MATH 732

Topics in Algebraic Geometry II: Introduction to singularities

- TuTh 1:00 pm-2:30 pm, EH 1866

- Course webpage: */www-personal.umich.edu/~mmustata/732-2022.html*

- Office hours: Tuesdays 11:00 am–12:00 pm and by appointment

- The goal of the course is to give an overview of several points of views on singularities of algebraic varieties and of the connections between these, either established or conjectural. The course will consist of four parts, of roughly equal lengths:

  1) **Multiplier ideals and the log canonical threshold**. To an ideal on a smooth complex algebraic variety and to a positive rational number one associates a *multiplier ideal* either via integrability conditions or via resolution of singularities. The *log canonical threshold* detects which such ideals are trivial. Multiplier ideals measure in a subtle way the singularities of the original ideal and they found many applications in higher-dimensional algebraic geometry due to their appearance in vanishing theorems.
  2) **Test ideals and the F-pure threshold**. Test ideals are positive characteristic analogues of multiplier ideals, whose definition makes use of the Frobenius morphism. Interesting results and conjectures link them to multiplier ideals via reduction mod p.
  3) **Spaces of arcs and motivic integration**. The space of arcs $J_\infty(X)$ of a complex variety X is the space $X\big(\mathbf{C}[[t]]\big)$ of formal arcs on X. Motivic integration is an analogue of p-adic integration that associates invariants to certain functions on $J_\infty(X)$. It allows translating properties of singularities of a subvariety Y of a smooth variety X in terms of the codimension of certain subspaces of $J_\infty(X)$. Furthermore, it allows defining a *motivic* version of Igusa's p-adic zeta function of Y, whose poles contain important information about the singularities of Y.
  4) **The Bernstein-Sato polynomial**. This is an invariant defined by Bernstein using the theory of $\mathcal{D}$-modules. While originally motivated by the problem of meromorphically extending complex powers, its roots turned out to be related to various other points of view on singularities.

- Prerequisites: I will assume familiarity with algebraic varieties, coherent sheaves, and cohomology, as covered in Math 631 and 632.

- The grade in the class will be based on a final project. This will be an expository paper of 8-16 pages, submitted by the last day of classes, on a subject related to the topics covered in the course. I will post a list of suggested topics around the middle of the semester, but people could also pick other topics.