the three cost apportionment rules that will be considered here result in a unique pure strategy equilibrium where both injurer and victim choose $\bar{x} = x^*$. Additionally, for any $\bar{x} < x^*$, injurer chooses $x_n = \bar{x}$, as at that level he does not bear the cost of the accident, and victim chooses $x_v = x^*$.

2.1 Negligence and liability distinguished

The terms negligent and liable are both used throughout the paper and should be distinguished. Where negligence simply relates a party's level of care to the standard of care, liability depends also on the cost apportionment rule. A party to an accident is considered negligent whenever his level of care falls below the legal standard. Alternatively, liability is assigned on the basis of each party's level of care, the legal standard, and the cost apportionment rule. The table below summarizes the way in which liability is assigned under each of the three cost apportionment rules considered:

Injurer	Victim	Simple Negligence	Contributory Negligence	Comparative Negligence
Negligent	Negligent	Injurer	Victim	Injurer pays share α
Negligent	Not Negligent	Injurer	Injurer	Injurer
Not Negligent	Negligent	Victim	Victim	Victim
Not Negligent	Not Negligent	Victim	Victim	Victim

3 The Introduction of Evidentiary Uncertainty

In what follows, I characterize the set of possible equilibria under both a simple and contributory negligence rule for small levels of uncertainty.[12] For each rule, I consider first the parties' equilibrium behavior when the legal standard of care is set at the efficient level with no uncertainty, i.e., x^*. I find that under contributory negligence, in equilibrium injurer and victim will both be overly cautious as each party has some incentive to avoid liability in

[12]The reader will note throughout the discussion that the level of uncertainty sufficient for existence of an efficient equilibrium is a function of the accident avoidance technology, monetary damages in the event of an accident and the form of the distribution of uncertainty.

the event of an accident. At sufficiently small levels of uncertainty, this incentive overshadows the parties' incentives to reduce the private cost of care and the likelihood of an accident.

This, however, is not the case under simple negligence, where injurer will be overly cautious and victim will not be cautious enough. Victim's only incentive is to minimize the sum of private cost of care and the likelihood of an accident, weighted by the probability that he will be liable for damages as he is barred from recovery whenever injurer is non-negligent, irrespective of his level of precaution.

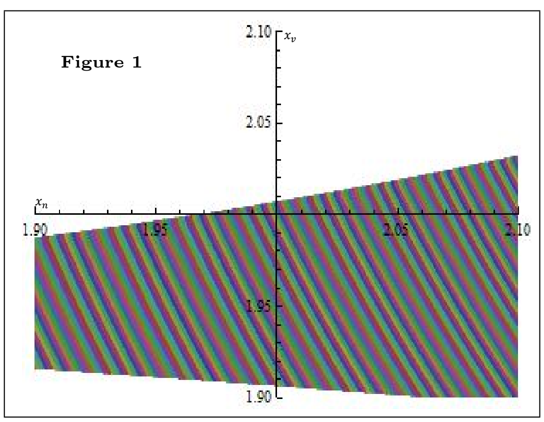
Given this equilibrium, I next consider the effect of progressive reductions in the standard of care, *noting that at the optimum the effect of a change in the standard of care induces a continuous set of new equilibria.* I find that under contributory negligence, injurer and victim will both be less cautious for progressively lower legal standards. Intuitively, injurer can reduce her private cost and maintain the same probability of being found non-negligent. As for increases in the likelihood of an accident, since injurer will be highly unlikely to be found liable in the first place, the incentive to reduce the likelihood of an accident will be overshadowed by her desire to avoid payment in the event an accident occurs. Under a simple negligence regime, injurer will reduce her care with a reduction in the legal standard. This in turn has an ambiguous affect on the victim's behavior.

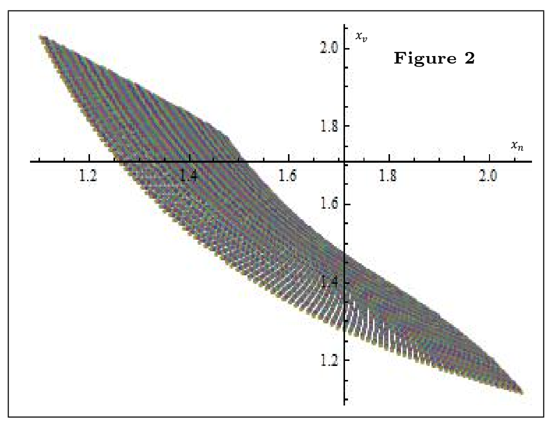
In doing so, I identify the set of feasible equilibria when the legal standards is set below the efficient level. I find that under contributory negligence no equilibrium is feasible where injurer is overly cautious relative to the efficient level and victim is less than efficiently cautious. Under simple negligence, no equilibrium is feasible where victim is overly cautious relative to the efficient level, irrespective of injurer's behavior. Using the characterization of equilibria in the parties' strategy space, Section 4 identifies feasible equilibria under a comparative negligence rule for all division of liability rules.

Before deriving the sufficient conditions for the existence of an efficient equilibrium under comparative negligence, a numerical example of the problem illustrates the absence of a general result.

Each plot below shows, for a fixed level of uncertainty, equilibria generated by each of the three negligence regimes (comparative negligence for all division of liability rules between 0 and 1) for various legal standards. The equilibria are plotted in the player's strategy space. In each figure the x and y axes represents the injurer's and victim's level of care, respectively. The origin represents the efficient equilibrium, that is, the equilibrium that solves the social planner's problem with no uncertainty. The level of uncertainty in each simulation is fixed at a ratio of 1 with the mean of the distribution of possible legal standards, which is distributed normally for purposes of the simulation. *Figure 1 represents an accident avoidance technology where the level of care of each party is a substitute for the other's, whereas Figure 2 represents a situation where they are complements.*[13]

The set of points that comprise the upper boundary in each figure represent equilibria under a contributory negligence regime for varying levels of the legal standard of care, and the set of points that make up the lower boundary represent equilibria under a simple negligence regime. Each point in the shaded region represents the equilibrium under a comparative negligence regime for a legal standard, division of liability rule pair. The reader will note that while comparative negligence is clearly most efficient in the numerical simulation in figure 1 (as there is some legal standard, division of liability rule pair that supports the efficient equilibrium), contributory negligence is relatively more efficient in the simulation in figure 2.





[13]Formally, the accident avoidance technology used to generate the equilibria in Figure 1 is the function $L(x_n, x_v) = \left(\frac{1}{1+x_n} + \frac{1}{1+x_v}\right)$ where $\lambda = 9$ is the ratio of monetary damage to a unit of cost of care, and x_i is party i's level of care. The accident avoidance technology function in Figure 2 is $L(x_n, x_v) = 9\left(\frac{1}{1+x_n x_v}\right)$.

3.1 Contributory negligence

In what follows the set of possible equilibria under a contributory negligence rule is characterized. Considered first is the equilibrium when the legal standard is set at the socially efficient level. When uncertainty is sufficiently small, both choose to be overly cautious. This finding is based on a formal characterization of the parties' best response functions for low levels of uncertainty. However, intuition would lead to a similar conclusion; when a small amount of additional care, which is almost costless at very small levels of uncertainty, can almost guarantee that injurer does not bear any of the cost of the accident, she will certainly expend the additional effort. From the victim's perspective, given that there is always some possibility that injurer is found negligent, a small enough level of uncertainty provides a strong enough incentive to exercise care so that he will not be found negligent under such a scenario. Additionally, victim has a strong incentive to choose some level of care near the efficient level as he will bear the burden of the accident with a high probability, implying that he should minimize the likelihood of one.

Considered next is the set of all possible equilibria induced by continuous reductions in the standard of care, noting that continuous reductions in the standard of care will induce a continuous set of equilibria as the parties' best response functions are continuous in $\bar{x}$. The central finding of this section is that certain equilibria cannot be sustained under a contributory negligence rule. Specifically, Section 3.2.2 shows that there exist no equilibria, relative to the socially efficient equilibrium (x^*, x^*) where

- Injurer is overly cautious and victim is efficiently cautious,
- Injurer and victim are both efficiently cautious, or
- Injurer is efficiently cautious and victim is not cautious enough

As demonstrated in this section, **since the set of equilibria induced by continuous reductions in the legal standard can not include any of these four regions of the players' strategy space, it also can not include any equilibrium where injurer**

is overly cautious and victim is not cautious enough as the set would have to include one of the three types of equilibria above due to its continuity.

The social planner's problem under a contributory negligence regime can formally be expressed as follows:

$$\min_{\bar{x}} \qquad \hat{x}_n + \hat{x}_v + \lambda \cdot L(\hat{x}_n, \hat{x}_v), \;\; s.t.$$

$$\hat{x}_n = \underset{x_n}{\operatorname{argmin}} \quad c_n = x_n + \lambda \cdot L(x_n, x_v) \cdot (1 - \Phi_n) \cdot \Phi_v \tag{1}$$

$$\hat{x}_v = \underset{x_v}{\operatorname{argmin}} \quad c_v = x_v + \lambda \cdot L(x_n, x_v)\Big(1 - \Phi_v \cdot (1 - \Phi_n)\Big) \tag{2}$$

where $\Phi_i \equiv \Phi_i(x_i; \bar{x}, \sigma)$, an everywhere twice differentiable (in all arguments) cumulative distribution function with mean $\bar{x}$ and variance σ, measures the likelihood that victim is found non-negligent by a jury given his level of care and the legal standard $\bar{x}$.[14] σ measures the evidentiary uncertainty faced by both parties and is exogenously determined.[15]

The problem generates the following first order conditions:

$$\frac{\partial c_n}{\partial x_n} = 1 + \lambda \cdot (1 - \Phi_n) \cdot \Phi_v \cdot L_n - \lambda \cdot \phi_n \cdot \Phi_v \cdot L = 0 \tag{3}$$

$$\frac{\partial c_v}{\partial x_v} = 1 + \lambda \cdot (1 - \Phi_v(1 - \Phi_n)) \cdot L_v - \lambda \cdot \phi_v \cdot (1 - \Phi_n) \cdot L = 0, \tag{4}$$

Letting $\tau = (1 - \Phi_n) \cdot \Phi_v$ and solving for $L(x_n, x_v)$ in (3) and (4) gives

$$L(x_n, x_v) = \frac{1 + \lambda \cdot \tau \cdot L_n}{\lambda \cdot \phi_n \cdot \Phi_v} \tag{5}$$

$$L(x_n, x_v) = \frac{1 + \lambda \cdot (1 - \tau) \cdot L_v}{\lambda \cdot \phi_v \cdot (1 - \Phi_n)} \tag{6}$$

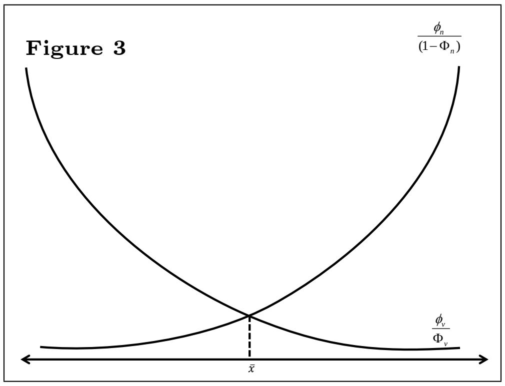
[14]Note that the subscript does not imply that the injurer and victim face different distributions, Φ_v is used as short-hand for $\Phi(x_v)$.

[15]Injurer and victim face the same level of uncertainty and standard of care as they are engaging in the same activity.

Setting the two terms equal and rearranging provides the following necessary condition for an equilibrium under contributory negligence:

$$\frac{1 + \lambda \cdot \tau \cdot L_n}{1 + \lambda \cdot (1 - \tau) \cdot L_v} = \frac{\frac{\phi_n}{(1-\Phi_n)}}{\frac{\phi_v}{\Phi_v}} \tag{7}$$

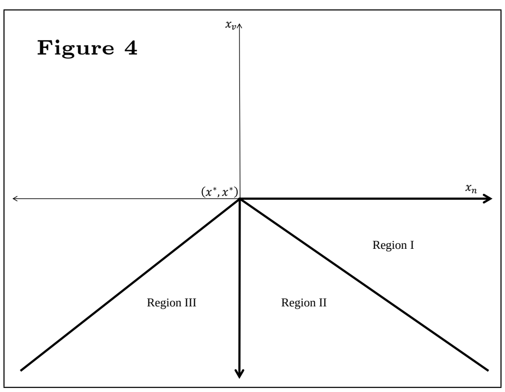
Figure 3 shows the relative values of $\frac{\phi_n}{(1-\Phi_n)}$ and $\frac{\phi_v}{\Phi_v}$ for any symmetric distribution. The reader will note, for example, that the right hand side of condition (7) takes on values of one when injurer and victim each observe the legal standard of care or symmetrically deviate from the standard in opposite directions.



3.2 Proof

3.2.1 Equilibrium at $\bar{x} = x^*$

For $\bar{x} = x^*$, consider the possibility of an equilibrium in the highlighted regions of the (x_n, x_v) space as indicated in Figure 4 below:



We note that when both victim and injurer are observing the standard of care, the RHS=1. This is also true for deviations from the standard of care that are (a) identical or (b) symmetric if in opposite directions. When injurer's deviation from the standard of care is greater than the victim's, the RHS is less than 1, and finally the RHS is greater than 1 whenever victim's deviation is greater than injurer's.

No equilibrium can exist in Regions II and III:

At all points in those regions, $x_n > x_v$, which implies that $|\lambda \cdot L_n| < |\lambda \cdot L_v|$ and $\tau < \frac{1}{2}$. Therefore, the value of the LHS must be greater than 1. However, as Figure 3 shows, the value of the RHS in that region must be less than 1 as victim's deviation from the efficient point is larger.

No equilibrium can exist in (a) Region I or (b) at any point where $x_n \geq x^*$ and $x_v = x^*$:

The following constraints must be satisfied in order for such an equilibrium to exist:

$$\lambda \cdot L_v \leq -1 \quad \leq \quad \lambda \cdot L_n < 0 \tag{8}$$

$$\Phi_v \leq \frac{1}{2} \quad \leq \quad \Phi_n < 1 \tag{9}$$

$$\phi_n \quad \leq \quad \phi_v \tag{10}$$

$$\frac{\phi_v}{\Phi_v} \quad \leq \quad \frac{\phi_n}{(1 - \Phi_n)} \tag{11}$$

Solving this system of inequalities numerically shows that condition (7) can not be satisfied for the set of solutions to (8) through (11).[16]

These findings imply that if a locally unique equilibrium where $x_n > x^*$ and $x_v > x^*$ exists, then it must be globally unique unless injurer has a profitable deviation to $x_n \leq x^*$, since there is no possible equilibrium in Regions I, II and III, i.e., victim would not deviate to a level of care below x^*. In what follows, I show that there must exist some equilibrium where

[16] The system is solved using Mathematica's Reduce command. The solution to the system of equations, including condition (7) is the null set.

$x_n > x^*$ and $x_v > x^*$ and that injurer would not deviate to $x_n \leq x^*$.[17]

Best response functions:
Consider injurer's best response to $x_v = x^*$ relative to the efficient point.

$$\left.\frac{\partial c_n}{\partial x_n}\right|_{(x^*,x^*)} \quad = \quad \frac{3}{4} - \frac{1}{2}\lambda \cdot L(x^*, x^*) \cdot \phi_n \tag{12}$$

Since $L(x_n, x_v) > 0$ for any action pair, there must exist some σ small enough such that (12) is negative, implying that when victim chooses x^*, injurer would choose $\hat{x}_n > x^*$. Since Φ_n is unimodal and symmetric, the value of the probability density function is unbounded at the mean and such a σ must exists as the Φ_n is continuous in σ.

The best response function,BR_n, of injurer is defined implicitly by the identity

$$\frac{\partial c_n(BR_n(x_v), x_v)}{\partial x_n} \equiv 0$$

Differentiating with respect to x_n and solving for BR_n' gives

$$BR_n' = -\frac{\frac{\partial^2 c_n}{\partial x_n \partial x_v}}{\frac{\partial^2 c_n}{\partial x_v^2}}$$

Differentiating (3) with respect to x_n and x_v, the slope of injurer's best response function at $(\hat{x}, x^*)$ can be expressed and evaluated as

$$BR_n'(x_n, x_v, \bar{x}, \sigma) = -\frac{\lambda \cdot L_n \cdot (1 - \Phi_n) \cdot \phi_v - \lambda \cdot L_v \cdot \Phi_v \cdot \phi_n - \lambda \cdot L \cdot \phi_v \cdot \phi_n}{\lambda \cdot L_{nn} \cdot (1 - \Phi_n) \cdot \Phi_v - 2\lambda \cdot L_n \cdot \Phi_v \cdot \phi_n - \lambda \cdot L \cdot \Phi_v \cdot \frac{\partial \phi_n}{\partial x_n}} \tag{13}$$

where BR_n is injurer's best response function. The denominator of (13) is always positive when $x_i > \bar{x}$, since $\frac{\partial \phi_i}{\partial x_i} < 0$. The numerator can be negative or positive. The numerator is negative, implying a positively sloped best response function, whenever

$$|\lambda \cdot L_n \cdot (1 - \Phi_n) \cdot \phi_v| > |\lambda \cdot L_v \cdot \Phi_v \cdot \phi_n| \tag{14}$$

[17] This implications is implicit in the construction of the injurer's best response function below

Since $x_v = x^*$, there must be some σ small enough such that this is true.

The slope of BR_n is not monotonic in x_v, as the first and third terms of the numerator are decreasing in x_v and the second term may increase.[18] However, in constructing BR_n it is sufficient to note that the sign of the slope will change at most once.

Turning to the injurer's best response as $x_v \to \infty$ relative to the efficient point, the additive separability of $L(x_n, x_v)$ implies that $\lim_{x_v \to \infty} L(x_n, x_v) = \underline{L} > 0$.

$$\left. \frac{\partial c_n}{\partial x_n} \right|_{(x^*, x_v)} = \frac{1}{2} - \lambda \underline{L} \phi_n \tag{15}$$

Finally, it is trivially true that as $x_v \to 0$, $\frac{\partial c_n}{\partial x_n} \to 1$, implying that injurer will choose some level of care less than x^*, for any level of uncertainty.

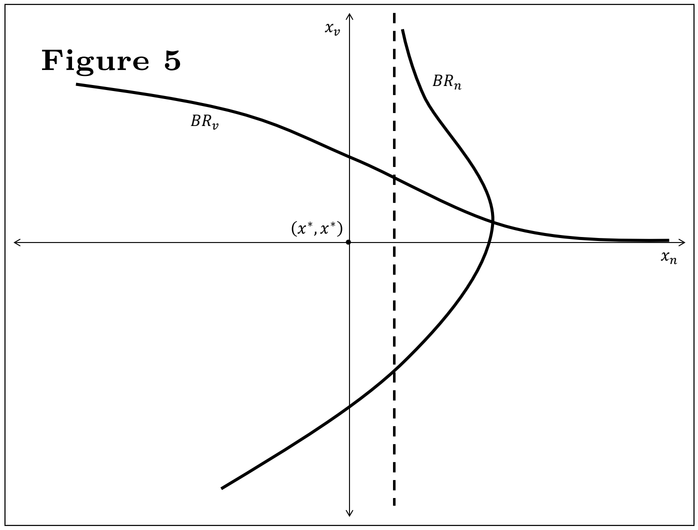
There must be some level of uncertainty small enough such that (12) and (15) are negative and that $|\lambda \cdot L_n \cdot (1 - \Phi_n) \cdot \phi_v| > |\lambda \cdot L_v \cdot \Phi_v \cdot \phi_n|$.

Considering next the victim's best response function, similar reasoning shows the following features of victim's best response function for small levels of uncertainty:

- above the x-axis when $x_n \to 0$
- above the x-axis when $x_n = x^*$
- Asymptotically goes to $x*$ as $x_n \to \infty$

Figure 5 below illustrates the injurer's and victim's best response functions for a sufficiently small level of uncertainty. The dotted line represents the value to which injurer's best response function asymptotes, i.e., the value of x_n that solves condition (15).

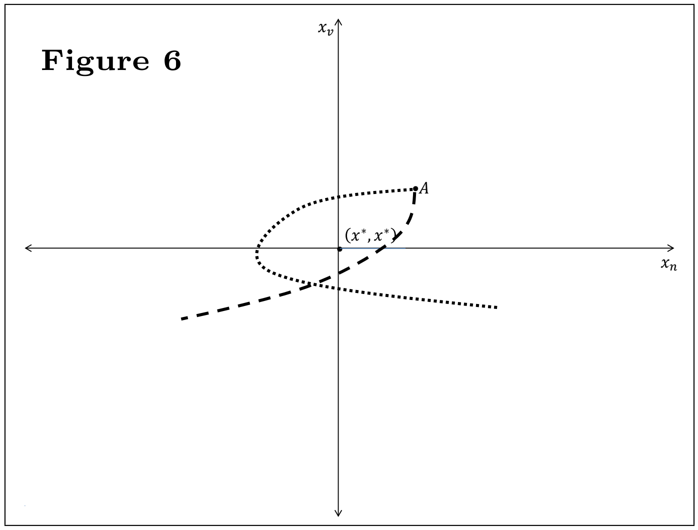
[18] Whether the absolute value of the second term increases before decreasing depends on the relative curvature of the accident technology function and the cumulative distribution function. For small values of σ, the second term will increase as Φ_v increases in value rapidly around the mean.



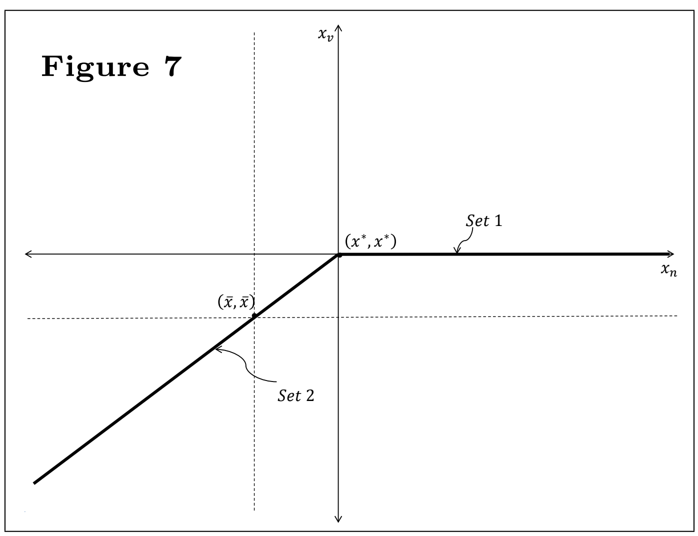
The intersection of the best response functions represents the equilibrium level of care of injurer and victim. Injurer and victim are both inefficiently too cautious at low levels of uncertainty whenever the standard of care is distributed according to a symmetric, unimodal distribution. In what follows, the paper examines equilibria for reductions in the standard of care below x^*.

3.2.2 Effect of decrease in standard of care

In this section the paper shows that in every new equilibrium for a given standard of care below x^*, (a) either injurer and victim are both overly cautious relative to the efficient level of care, (b) victim is overly cautious and injurer is at or below x^*, or (c) victim is at or below x^* and injurer is below x^*. That is, there is no equilibrium where injurer is overly cautious and injurer is at or below x^*. Turning once again to the players' action space, I argue that starting from point A (which represents the equilibrium at $\bar{x} = x^*$) in Figure 6 below, the set of equilibria represented by the dotted curves is impossible. That is, a reduction in the standard of care, while it reduces the level of care for both injurer and victim, can not result in any equilibrium where injurer is overly cautious and injurer is at or below x^*.



Since both parties' best response functions are continuous in the standard of care (since it is assumed that Φ is everywhere twice differentiable), continuous changes in the standard of care will induce a continuous set of equilibria. Given this continuity, that there are regions in the action space where an equilibrium cannot exist as an equilibrium in one of the regions highlighted in Figure 7 below would necessarily have to be supportable under a contributory negligence cost apportionment rule.



Let us consider the efficient point, where $x_n = x_v = x^*$. For any $\bar{x} < x^*$, $\Phi_n = \Phi_v = \Phi > \frac{1}{2}$, $\phi_n = \phi_v = \phi$, and $\lambda L_n = \lambda L_v = -1$.

Substituting these conditions into (14), we have

$$\begin{aligned}
\frac{(1-(1-\Phi)\Phi)}{(1-(1-(1-\Phi)\Phi))} &= \frac{\frac{\phi}{(1-\Phi)}}{\frac{\phi}{\Phi}} \\
\frac{(1-(1-\Phi)\Phi)}{(1-(1-(1-\Phi)\Phi))} &= \frac{\Phi}{(1-\Phi)} \\
\frac{1-\Phi+\Phi^2}{\Phi-\Phi^2} &= \frac{\Phi}{(1-\Phi)} \\
1-2\Phi+\Phi^2 &= 0 \\
\Phi &= 1
\end{aligned}$$

Which contradicts the property that $\Phi_i < 1$ for all actions of injurer and victim.

Consider next some equilibrium where $x_n = x_v < x^*$, i.e., an equilibrium in the region Set 2 identified in Figure 7. At any point in that set, $\Phi_n = \Phi_v = \Phi$, $\phi_n = \phi_v = \phi$ and $\lambda \cdot L_n = \lambda \cdot L_v < -1$. Letting $\lambda \cdot L_i = \gamma$ and substituting these conditions into condition (14) we have

$$\begin{aligned}
\frac{1+\gamma(1-\Phi)\Phi}{1+\gamma(1-\Phi+\Phi^2)} &= \frac{\frac{\phi}{(1-\Phi)}}{\frac{\phi}{\Phi}} \\
\frac{1+\gamma(1-\Phi)\Phi}{1+\gamma(1-\Phi+\Phi^2)} &= \frac{\Phi}{(1-\Phi)} \\
1-\Phi+\gamma(1-\Phi)^2\Phi &= \Phi+\gamma(1-\Phi+\Phi^2)\Phi \\
1-\Phi+\gamma(\Phi-2\Phi^2+\Phi^3) &= \Phi+\gamma(\Phi-\Phi^2+\Phi^3) \\
1-\Phi-2\gamma\Phi^2 &= \Phi-\gamma\Phi^2 \\
3\gamma\Phi^2+2\Phi-1 &= 0
\end{aligned}$$

which implies that an equilibrium can exist only when $-\frac{1}{3} \leq \gamma < 0$, contradicting the condition that $\lambda \cdot L_n = \lambda \cdot L_v < -1$.

Finally, consider some equilibrium where $x_v = x^*$ and $x_n > x^*$, i.e., an equilibrium in the region Set 1. The following constraints must all hold at such an equilibrium:

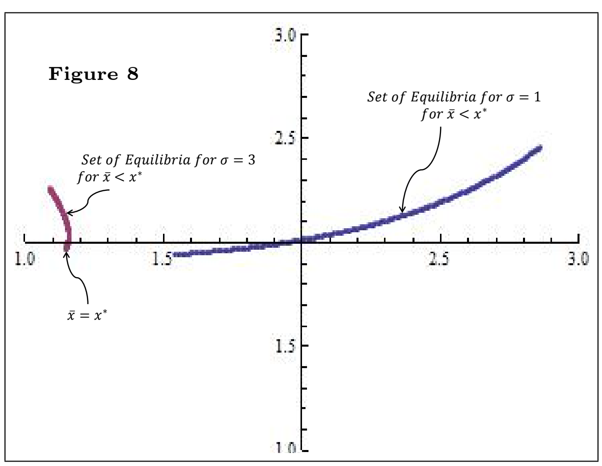
$$
\begin{aligned}
-1 < \lambda \cdot L_n &< 0 \\
\lambda \cdot L_v &= -1 \\
\frac{1}{2} < \Phi_v &< \Phi_n \\
\phi_n &< \phi_v \\
\frac{\phi_v}{\Phi_v} &< \frac{\phi_n}{(1-\Phi_n)}
\end{aligned}
$$

Using Mathematica, I find that there are no real values that simultaneously satisfy condition (7) and the constraints above.

3.2.3 Summary and numerical example

The foregoing results are illustrated numerically in the player's action space in the figure below. Using the functional forms and parameter values from the previous example, the problem is mapped into the (x_n, x_v) space, holding fixed the level of uncertainty. The two loci in Figure 8 represent the set of equilibria for high and low levels of uncertainty (specifically $\sigma = 1$ and $\sigma = 3$), as marked for various legal standards $\bar{x} \leq x^*$.

The origin of Figure 8 represents the socially efficient outcome $(x^*, x^*) = (2, 2)$ for the given accident prevention technology and level of damages. The x-axis represents the Injurer's level of care in equilibrium and the y-axis represents same for Victim.



The figure demonstrates the findings in the previous subsection. For sufficiently low levels of uncertainty, the social planner can choose the standard of care such that (i) injurer and victim are both overly cautious (e.g., the reader will note that when $\bar{x} = x^*$, both are inefficiently over-cautious), (ii) injurer is efficiently cautious but victim is overly cautious, (iii) injurer is not cautious enough and victim is overly cautious, (iv) injurer is not cautious enough and victim is efficiently cautious and (v) neither is cautious enough. In contrast, when uncertainty is high, the injurer will not be sufficiently cautious as the payoff to care is weak. Injurer knows that additional care will have a small impact on a finding of negligence, and would therefore lower her private cost of care. The victim will choose to be either overly cautious or not cautious enough.

An additional feature of Figure 8 which the reader should note is the players' responsiveness to changes in the standard of care. While both are quite responsive to changes in the legal standard when uncertainty is low, changes in the legal standard do not affect equilibrium care with the same magnitude when signals of the standard are weak from the individuals' perspective.

3.3 Simple Negligence

In this subsection I characterize the set of equilibria that can be induced by the social planner under a simple negligence cost apportionment rule for small levels of uncertainty. Not surprisingly, irrespective of the accident avoidance technology, distributional form of uncertainty, and level of uncertainty, there is no standard of care that would induce the victim to choose a level of care at or above the efficient level. As the victim's cautiousness does not affect his liability, his only incentives are his private cost of care and reducing his share of the social cost of care, but can not affect that share with additional care. The injurer on the other hand will be responsive to changes in the standard of care for small enough levels of uncertainty, as the cost of reducing his expected share of liability diminishes with a decrease in uncertainty.[19]

[19] The injurer's response however is generally ambiguous. Analysis of the second order conditions of the problem reveal that there exist, in equilibrium, functional forms of the accident technology and high enough dispersion of uncertainty that could induce the injurer and victim to jointly stop reacting to an increase in the standard of care as their only incentive is to minimize the likelihood of an accident. The injurer may have

I find that sufficiently small levels of exogenous uncertainty imply that the social planner can effectively influence the injurer's behavior and induce equilibria that range from injurer being not cautious enough to overly cautious, while victim is always under-cautious. I start with the standard of care set at the socially efficient level with no uncertainty, which implies an equilibrium with $(x_n, x_v) = (x^*, x^*)$. I then introduce a small level of uncertainty and predict the behavior of each agent. I characterize some features of the new equilibrium and ask how injurer will respond to a change in the standard of care. Through this line of reasoning, I map out a subset of equilibria that the social planner can induce in the neighborhood of the socially efficient level.

The social planner's problem under a simple negligence regime with evidentiary uncertainty can be expressed as

$$\min_{\bar{x}} \qquad \hat{x}_n + \hat{x}_v + \lambda \cdot L(\hat{x}_n, \hat{x}_v) \;\; s.t.$$

$$\begin{aligned} \hat{x}_n &= \underset{x_n}{\text{argmin}} \qquad {c_n}^{SN} = x_n + \lambda \cdot (1 - \Phi_n) \cdot L \\ \hat{x}_v &= \underset{x_v}{\text{argmin}} \qquad {c_v}^{SN} = x_v + \lambda \cdot \Phi_n \cdot L \end{aligned}$$

where ${c_n}^{SN}$ and ${c_v}^{SN}$ are injurer and victim's individual cost functions. $\Phi_n(\cdot) \equiv \Phi_n(x_n; \bar{x}, \sigma)$ is a twice differentiable, symmetric, and unimodal cumulative distribution function with full support with mean $\bar{x}$ and variance σ. Φ_n represents the likelihood that Injurer will be found negligent by a jury given the standard of care $\bar{x}$ and her level of care x_n. Suppose that the activity in question is driving and the legal standard of care is the speed limit. The Injurer views the speed limit as the mean of the distribution of all possible speed limits that a jury will apply in a tort suit. If the speed limit were 55, if for example, half of the randomly drawn

no incentive to keep up with progressively increasing standards of care as the probability of liability does not change and an increase in care on his part would induce a decrease in care from victim, which would serve only to increase the likelihood of the accident. Therefore, there it is not generally true that the social planner can induce equilibria where $x_n > x^*$. The intuition behind this is that with enough evidentiary uncertainty (which diminishes the value of additional care in reducing share of accident), progressive increases in the standard of care that induce less care from the victim may actually make it beneficial for injurer to reduce his level of care in response as he cares only about minimizing private plus social cost and not his share of the cost.

juries will find the injurer negligent if he observes the speed limit, while 50 percent of the time a jury will impose a stricter standard of care as they believe that the reasonable person standard dictates more prudence. As such, additional care on the part of the injurer will reduce the probability that she is found negligent, and therefore liable under simple negligence.

The problem leads to the following first order conditions:

$$\frac{\partial {c_n}^{SN}}{\partial x_n} = \quad 1 + \lambda \cdot (1 - \Phi_n) \cdot L_n - \lambda \cdot \phi_n \cdot L = 0 \tag{16}$$

$$\frac{\partial {c_v}^{SN}}{\partial x_v} = \quad 1 + \lambda \cdot \Phi_n \cdot L_v = 0 \tag{17}$$

where $\phi_n \equiv \phi_n(x_n; \bar{x}, \sigma)$ is the probability distribution function of Φ_n. The first term in the Injurer's first order condition is the marginal cost of a unit of care, which the paper assumes increases at a constant rate with care without loss of generality. [20] As indicated, the second term represents the marginal reduction in the Injurer's share of the social cost of his activity for an increase in care. The second term represents the measure of the Injurer's incentive for care based on the benefit of reducing the likelihood of the accident with additional care. Finally, the third term represents the marginal reduction in the likelihood of being found negligent in the event of an accident. This term measures the Injurer's incentive to escape a finding of negligence by a jury.

The victim's first order condition on the other hand consists of only two terms. The first term is the marginal cost of care and the second is the marginal reduction in his share of the social cost of his activity for an increase in care. We note at that since the simple negligence rule ignores the victim's behavior in assigning liability, the victim here has no secondary incentive to increase care as any such increase would not affect the likelihood that the injurer is found negligent.

[20] As the paper examines the behavior of the parties in the neighborhood of the efficient level of care for small levels of uncertainty, the parties essentially ignore all incentives other than the incentive to escape liability. Therefore the curvature of the private cost function becomes irrelevant. Numerical simulations in the appendix include examples with non-linear private cost functions.

3.3.1 Effect of evidentiary uncertainty on level of care

For any social cost function, distributional form of uncertainty, and level of evidentiary uncertainty, there exists no equilibrium where victim's level of care is greater than the efficient level x^*:

Let $(\hat{x}_n, \hat{x}_y)$ be an equilibrium. Condition (17) above must hold in equilibrium, implying that $L_v(\hat{x}_v) = -\frac{1}{\lambda \cdot \Phi_n(\hat{x}_n)} > -\frac{1}{\lambda}$ since $\Phi_n < 1$ by definition. This in turn implies that $x_v < x^*$ as L_v is decreasing in care.

Therefore, victim will always reduce his level of care with the introduction of uncertainty. Intuitively, the introduction of any uncertainty gives victim some chance of escaping liability, which induces a reduction in care.

For any social cost function and distributional form of uncertainty, there is some level of evidentiary uncertainty small enough to induce injurer to chooses a level of care above the standard of care.

By assumption, $L(x_n, x_v) > 0$ for any action pair. For any standard of care at or below the socially efficient level, i.e., $\bar{x} \leq x^*$, in equilibrium with no uncertainty injurer will choose $\bar{x}$ in equilibrium. I evaluate condition (16) above at such an equilibrium, noting that $(1 - \Phi_n) = \frac{1}{2}$ and that $\lambda \cdot L_n \leq -1$. I let $\Theta = \lambda \cdot (1 - \Phi_n) \cdot L_n \leq -\frac{1}{2}$. Fixing $\tilde{x}_v < \bar{x}$, any equilibrium, if it exists, must meet the following condition:

$$\left.\frac{\partial {c_n}^{SN}}{\partial x_n}\right|_{(x_n, \tilde{x}_v)} = 1 - \Theta - \lambda \cdot \phi_n \cdot L(x_n, \tilde{x}_v) \tag{18}$$

For any standard of care below the efficient level, if $\Theta < -1$, injurer will increase care. Intuitively, if the standard of care is set too low, the likelihood of an accident may be great and since injurer is responsible for half of the cost of an accident if it occurs (assuming that he does not respond), he may have strong incentives to increase his level of care. Supposes that $\frac{1}{2} < \Theta < 1$. Since $\lambda L > 0$ for any action pair, injurer would only increase his care if he could sufficiently reduce the probability that he will not bear the cost of the accident. This incentive is captured by the value of ϕ_n at the equilibrium. But given the symmetric, unimodal properties of the distributional form of evidentiary uncertainty, a decrease in uncertainty increases the value of ϕ_n at the mean without bound, implying that there must be some σ small enough to induce Injurer to increase his level of care for any action of the victim.

To show that an equilibrium exists such that injurer takes too much caution and victim takes too little caution, I think about the problem dynamically. Injurer's increase in care incudes a decrease in care from victim, which in turn induces an increase from injurer, etc. To show that the system is dynamically stable, i.e., is increases in care by injurer and decreases in care by victim get sequentially smaller, I show that whenever $x_n > \bar{x}$, the Hessian $\begin{pmatrix} \frac{\partial^2 c_n}{\partial x_n^2} & \frac{\partial^2 c_n}{\partial x_n \partial x_v} \\ \frac{\partial^2 c_v}{\partial x_n \partial x_v} & \frac{\partial^2 c_v}{\partial x_v^2} \end{pmatrix}$ is positive semi-definite, i.e., that $\frac{\partial^2 c_n}{\partial x_n^2} > 0$ and that the Determinant of the Hessian is positive. The relevant second derivatives, given that $L_{nv} = 0$, are

$$\frac{\partial^2 {c_n}^{SN}}{\partial x_n^2} = \lambda \cdot (1 - \Phi_n) \cdot L_{nn} - 2 \cdot \lambda \cdot \phi_n \cdot L_n - \lambda \cdot \frac{\partial \phi_n}{\partial x_n} \cdot L \tag{19}$$

$$\frac{\partial^2 {c_n}^{SN}}{\partial x_n \partial x_v} = -\lambda \cdot \phi_n \cdot L_v < 0 \tag{20}$$

$$\frac{\partial^2 {c_v}^{SN}}{\partial x_v^2} = \lambda \cdot \Phi_n \cdot L_{vv} > 0 \tag{21}$$

$$\frac{\partial^2 {c_v}^{SN}}{\partial x_v \partial x_n} = \lambda \cdot \phi_n \cdot L_y < 0 \tag{22}$$

Note that the sign of $\frac{\partial^2 c_n}{\partial x_n^2}$ is ambiguous as $\frac{\partial \phi_n}{\partial x_n}$ can not generally be signed. However, $x_n > \bar{x}$ and the symmetric, uni-modal nature of Φ_n, imply that $\frac{\partial^2 c_n}{\partial x_n^2} < 0$ and that in turn $\frac{\partial^2 c_n}{\partial x_n^2} > 0$.

Given that this general result holds for any standard of care, it follows that whenever $\bar{x} = x^*$, for a sufficiently small level of uncertainty, there exists an equilibrium where $x_n > x^*$ and $x_v < x^*$. I next show that there must also exist some $\bar{x} < x^*$ that induces injurer to take less caution than is socially efficient. This is not trivially true (although intuition would suggest that lowering the standard of care progressively should induce smaller and smaller levels of effort from injurer) as injurer may always want to choose a level of care greater than the efficient level. I next show that small enough levels of uncertainty guarantee against that.

First we note that at the equilibrium with no uncertainty for a standard of care lower than the efficient level, the value of $L(x_n, x_v) > L(x^*, x^*)$. This implies that everything else being equal (including the level of uncertainty), injurer has a stronger incentive to increase his level of care above the standard of care relative to when $\bar{x} < x^*$.[21] Next we note that decreases in the standard of care have a dual effect on injurer's behavior. I abstract away from the concept of an equilibrium to illustrate this dual effect numerically:

Let $L = \frac{1}{1+x_n} + \frac{1}{1+x_v}$, $\Phi \sim N(2, \sigma)$,$\lambda = 9$, and $x_v = 2$. Evaluating $\frac{\partial c_n}{\partial x_n}$ when injurer choosing the standard of care, for $\sigma = 0.2$ and $\sigma = 1$, I find

$$\left.\frac{\partial c_n}{\partial x_n}\right|_{\sigma=0.2} = -11.83$$
$$\left.\frac{\partial c_n}{\partial x_n}\right|_{\sigma=1} = -1.893$$

In order to satisfy the first order condition when $\sigma = 0.2$ and $\sigma = 1$, injurer chooses $x_n = 2.44$ and $x_n = 3.2408$, respectively. So while injurer is more likely to be over-cautious for progressively lower levels of uncertainty, his response is smaller, fixing victim's action. In addition, at the lower level of uncertainty, the smaller increase in care ensures a smaller probability of being found negligent. In the above example $(1-\Phi(2.44; 2, 0.2)) = .0137$ while $(1 - \Phi(3.2408; 2, 1)) = .1073$. Since victim's response to injurer is smaller whenever he faces a greater probability of bearing the cost of the accident (which in turn affects injurer's subsequent deviation, etc), we are ensured that in equilibrium injurer chooses a smaller deviation from the mean for progressively smaller levels of uncertainty since we have already shown that the system is dynamically stable for any standard of care and uncertainty that induces additional care from injurer.[22]

It therefore follows that for any standard of care below the efficient level, there is some level of uncertainty small enough such that injurer chooses to be overcautious but less than efficiently cautious in equilibrium.

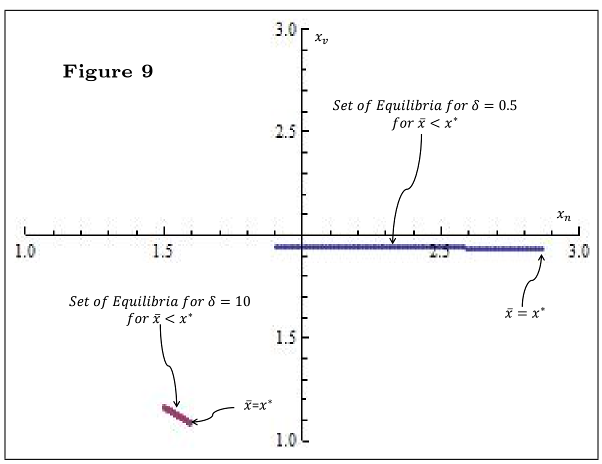
[21] This assertion is based on condition (1).
[22] This dual effect is present for any symmetric, uni-modal distribution

These results collectively imply that whenever evidentiary uncertainty is distributed with infinite support, i.e., $\Phi_n < 1$ for all x_n, the socially efficient outcome is unattainable under the simple negligence regime as the victim chooses to be less cautious than the socially efficient level of care. More importantly, for sufficiently low levels of uncertainty, the social planner can induce equilibria where the injurer is both overly cautious, optimally cautious and not cautious enough by reducing the standard of care below the socially efficient standard.

3.3.2 Summary and numerical example

In this section I illustrate these results numerically in the player's action space. Using the functional forms and parameter values from the numerical example above, I map the problem into the (x_n, x_v) space, holding fixed the level of uncertainty. The locus of points in each graph represents the equilibria induced by various standards of care.

The origin of Figure 9 represents the socially efficient outcome $(x^*, x^*) = (2, 2)$ for the given accident prevention technology. The x-axis represents the Injurer's level of care in equilibrium and the y-axis represents same for Victim. The Figure illustrates the set of equilibria that the social planner can induce for $\sigma = 0.5$ and $\sigma = 10$:



The figure demonstrates the findings in the previous subsection. For sufficiently low levels of uncertainty, the social planner can choose the standard of care such that Injurer is overly cautious (e.g., the reader will note that when $\bar{x} = x^*$, she is inefficiently over-cautious), not cautious enough and perfectly cautious by lowering the legal standard of care. In contrast, when uncertainty is high, the injurer will not be sufficiently cautious as the payoff to care is weak. Injurer knows that additional care will have a small impact on a finding of negligence, and would therefore lower her private cost of care. The victim will choose to be under-cautious irrespective of the level of uncertainty.

An additional feature of Figure 9 which the reader should note is the players' responsiveness to changes in the standard of care. While injurer is quite responsive to changes in the legal standard when uncertainty is low, changes in the legal standard do not affect equilibrium care with the same magnitude when signals of the standard are weak from the individuals' perspective.

4 Equilibrium under comparative negligence

In this section, I examine, for a fixed standard of care and level of uncertainty, the equilibrium level of care of injurer and victim relative to both contributory and simple negligence for any division of liability rule. As expected, the injurer is most cautious under a simple negligence regime and least cautious under contributory negligence, while the victim is most cautious under contributory negligence and least cautious under simple negligence. This ordering is what led to conclusions that comparative negligence must be most efficient. However, this is only part of the story as relative efficiency depends on the equilibria induced in the first place. This result must be combined some characterization of the possible set of equilibria for a given level of uncertainty for any relative efficiency arguments to be valid.

Having shown the relative equilibria for a fixed standard of care and level of uncertainty, I combine this result from the findings of Sections 2.1 and 2.2, in which I characterized the set of equilibria for sufficiently small levels of uncertainty. I show that given the set of equilibria induced under contributory and simple negligence (more importantly given such equilibria relative to the efficient point), and the relation of comparative negligence to those rules, there must exist some standard of care and division of liability rule that induces the efficient outcome.

4.1 Comparative negligence relative to contributory and simple negligence

The social planner's problem under a comparative negligence cost apportionment rule, where α is the division of liability rule that determines each party's share of liability in the event both are found negligent, follows:

$$\min_{\bar{x}} \qquad \hat{x}_n + \hat{x}_v + \lambda L(\hat{x}_n, \hat{x}_v), \;\; s.t.$$

$$\hat{x}_n = \underset{x_n}{\text{argmin}} \quad {c_n}^{PN} = x_n + \lambda L(\cdot)(1-\Phi_n)\Big[\Phi_v + \alpha(1-\Phi_v)\Big] \tag{23}$$

$$\hat{x}_v = \underset{x_v}{\text{argmin}} \quad {c_n}^{PN} = x_v + \lambda L(\cdot)\Big[1-(1-\Phi_n)[\Phi_v + \alpha(1-\Phi_v)]\Big] \tag{24}$$

Differentiating (23) and (24) with respect to x_n and x_v respectively, we have

$$\frac{\partial c_n}{\partial x_n}^{PN} = 1 + \lambda L_n(1-\Phi_n)\Big[\Phi_v + \alpha(1-\Phi_v)\Big] - \lambda L\phi_n\Big[\Phi_v + \alpha(1-\Phi_v)\Big] \tag{25}$$

$$\frac{\partial c_v}{\partial x_v}^{PN} = 1 + \lambda L_v\Big[1-(1-\Phi_n)[\Phi_v + \alpha(1-\Phi_v)]\Big] - \lambda L(1-\Phi_n)(1-\alpha)\phi_v \tag{26}$$

where we note that (a) $\Phi_v + \alpha(1-\Phi_v) > \Phi_v$ since $\alpha(1-\Phi_v) > 0$ and (b) $\Phi_v + \alpha(1-\Phi_v) = \Phi_v(1-\alpha) + \alpha < 1$ since $0 < \alpha < 1$.

<u>Equilibrium under comparative negligence relative to simple negligence for any $(\bar{x}, \sigma)$:</u>

Suppose that for some standard of care $\bar{x}$, in equilibrium injurer and victim choose $({x_n}^{SN}, {x_v}^{SN})$ under a simple negligence regime. The following first order conditions must be satisfied in equilibrium:

$$1 + \lambda L_n({x_n}^{SN})(1-\Phi_n) - \lambda L({x_n}^{SN}, {x_v}^{SN})\phi_n = 0 \tag{27}$$

$$1 + \lambda L_v({x_v}^{SN})\Phi_n = 0 \tag{28}$$

Comparing the second and third terms of (25) and (27), we note that since $\Phi_v + \alpha(1-\Phi_v) < 1$, and since both terms of negative, $\frac{\partial c_n}{\partial x_n}^{PN}\Big|_{({x_n}^{SN}, {x_v}^{SN})} > 0$, implying that *everything else being equal injurer will choose a lower level of care under comparative negligence relative to simple negligence*. Intuitively, this formalizes the notion that injurer has a weaker incentives for care under comparative negligence because there is some non-zero chance that victim will be negligent even if she is herself negligent. Under similar reasoning, Victim will be more cautious under comparative negligence relative to simple negligence.

Equilibrium under comparative negligence relative to contributory negligence for any $(\bar{x}, \sigma)$:

Suppose that for some standard of care $\bar{x}$, in equilibrium injurer and victim choose $({x_n}^{CN}, {x_v}^{CN})$ under a contributory negligence regime. It must follow that the following conditions are met at the equilibrium:

$$
\begin{aligned}
1 + \lambda L_n({x_n}^{CN})(1-\Phi_n)\Phi_v - \lambda L({x_n}^{CN}, {x_v}^{CN})\Phi_v\phi_n &= 0 \qquad (29) \\
1 + \lambda L_v({x_v}^{CN})(1-(1-\Phi_n)\Phi_v) - \lambda L({x_n}^{CN}, {x_v}^{CN})(\phi_v - \Phi_n\phi_v) &= 0 \qquad (30)
\end{aligned}
$$

Given that $0 < \alpha < 1$, we note that $\alpha\lambda L_n(1-\Phi_n)(1-\Phi_v) < 0$ and $\alpha\lambda L(1-\Phi_v)\phi_n > 0$ for all (x_n, x_v), implying that if $({x_n}^{CN}, {x_v}^{CN})$ solves (31), $\frac{\partial c_n}{\partial x_n}\Big|_{({x_n}^{CN}, {x_v}^{CN})}$ must be negative.

Therefore, *injurer will always be more cautious under comparative negligence*. The reader will also note that the additional caution is increasing in α, everything else being equal. Under similar reasoning, victim will be less cautious under comparative negligence.

4.2 Efficiency under comparative negligence

The main result of this paper can best be illustrated graphically. Consider Figure 10 below, where the origin represents the point where both parties exercise the socially efficient level of care:

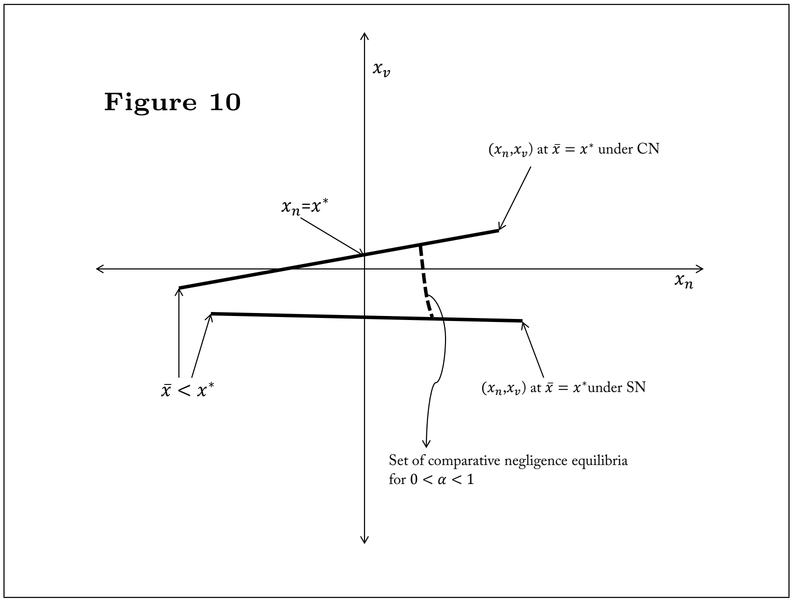


Figure 10

The figure above illustrates the set of equilibria that can be sustained for values of $\bar{x} \leq x^*$ in the neighborhood of the efficient point under both contributory and simple negligence for sufficiently small levels of uncertainty. The set of points below the origin represent equilibria under a simple negligence rule. At $\bar{x} = x^*$, injurer is overly cautious and victim is not cautious enough. Reductions in the standard of care induce injurer to reduce his level of care and victim never exercises the efficient level of caution. The set of points above the origin represent equilibria under a simple negligence regime. At $\bar{x} = x^*$, injurer and victim are both overly cautious. Reductions in the standard of care induce injurer to reduce his level of care and victim is always overly cautious whenever injurer is at least as cautious as the efficient level.

The dotted curve represent the set of possible equilibria under comparative negligence for all values of α, for an arbitrary standard of care. We note that for any standard of care the equilibrium point moves down and to the right, relative to the contributory negligence equilibrium, in the (x_n, x_v) space. Given the continuity of care in α and the standard of care, there must be some $\bar{x}, \alpha$ pair that supports an equilibrium at (x^*, x^*).

5 Numerical simulation: Who pays what share in equilibrium?

In this section, I illustrate the theoretical results of sections 3 and 4 using numerical solutions of the model. First, using the functional form used throughout the paper, I solve for the optimal $(\bar{x}, \alpha)$ pair for various exogenous levels of uncertainty and summarize the results. Next, I demonstrate more generally the effects of uncertainty, as it asymptotes from zero to infinity, on the injurer's share of the accident under three cost apportionment rules.

The table below illustrates, for a specified level of uncertainty, the optimal standard and division of liability rule in addition to the deadweight loss and the injurer's share of damages at the optimum. The reader should note that for low to intermediate levels of uncertainty, injurer and victim choose the efficient level of care (thus, there is no deadweight loss), but also that victim is not barred from complete recovery in expectation. The last column of the table shows injurer's expected share of damages for the various levels of uncertainty.

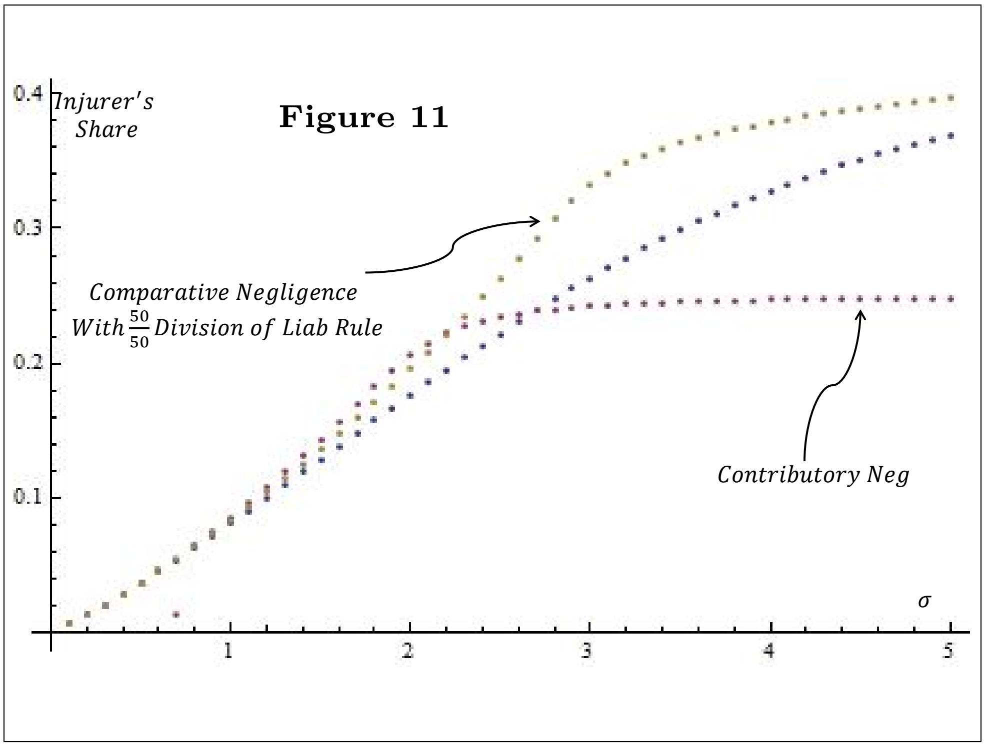
σ	α	Optimal Standard of Care	Deadweight Loss	Injurer's Share of Damages
0.1	0.005832	1.747992	0	0.005832
0.2	0.012771	1.554376	0	0.012771
0.5	0.036967	1.115136	0	0.0369672
1	0.085265	0.678773	0	0.085265
2	0.215338	0.801088	0	0.215338
3	0.417716	1.793386	0.0275	0.351022
4	0.551794	1.536766	0.1092	0.376828

In the absence of evidentiary uncertainty, Victim always bears the entire social cost in equilibrium as it is incentive compatible for Injurer to choose the standard of care assigned by the social planner. [23] With the introduction of uncertainty however, Injurer cannot avoid some probability of bearing the cost of the accident. [24]

[23] Of course if the standard of care is greater than x^*, Injurer will choose x^* and bear the burden of the social cost while Victim chooses to exercise no care, but $\bar{x} > x^*$ is suboptimal from the social planner's perspective and will not be the standard of care.

[24] This is true if the uncertainty is distributed according to a distribution with infinite support such that $0 < \Phi_n < 1$ for all x_n.

Next, I identify the share of the damages that Injurer pays in expectation as evidentiary uncertainty goes from zero and to infinity. Using numerical simulations, I plot the effect of increasing levels of uncertainty on the Injurer's equilibrium expected share of social cost under three regimes: simple negligence, contributory negligence and comparative negligence with a 50/50 division of liability rule. Figure 11 illustrates the relationship of evidentiary uncertainty to the likelihood that Injurer will bear, in equilibrium, the social cost in the event of an accident for all regimes when $\bar{x}$ is not fixed, but set optimally for each given level of uncertainty. That is, given a fixed level of evidentiary uncertainty and cost allocation regime, the social planner chooses the standard of care that in equilibrium will induce the minimum social cost.



The implications of the the injurer's share of liability in equilibrium are discussed in Section 6.

6 The social planner's welfare maximization problem

In his seminal paper on the subject, Shavell showed that irrespective of the rule of tort law, be it strict liability or any of the forms of negligence discussed above, only party adopts an optimal level of activity. The Shavell Theorem as it is known proves the non-existence of a rule of law which can achieve both the optimal level of care and optimal level of activity. Essentially, if the standard of care is set optimally, it will induce both Injurer and Victim to choose the level of care in equilibrium. However, doing so will guarantee the injurer is not the residual bearer of the cost of an accident. As such, the injurer will fail to internalize the externality generated by excesses engagement of the activity.

The objective of this section therefore is to show that a sufficiently small levels of uncertainty can be welfare improving under the assumptions on the accident technology and distributional form of uncertainty made throughout. Formally, the social planner's problem can be written as

$$\min_{\bar{x},\Gamma} \qquad W(a_n, a_v, x_n, x_v) = U_n(a_n) + U_v(a_v) - a_n * \hat{x_n} - a_v * \hat{x_v} - F(a_n, a_v)\Big[\lambda L(\hat{x}_n, \hat{x}_v)\Big], \quad s.t.$$

$$\begin{aligned}
\hat{x}_n &= \operatorname*{argmin}_{x_n, a_n} \qquad W_n = U_n(a_n) - a_n * x_n - F(a_n, a_v)\Big[\lambda L(x_n, x_v)\Gamma(x_n, x_v; \bar{x})\Big] \\
\hat{x}_v &= \operatorname*{argmin}_{x_v, a_v} \qquad W_v = U_v(a_v) - a_v * x_v - F(a_n, a_v)\Big[\lambda L(x_n, x_v)(1 - \Gamma(x_n, x_v; \bar{x}))\Big],
\end{aligned}$$

where a_i is party i's level of activity, U_i is party i's utility function and $F(a_n, a_v)$ is a function that measures the aggregate frequency of the level of activity.[25]. Assuming that agents have identical preferences over the activity level and that U_i is concave, $\frac{\partial U_i}{\partial a_i} < 0$. I further assume that the marginal utility of the activity approaches ∞ as activity approaches zero. These assumption on the utility function of the parties essentially guarantees that at the optimum $a_i > 0$. Further, analytical tractability I assume that $F(a_n, a_v) = a + b$. Although the injurer and victim do not independently impose a risk of an accident, the additive function is a proxy for the fact that a additional engagement in the activity by either increases the chance that an accident will occur. Differentiating W with respect to a_n, a_b, x_n and x_v, we have

[25] I assume a utilitarian social welfare function without loss of generality

$$
\begin{aligned}
\frac{\partial W}{\partial a_n} = U_n' - x_n - \lambda L &= 0 \qquad (31)\\
\frac{\partial W}{\partial a_v} = U_v' - x_v - \lambda L &= 0 \qquad (32)\\
\frac{\partial W}{\partial x_n} = -a_n - (a_n + a_v)\Big[\lambda L_n\Big] &= 0 \qquad (33)\\
\frac{\partial W}{\partial x_v} = -a_v - (a_n + a_v)\Big[\lambda L_v\Big] &= 0 \qquad (34)
\end{aligned}
$$

Given the concave, symmetric nature of the problem, there must be some $(a_n^*, a_v^*, x_n^*, x_v^*)$ that solves the social planner's problem.

In the absence of uncertainty, injurer and victim will never both choose the efficient level of care if cost is increasing in the activity level of injurer

For an equilibrium where both parties choose the efficient level of care to exist, I let $x_n = \bar{x}$ and show that victim will never choose $x_n = \bar{x}$, as injurer would not choose the optimal level of activity whenever he is non-negligent. Let $x_n = \bar{x}$. Injurer's problem can now be written as

$$
\max_{a_n} \qquad W_n(a_n) = U_n(a_n) - a_n \cdot \bar{x}, \qquad (35)
$$

Assuming that victim is behaving optimally, i.e., choosing $x_v = \bar{x}$, we note that injurer is no longer solving the social planner's problem. In fact, his first order condition is

$$
\frac{\partial W_n}{\partial a_n} = U_n' - \bar{x} = 0 \qquad (36)
$$

We note that the first order condition for the social planner's problem, (32), differs in that λL is always positive, implying that injurer will choose a higher level of activity in equilibrium than the level of activity that solves the social planner's problem. We now turn to the victim's problem, given injurer's choice of $x_n = \bar{x}$ and $a_n > a^*$.

I next turn to the victim's problem. Given the equilibrium choices by injurer, victim knows with certainty that he will bear the cost of any accident, leading to the following problem:

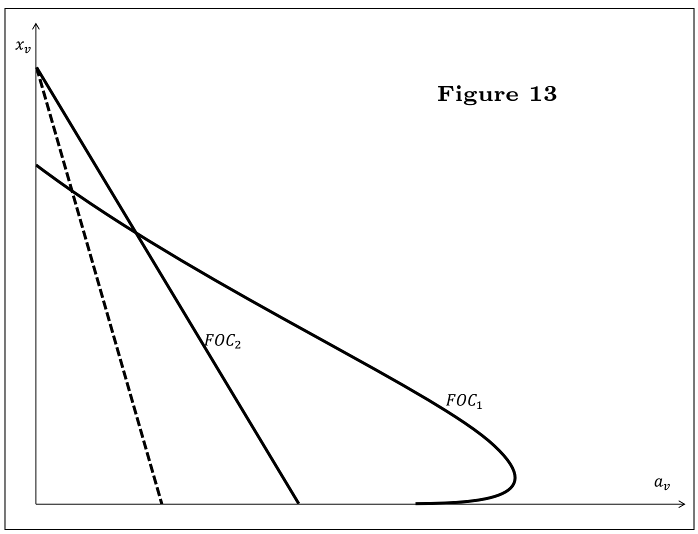
$$\max_{a_v, x_v} \qquad W_v(a_v, x_v) = U_n(a_n) - a_v * x_v - (a_n + a_v) * \Big[\lambda L(x^*, x_v)\Big] \tag{37}$$

We note that while x_n is fixed at $\bar{x}$, we allow a_n to vary. Differentiating (38) with respect to x_v and a_v, we have

$$\frac{\partial W_v}{\partial a_v} = U_v'(a_v) - x_v - \Big[\lambda L(x^*, x_v)\Big] \quad = \quad 0 \tag{38}$$

$$\frac{\partial W_v}{\partial x_v} = a_v - (a_n + a_v) * \Big[\lambda L_v\Big] \quad = \quad 0 \tag{39}$$

Comparing these first order conditions to (33) and (35), Figure 5 below shows the plot of these two first order conditions when $a_n = a^*$. Given that assumption, the victim would be solving the social planner's problem since $x_n = x^*$, and therefore represent the point where both parties choose efficient level of care and activity.



Firstly, given the conditions on the social planner's problem we know that there is a unique intersection between graphs of the two first order conditions. The intersection between the solid lines represents the equilibrium where both injurer and victim choose the efficient level of care and activity. The dashed line represents the shift in the victim's problem for $a_n > a^*$. Since injurer's activity level only affects one of the two first order conditions, there is no change in the graph of (39) and as the Figure shows, in equilibrium victim will always choose a higher level of care. In other words, *victim anticipates the inefficiently high level* *of activity by injurer and given that he bears all the cost of the accident in equilibrium he compensates by increasing his level of care and decreasing his level of activity relative to the efficient levels.* If injurer chose the optimal level of activity, victim would behave optimally as well.

6.1 The introduction of uncertainty: Discussion and numerical example

The results of Sections 3 and 4 imply that for sufficiently small levels of uncertainty, there exists some comparative negligence rule that would induce efficient levels of care from both injurer and victim in equilibrium. Turning once again to the injurer's problem and letting $x_n = x^*$[26], we have

$$\max_{a_n} \qquad W_n(a_n) = U_n(a_n) - a_n \cdot \bar{x} - \rho F(a_n, a_v) \cdot \lambda L(x^*, x^*), \tag{40}$$

Where ρ is some fraction of the damages that injurer expects to pay in expectation. Differentiating (41) with respect to a_n, we have

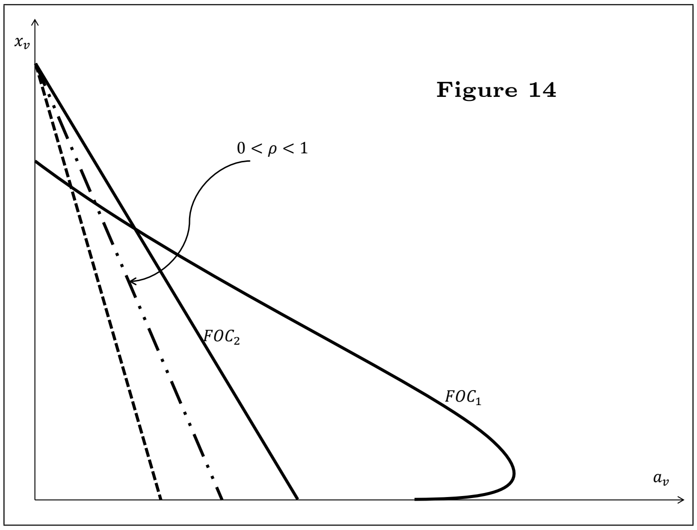
$$\frac{\partial W_n}{\partial a_n} = U_n' - \bar{x} - \rho \cdot \lambda L(x^*, x^*) = 0 \tag{41}$$

Comparing (32),(37) and (42) clearly implies that injurer will choose to engage in the activity less frequently relatively to the problem with no uncertainty, but more frequently than the solution to the social planner's problem. Given the inefficiently high level of engagement, uncertainty has an unambiguously positive welfare affect from the perspective of the injurer's behavior as she internalizes some portion ρ of the social cost of her activity.

Turning to the victim's equilibrium behavior, Figure 14 below compares the victim's first order conditions for the social planner's problem, the problem with no uncertainty and the problem with uncertainty. Comparing the figure to Figure 13, the dashed line is the shift in the victim's best response function relative to the social planner's problem for some

[26] This assumes that the standard of care and division of liability are set such that injurer chooses the efficient level of care. As shown Sections 3 and 4, such a rule must exist for sufficiently small levels of uncertainty whenever the parties' care reduces the likelihood of an accident independently of the others'

$0 < \rho < 1$. Figure 14 shows that in solving the problem with uncertainty, the victim will choose a level of caution and level of activity closer to the efficient level, both of which are unambiguously welfare improving.



Numerical example

The table below represents numerical solutions to the general welfare problem for various levels of uncertainty using the following functional forms: $U_i = 50{a_i}^{\frac{1}{2}} - 15a_i$,$F(a_n, a_v) = a_n + a_v$, $\lambda = 9$, $L(x_n, x_v) = \frac{1}{1+x_n} + \frac{1}{1+x_v}$, Φ_i is a normal distribution.

Solving for the solution of the social planner's problem, the optimal level of care is $a_i = 1.23618$ and the optimal level of care is $x_i = 3.24264$, for net utility of 55.5919. The table below contains the solution to the problem first with no uncertainty followed by various levels of uncertainty.

Variance	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
-	-	-	3.24264	3.80157	0	1.87804	1.20258	54.17185
1	1.7694	0.503	3.24257	3.24205	0.06789	1.81301	1.27266	54.34784
2	1.3271	0.549	3.24257	3.24653	0.15624	1.74202	1.31653	54.59134
3	1.766	0.481	3.24221	3.24224	0.26102	1.66927	1.36759	54.78747

(a) Optimal standard of care

(b) Division of liability rule

(c) Injurer level of care
(d) Victim level of care
(e) Injurer expected share of damages
(f) Injurer level of activity
(g) Victim level of activity
(h) Net welfare

The reader should note the correlation between column (e), the injurer's share of damages, to the decrease in activity level of injurer. As analytically shown in the previous section, the introduction of uncertainty incentivizes injurer to internalize some portion of the externality generated by repeated engagement in the risky activity.

7 Conclusion

The current literature on the bilateral accident problem focuses either on the cautionary behavior of parties in the face of uncertainty or the externality created by the frequency of their engagement in the activity. For a better understanding of the optimal design of tort laws, both problems should be considered simultaneously as such rules affect agents' behavior on both dimensions.

The possibility of a cost apportionment rule that induces injurers and victims to take the efficient level of care and simultaneously internalize some portion of the externality of repeated engagement in the activity has been ruled out in the perfect information setting. The problem has not been addressed in a setting with evidentiary uncertainty because uncertainty generally creates inefficiency in care, generating a trade-off between costly distortions in the parties' level of care and possible gains from more efficient levels of activity. As this paper proves, however, for a general class of accidents efficient care can be induced for sufficiently low levels of uncertainty if the standard of care and division of liability under comparative negligence are optimally chosen. Moreover, under such a rule, in equilibrium injurer is in expectation liable for some non-zero portion of damages, inducing her to internalize a fraction of the social cost of her behavior when she otherwise does not do so with perfect information. As such, the paper shows that less rigid legal standards of care, such as the reasonable care standard, combined with an optimally designed comparative negligence rule are unambiguously welfare improving relative to the perfect information setting.

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