Between characteristic zero and characteristic p

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Abstracts

Lecture 1. One of the primary goals of algebraic topology is to obtain information about topological spaces X by studying algebraic invariants of X, such as the cohomology $H^*(X;R)$ with coefficients in a commutative ring R. For many applications, it is useful to consider more exotic invariants given by "extraordinary" cohomology theories (such as K-theory). In this talk, I'll give an informal introduction to the study of extraordinary cohomology theories, emphasizing their algebraic role as generalized commutative rings.

Lectures 2 and 3. Let G be a finite group. One can study representations of G over any field k: that is, vector spaces over k equipped with an action of G. In general, such representations behave very differently in characteristic zero (where all representations are completely reducible) and in characteristic p (where, if G is a p-group, there are no irreducible representations other than the trivial representation). In these talks, I will discuss representation theory over more exotic "fields" known as Morava K-theories, which in some sense interpolate between fields of characteristic zero and fields of characteristic p and share many pleasant features of both.

Lecture 4. In classical algebraic geometry, there is often a stark difference between the behavior of fields of characteristic zero (such as the complex numbers) and fields of characteristic p (such as finite fields). For example, the equation $x^p = 1$ has p distinct solutions over the field of complex numbers, but only one solution over any field of characteristic p. In this talk, I'll introduce the subject of $K(n)$-local homotopy theory, which in some sense interpolates between characteristic zero and characteristic p, and describe the curious behavior of roots of unity in this intermediate regime.