Preliminary Exam Syllabus
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Model categories

1. Definition of model categories. Basic consequences of the axioms: the retract argument, closure properties of the distinguished classes of morphisms, lifts and extensions, Ken Brown’s lemma.

2. Homotopies in a model category. The Whitehead theorem in a model category. Construction of the derived category (i.e. homotopy category) of a model category.


5. Definition of cofibrantly generated model categories.

6. Examples:
   • The standard model structure on topological spaces.
   • The projective and injective model structures on unbounded chain complexes.
   • The standard model structure on simplicial sets.
   • The stable homotopy category:
     – Definitions of May spectra, L-spectra and S-modules.
     – Construction of the smash product of each of the above.
     – Definitions of ring and module spectra.
     – The model structure on May spectra and the induced model structure on S-modules.

References:

Other contexts for homotopy theory. Models for (\infty, 1)-categories

1. Homotopical categories.
   • Definition of homotopical categories.
   • The underlying homotopical category of a model category.
   • Derived functors on homotopical categories via deformations.

2. Simplicially enriched categories.
   • Definition of simplicially enriched categories.
   • The Dwyer-Kan hammock localization of a homotopical category producing a simplicially enriched category.
3. Quasicategories.

- Definition of quasicategories.
- The homotopy coherent nerve functor from simplicially enriched categories to quasicategories.
- Examples:
  - Nerves of ordinary categories are quasicategories.
  - Singular complexes of topological spaces are quasicategories.
- The homotopy category of a quasicategory. Adjoint relationship to the nerve functor.
- Mapping spaces in a quasicategory. Mapping spaces are quasigroupoids.

References: