Feedforward (FF) control uses a priori knowledge about a given system and its disturbances to influence the system’s behavior in a pre-defined way. However, unlike feedback (FB) control, it does not adjust the control signal (or manipulated variable) in response to how the system actually reacts. In other words, FF control is proactive while FB control is reactive.

Model-inversion-based FF (MIBFF) control is a specific type of FF control where the control variable is determined by inverting a model of the system to be controlled. It is very popular for its use in output tracking control problems, where the goal is to force the system’s output to follow a desired trajectory. Output tracking is critical in several different applications, e.g., in manufacturing, robotics, automotive, aerospace, and is typically achieved using a combination of FF and FB control. As motivating examples, consider MIBFF in ultra-precise wafer scanners used in photolithography (see Sidebar S1) and in desktop 3D printing (see Sidebar S2). As these motivating examples illustrate, non-minimum phase (NMP) zeros often arise in practical engineering applications due to non-collocated sensors and actuators, fast sampling, etc. MIBFF for
systems with NMP zeros is arguably the most important (and certainly the most researched) issue related to MIBFF.

**METHODS FOR FF CONTROL OF SYSTEMS WITH NMP ZEROS**

The FF output tracking problem is illustrated in block diagram form in Figure 3. Consider the simple case of a linear time invariant (LTI) single-input single-output (SISO) discrete-time system (with transfer function $G(q)$), controlled by a tracking controller $C(q)$, where $q$ is the forward shift operator. The system $G(q)$ could represent the model of an open-loop plant or that of a closed-loop controlled system. Given a desired trajectory, $y_d(k)$, where $0 \leq k \leq M$, $k \in \mathbb{Z}$ and $M+1$ is the number of discrete points in the trajectory, the objective of tracking control is to design $C(q)$ such that its output (i.e., the control trajectory) $u(k)$, after passing through $G(q)$, results in an output trajectory $y(k)$ that is sufficiently close to $y_d(k)$. Ideally, $C(q) = G(q)^{-1}$ should be selected such that the overall transfer function $L(q) = C(q)G(q)$, from $y_d(k)$ to $y(k)$, is unity over the entire range of frequencies, resulting in perfect tracking. However, $C(q) = G(q)^{-1}$ yields unbounded or highly-oscillatory control signal, $u(k)$, if $G(q)$ contains NMP zeros.

A lot of research has been done on developing methods for tracking control of systems with NMP zeros [3]-[5]. The simplest methods are NMP zero ignore (NPZ-ignore), zero phase error tracking controller (ZPETC), and zero magnitude error controller (ZMETC) [4]. NPZ-ignore cancels all poles and cancellable zeros while ignoring the NMP and poorly-damped zeros; it results in a controlled system that exhibits magnitude and phase errors between its desired and output trajectories. ZMETC focuses on cancelling magnitude errors across all frequencies, whereas zero phase error tracking control (ZPETC) focuses on cancelling phase errors across all frequencies at the expense of magnitude errors.

However, depending on the system and the performance specifications, NPZ-ignore, ZMETC and ZPETC may not yield

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**S1: BEYOND RIGID BODY FF CONTROL OF WAFER SCANNERS FOR PHOTOLITHOGRAPHY**

A wafer scanner (see Figure 1) is a device used to deliver ultra-precise motion needed for the photolithography process in integrated circuit manufacturing. Wafer scanners must achieve nanometer-level servo errors in tracking motion trajectories with velocities of up to 1 m/s and 10 g acceleration [1]. Over 99% of the control effort required for generating the desired motion is contributed by FF control [2], because wafer scanners are specially designed to minimize uncertainties and unknown disturbances. In a typical wafer scanner, MIBFF is achieved by inverting a rigid body model of a moving stage of mass $m$ (i.e., control input, $u = ma = m\ddot{y}$). However, as $\ddot{y}$ (desired acceleration) increases, there is a push to reduce $m$ so that the required $u$ is not excessive. An unintended result of reducing $m$ is that the stiffness of the stage reduces, thus invalidating the rigid body assumption used for MIBFF. This has given rise to so-called beyond rigid body (BRB) FF control, which considers the flexible modes of the stage in MIBFF control of wafer scanners [1][2]. A major concern that arises with the inclusion of flexibilities in MIBFF control is the presence of NMP zeros, which are almost certain to occur when sensors and actuators are non-collocated as in wafer scanners. Moreover, model uncertainties increase as high-frequency vibration modes are introduced into MIBFF. These issues are further complicated by the fact that the structural dynamics vary from location to location on the stage, hence a position (and time) varying model is required for MIBFF control of wafer scanners.

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**FIGURE 1** Schematic of a wafer scanner; it is critical to integrated circuit manufacturing and relies >99% on FF control.
satisfactory tracking accuracy due to the approximations involved [4]. To improve tracking accuracy, advanced methods have been developed, e.g., extended bandwidth ZPETC, truncated series (TS), direct inversion with bounded reference trajectories, approximate frequency domain inversion, H∞ matching, B-spline-based tracking with preview using iterative learning control, spline filtering with feedback, causal/anti-causal dynamics decomposition, etc. (as summarized in excellent review papers like [3]-[5]). A major shortcoming of most advanced methods is that they are not versatile in terms of the systems and/or the desired trajectories to which they are applicable (e.g., several of the methods cannot be applied to nonhyperbolic systems, i.e., systems with zeros on the unit circle) – see [7] for a more detailed discussion of this matter. Moreover, their tracking accuracy varies significantly depending on NMP zero location (in the complex plane) [7][8].

The filtered basis function (FBF) approach has been proposed by the authors as a means for achieving consistent feedforward tracking accuracy regardless of the location of the system NMP zeros [6][7]. The FBF approach traces its origins to the work of Frueh and Phan on Inverse Linear Quadratic Learning in the context of Iterative Learning Control [9], and also bears some similarities to less-versatile techniques proposed by Lunenberg et al. [1] and Jetto et al. [10]. Without any system inversion, the FBF approach approximates \( u(k) \) in Figure 3 as:

\[
    u(k) = \sum_{i=0}^{n} \gamma_i \phi_i(k)
\]

where \( \{\phi_i(k)\} \) is a set of \( n+1 \) linearly independent basis functions and \( \{\gamma_i\} \) is a set of coefficients, \( \gamma_i \in \mathbb{R} \). Based on Eq. (1), the control input and output trajectory can be expressed as

\[
    y(k) = Gu(k) = \sum_{i=0}^{n} \gamma_i \bar{\phi}_i(k),
\]

where \( \bar{\phi}_i(k) = G \phi_i(k) \)

In Eq. (2), \( \bar{\phi}_i \) represents the filtered basis functions, i.e., forward filtered by the system \( G \) (or its accurate model). For perfect tracking (i.e., \( y(k) = y_d(k) \) for all \( k \)), the coefficients \( \{\gamma_i\} \) should satisfy:

\[
    \begin{bmatrix}
        \bar{\phi}_0(0) & \bar{\phi}_1(0) & \ldots & \bar{\phi}_N(0) \\
        \bar{\phi}_0(1) & \bar{\phi}_1(1) & \ldots & \bar{\phi}_N(1) \\
        \vdots & \vdots & \ddots & \vdots \\
        \bar{\phi}_0(M) & \bar{\phi}_1(M) & \ldots & \bar{\phi}_N(M)
    \end{bmatrix}
    \begin{bmatrix}
        \gamma_0 \\
        \gamma_1 \\
        \vdots \\
        \gamma_N
    \end{bmatrix}
    =
    \begin{bmatrix}
        y_0(0) \\
        y_1(0) \\
        \vdots \\
        y_N(0)
    \end{bmatrix}
\]

**S2: BEYOND RIGID BODY FF CONTROL OF DESKTOP 3D PRINTERS**

There is a push to make desktop 3D printers as widely available as desktop office printers, in a bid to democratize manufacturing. To achieve this goal, desktop 3D printers must be low-cost, lightweight, and have reasonable print speeds and print quality (see Figure 2a). To keep costs low, desktop 3D printers are typically controlled 100% in open loop (i.e., FF) using stepper motors. However, given their lightweight structures, they have structural flexibilities, which cause them to vibrate excessively due to the motion of their print head and/or platform. Excessive vibration leads to poor surface quality or, even worse, causes the stepper motors to skip counts leading to scrapped parts (see Figure 2b). These issues are currently dealt with by reducing the speed of the printers, which significantly slows down the printing process (already notorious for its low throughput). Without unduly sacrificing throughput, the vibration-induced problems of desktop 3D printers could be resolved using FB control, but this would require servo motors, sensors, and high-performance drives to run active vibration control algorithms, all adding significant cost to the 3D printers. Moreover, to be effective, such sensors would have to be placed close to the print head where they can properly observe the unwanted vibration; this may not be practical. A more practical alternative, which can be implemented via stepper motors without extra hardware investments, is MIBFF control. However, much like wafer scanners, NMP zeros, and position-varying dynamics, are also issues in BRB FF control of 3D printers that must be addressed.

**FIGURE 2**

(a) Desktop 3D printer, and (b) Scrapped 3D printed part due to vibration of printer. Desktop 3D printers rely 100% on FF control.
An exact solution for Eq. (3) exists if and only if the rank of $\Phi$ is equal to the rank of $[\Phi | y_d]$, i.e., if $y_d$ belongs to the span of the filtered basis functions $\{\tilde{\varphi}_i\}$. However, for the general situation where $n < M$, Eq. (3) is overdetermined, and the solution to Eq. (3) is approximated as [6][7]:

$$\gamma = (\tilde{\Phi}^T \tilde{\Phi})^{-1} \tilde{\Phi}^T y_d$$

This approximation is the least square solution minimizing $||e||_2^2$, where $e = [e(0) e(1) ... e(E)]^T$ is the tracking error vector, based on the definition $e(k) = y_d(k) - y(k)$. Note that the solution to Eq. (4) exists and is unique, if and only if the filtered basis functions $\{\tilde{\varphi}_i\}$ are linearly-independent (i.e., if $\tilde{\Phi}$ is of full rank $n+1$). It is shown in [7] that this linear-independence condition can be readily achieved, given the flexibility in selecting the basis functions and their initial conditions in filtering.

**Figure 3** depicts the FBF method's generation of control input $u = [u(0) u(1) ... u(M)]^T$ from $y_d$ and BFs ($\Phi$), as well as its application to system $G(q)$ to output $y = [y(0) y(1) ... y(M)]^T$. Following the tracking control block diagram of **Figure 3**, the overall dynamics $L$ and tracking controller $C$ have functionally-equivalent counterparts in the FBF framework, given by the matrices $L_{FBF}$ and $C_{FBF}$:

$$\begin{align*}
y &= \tilde{\Phi}(\tilde{\Phi}^T \tilde{\Phi})^{-1} \tilde{\Phi}^T y_d, \\
u &= \Phi(\Phi^T \Phi)^{-1} \Phi^T y_d
\end{align*}$$

**Figure 4** Flowchart of FBF approach for FF tracking of NMP systems.

**Figure 5** Normalized tracking error over different zero locations for FBF-BPF, FBF-DCT, ZPETC and TS. Notice that both cases of FBF maintain consistent tracking accuracy irrespective of zero location, while the accuracies of ZPETC and TS vary significantly. Note also that FBF is applicable to all zero locations, while ZPETC and TS are not applicable at $a = 1$ and $|a| = 1$, respectively.

**Figure 6** Comparison of photographs and measured surface profiles ($h$) of the highlighted surfaces of blocks printed using (a) baseline approach (no vibration compensation) and (b) LPFBS method. LPFBS stands for limited-preview filtered B-splines. It is a low-computational-cost implementation of the FBF method using B-splines as basis functions.
Note that the $L(q)$ and $C(q)$ of any linear MIBFF method can be expressed as matrices $L$ and $C$, respectively, via the so-called lifted system representation [7]. It is interesting to note that $L_{qpp}$ and $C_{qpp}$ are not generally Toeplitz matrices [7] (i.e., matrices having constant entries along their diagonals). This constitutes a fundamental difference between the FBF method and most other MIBFF methods, whose $L$ and $C$ matrices are always Toeplitz. The implication is that while most other methods result in LTI controllers, *the FBF method generally yields an LTV controller*. It is this fundamental difference that enables the improvements over other MIBFF methods, in terms of consistent performance regardless of NMP zero location, that is evident from the comparisons in the next section. Note also that, though discussed in the context of LTI SISO systems, the FBF approach is applicable to any discrete-time linear system — including multi-input, multi-output and linear time-varying systems (see, e.g., [11])

**COMPARISON OF FBF TO ZPETC AND TS**

This section evaluates the FBF method in comparison with the ZPETC and TS methods using the first-order NMP system studied by Butterworth et al. [8]; it is defined as:

$$G(q) = \frac{q - a}{q - p} \quad (6)$$

where $a$ and $p = 0.5$ are the zero and pole of the system, respectively. TS is selected for the comparison because, among the advanced techniques for approximate inversion, it is very versatile with regards to its applicability irrespective of the location of the NMP zeros in the $z$-plane; ZPETC is considered because of its popularity. There is a wide range of basis functions available for use with the FBF method; e.g., Laguerre functions, wavelets, B-splines, etc. In the simulation studies, two basic basis functions — the block pulse function (BPF) and the discrete cosine transform (DCT) — are adopted [7]. The desired trajectory ($y_d$) used for the simulations is a zero-mean white noise signal with unity variance, $M = 1000$.

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TABLE 1 3D printed models of US Capitol using different acceleration limits (which influences total printing time). The LPFBS method eliminates the vibration-induced distortions observed in the baseline case at higher acceleration limits; the feedrate is 60 mm/s for all cases. LPFBS stands for limited-preview filtered B-splines. It is a low-computational-cost implementation of the FBF method using B-splines as basis functions.

<table>
<thead>
<tr>
<th>Acceleration Limit</th>
<th>1 m/s²</th>
<th>3 m/s²</th>
<th>5 m/s²</th>
<th>7 m/s²</th>
<th>10 m/s²</th>
<th>30 m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Printing Time</td>
<td>3:59 h</td>
<td>2:41 h</td>
<td>2:21 h</td>
<td>2:12 h</td>
<td>2:06 h</td>
<td>1:50 h</td>
</tr>
<tr>
<td>Baseline (No LPFBS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LPFBS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Images of 3D printed models of US Capitol using different acceleration limits]

> 47% (1:53 h) time saving using conservative accelerations for both Baseline and LPFBS

> 31% (0:51 h) time saving using aggressive accelerations for both Baseline and LPFBS (it is unclear at which acceleration LPFBS fails)

**EXPERIMENTAL VALIDATION: VIBRATION COMPENSATION OF A DESKTOP 3D PRINTER**

The FBF method has been validated in experiments reported in [13] using the commercially-available HICTOP Prusa i3 3D printer shown in Figure 2a. It is utilized for FF compensation of tracking errors induced by vibrations of the printer as a result of motions of its build platform and nozzle in the x and y directions, respectively. The axis frequency response functions (FRFs) are measured by applying swept sine acceleration signals (with amplitudes ranging from 1.5 m/s² to 3.0 m/s²) to the printer’s stepper motors and measuring the relative acceleration of the build platform and print head using accelerometers. The curve fit models are generated using MATLAB®’s invfreqs function applied to the 3.0 m/s² data, in order to better compensate for vibrations induced by more aggressive motions. The resulting discrete-time transfer function for the x-axis is

\[
G(q) = \frac{0.026q^4 - 0.048q^3 - 0.003q^2 + 0.048q - 0.023}{q^5 - 4.792q^4 + 9.274q^3 - 9.060q^2 + 4.466q - 0.889}
\]  

with sampling time, \(T_s = 1 \text{ ms}\). Note that \(G(q)\) has a zero at \(q = -1\); most MIBFF control methods cannot handle such systems.

The FBF method is implemented with B-splines as the basis functions. The local property of B-splines is exploited to implement the FBF method with limited preview (i.e., in small batches) to drastically reduce computational costs [13]. Figure 6 compares the roughness \((h)\) of the highlighted surfaces of two identical blocks, both printed at 60 mm/s feedrate and 7 m/s² maximum acceleration. Notice that the RMS value of \(h\) for the block printed with FBF is 77% smaller than that of the baseline case without FBF. Table 1 shows scale models of the US Capitol printed at 60 mm/s feedrate with various acceleration limits. The FBF method allows much higher accelerations (i.e., by a factor of ten), thereby shortening print time significantly.
(i.e., by a factor of two) without sacrificing print quality. Full details of the limited-preview implementation of the FBF approach and the experimental investigation can be found in [13].

CONCLUDING REMARKS

controllers play an important role in many engineering applications, but have not received as much research attention as FB controllers. A major issue in FF controller design arises when system models with uncancellable (e.g., NMP) zeros are inverted. We have presented the filtered basis functions (FBF) approach for MIBFF tracking control and compared it to well-known methods such as ZPETC and TS. The proposed FBF method uses basis functions to represent the control signal and determines the coefficients of those basis functions to minimize the error between the desired and actual trajectories in a least-square sense. The FBF design can be applied to any linear system and yields a LTV controller even for a LTI system.

For a first-order LTI plant with a NMP zero, the simulation results show that the FBF approach, unlike ZPETC and TS, can be used and yields consistent tracking performance regardless of NMP zero location. We have also validated the FBF approach experimentally for tracking control of a desktop 3D printer with stepper motors. The results show that the print time can be cut in half using an FBF FF tracking controller, while other methods may not be applicable due to an NMP zero on the unit circle.

The main conclusions are:

- The proposed FBF approach, which yields a LTV controller, is applicable to any linear system, and yields consistent tracking accuracy irrespective of NMP zero location.
- FBF stands out in situations where other methods struggle or fail (e.g., zeros on or near the unit circle).

We are considering various extensions of the current research to more general LTV tracking controllers, to multi-input multi-output systems, nonlinear systems, and to combined FP and FB control.

REFERENCES