Abstracts Week 1

Lecture series

Linquan Ma

Title: Perfectoid algebras and singularities in mixed characteristic

Abstract: In this lecture series, we give an introduction to recent applications of p-adic methods to commutative algebra. We first prove a p-adic version of Kunz's theorem characterizing regular local rings via perfectoid algebras. We will discuss André's flatness lemma and use it to sketch a proof of the direct summand theorem and the existence of big Cohen-Macaulay algebras. Finally, we will use perfectoid big Cohen-Macaulay algebras to define and study singularities in mixed characteristic, and we will introduce a numerical invariant called perfectoid signature, which is defined using Bhatt-Scholze's perfectoidization functor and Faltings' normalized length function that measures these singularities.

Karl Schwede

Title: Global and local applications of perfectoid algebras in mixed characteristic

Abstract: In the first lecture, we will discuss some motivation related to Kodaira vanishing in characteristic zero, Frobenius in positive characteristic, and how finite covers and perfectoid methods can be applied in mixed characteristic to obtain similar results. In the second lecture we will discuss lifting sections via such methods in global and local settings (for instance inversion of adjunction), and how that can be used in various applications. In the third lecture, we will discuss global generation properties and applications that these methods can be used to deduce. Finally, in the fourth lecture we will discuss forthcoming work unifying various notions of mixed characteristic test ideals with multiplier ideals in characteristic zero.

Research talks

Emelie Arvidsson

Title: Properties of log canonical singularities in positive characteristic

Abstract: We will investigate if some well known properties of log canonical singularities over the complex numbers still hold true over perfect fields of positive characteristic and over excellent rings with perfect residue fields. We will discuss both pathological behavior in characteristic p as well as some positive results for threefolds. We will see that the pathological behavior of these singularities in positive characteristic is closely linked to the failure of Kodaira-type vanishing theorems in positive characteristic. Additionally, we will explore how these questions are related to the moduli theory of varieties of general type.

This is based on joint work with F. Bernasconi and Zs. Patakfalvi, as well as joint work with Q. Posva.

Manuel Blickle

Title: F-finite schemes have a dualizing complex

Abstract: I will present joint work with Bhatt, Schwede and Tucker where we show that by surprisingly elementary means one can equip the category of F-finite schemes with a canonical dualizing complex $\omega^\bullet$ such that for all essentially finite type maps $f\colon X \to Y$ there is a canonical isomorphism $\omega_X^\bullet \cong f^! \omega_Y^\bullet$. This is in particular true for the finite Frobenius morphism yielding a canonical isomorphism $\omega_X^\bullet \cong F^! \omega_X^\bullet$.

Kevin Tucker

Title: On F-pure Inversion of Adjunction

Abstract: Suppose R is a local ring of prime characteristic $p > 0$ and f is a regular element. Viewed through the lens of reduction to characteristic $p > 0$, Kawakita's inversion of adjunction for log canonical singularities roughly predicts that R/fR is F-pure only if the pair $(R, \mathrm{div}(f))$ is as well, i.e., $R \xrightarrow{1 \mapsto F_* f^{p-1}} F_* R$ is pure. In this talk, I will discuss joint work with Thomas Polstra and Austyn Simpson showing this equivalence provided R is $\mathbb{Q}$-Gorenstein with arbitrary index. In particular, we recover previous results of Schwede (who showed this equivalence provided the index is prime to p) and Polstra-Simpson (who showed F-purity deforms in $\mathbb{Q}$-Gorenstein rings). Time permitting, we will also discuss additional results relating the F -purity of divisor pairs $(R, S + B)$ and that of $(S, \mathrm{Diff}_S(B))$.