Why and How to Differentiate Math Instruction

STUDENTS IN ANY CLASSROOM differ in many ways, only some of which the teacher can reasonably attend to in developing instructional plans. Some differences will be cognitive—for example, what previous concepts and skills students can call upon. Some will be more about learning style and preferences, including behaviors such as persistence or inquisitiveness or the lack thereof; whether the student learns better through auditory, visual, or kinesthetic approaches; and personal interests.

THE CHALLENGE IN MATH CLASSROOMS

Although many teachers of language arts recognize that different students need different reading material, depending on their reading level, it is much less likely that teachers vary the material they ask their students to work with in mathematics. The math teacher will more frequently teach all students based on a fairly narrow curriculum goal presented in a textbook. The teacher will recognize that some students need additional help and will provide as much support as possible to those students while the other students are working independently. Perhaps this occurs because differentiating instruction in mathematics is a relatively new idea. Perhaps it is because it is not easy in mathematics to simply provide an alternate book to read (as can be done in language arts). And perhaps it is because teachers may never have been trained to really understand how students differ mathematically. However, students in the same grade level clearly do differ mathematically in significant ways. Teachers want to be successful in their instruction of all students, and under new laws they are mandated to do so. Understanding differences and differentiating instruction are important processes for achievement of that goal.

The National Council of Teachers of Mathematics (NCTM), the professional organization whose mission it is to promote, articulate, and support the best possible teaching and learning in mathematics, recognizes the need for differentiation. The first principle of the NCTM Principles and Standards for School Mathematics reads, “Excellence in mathematics education requires equity—high expectations and strong support for all students” (NCTM, 2000, p. 12).
In particular, NCTM recognizes the need for accommodating differences among students, taking into account prior knowledge and intellectual strengths, to ensure that each student can learn important mathematics. "Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students" (NCTM, 2000, p. 12).

**How Students Might Differ**

One way that we see the differences in students is through their responses to the mathematical questions and problems that are put to them. For example, consider the task below, which might be asked of 3rd-grade students:

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In one cupboard, you have three shelves with five boxes on each shelf. There are three of those cupboards in the room. How many boxes are stored in all three cupboards?
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Students might approach the task in very different ways. Here are some examples:

- Liam immediately raises his hand and simply waits for the teacher to help him.
- Angelita draws a picture of the cupboards, the shelves, and the boxes and counts each box.
- Tara uses addition and writes $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$.
- Dejohn uses addition and writes $5 + 5 + 5 = 15$, then adds again, writing $15 + 15 + 15 = 45$.
- Rebecca uses a combination of multiplication and addition and writes $3 \times 5 = 15$, then $15 + 15 + 15 = 45$.

**The Teacher's Response**

What do all these different student approaches mean for the teacher? They demonstrate that quite different forms of feedback from the teacher are needed to support the individual students. For example, the teacher might wish to:

- Follow up with Tara and Dejohn by introducing the benefits of using a multiplication expression to record their thinking.
- Help Rebecca extend what she already knows about multiplication to more situations.
- Encourage Liam to be more independent, or set out a problem that is more suitable to his developmental level.
- Open Angelita up to the value of using more sophisticated strategies by setting out a problem in which counting becomes even more cumbersome.
These differences in student approaches and appropriate feedback underscore the need for a teacher to know where his or her students are developmentally to be able to meet each one's educational needs. The goal is to remove barriers to learning while still challenging each student to take risks and responsibility for learning (Karp & Howell, 2004).

WHAT IT MEANS TO MEET STUDENT NEEDS

One approach to meeting each student's needs is to provide tasks within each student's zone of proximal development and to ensure that each student in the class has the opportunity to make a meaningful contribution to the class community of learners. Zone of proximal development is a term used to describe the "distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86).

Instruction within the zone of proximal development allows students, whether through guidance from the teacher or through working with other students, to access new ideas that are close enough to what they already know to make the access feasible. Teachers are not using educational time optimally if they either are teaching beyond a student's zone of proximal development or are providing instruction on material the student already can handle independently. Although other students in the classroom may be progressing, the student operating outside his or her zone of proximal development is often not benefiting from the instruction.

For example, a teacher might be planning a lesson on multiplying a decimal by a whole number. Although the skill that is the goal of the lesson is to perform a computation such as $3 \times 1.5$, there are three underlying mathematical concepts that a teacher would want to ensure that students understand. Students working on this question should know:

- What multiplication means (whether repeated addition, or the counting of equal groups, or calculating the area of a rectangle)
- That multiplication has those same meanings regardless of what number 3 is multiplying
- That multiplication can be accomplished in parts (the distributive principle), for example, $3 \times 1.5 = 3 \times 1 + 3 \times 0.5$

Although the planned lesson is likely to depend on the fact that students understand that 1.5 is 15 tenths or 1 and 5 tenths, a teacher could effectively teach the same lesson even to students who do not have that understanding or who simply are not ready to deal with decimals. The teacher could allow the less developed students to explore the concepts using whole numbers while the more advanced students are using decimals. Only when the teacher felt that the use of decimals was in an individual student's zone of proximal development would the teacher ask that student to work with decimals. Thus, by making this adjustment, the teacher differentiates the task to locate it within each student's zone of proximal development.
ASSESSING STUDENTS' NEEDS

For a teacher to teach to a student’s zone of proximal development, first the teacher must determine what that zone is by gathering diagnostic information to assess the student’s mathematical developmental level. For example, to determine a 3rd- or 4th-grade student’s developmental level in multiplication, a teacher might use a set of questions to find out whether the student knows various meanings of multiplication, knows to which situations multiplication applies, can solve simple problems involving multiplication, and can multiply single-digit numbers, using either memorized facts or strategies that relate known facts to unknown facts (e.g., knowing that 6 × 7 must be 7 more than 5 × 7).

Some tools to accomplish this sort of evaluation are tied to developmental continua that have been established to describe students’ mathematical growth (Small, 2005a, 2005b, 2006, 2007). Teachers might also use locally or personally developed diagnostic tools. Only after a teacher has determined a student’s level of mathematical sophistication, can he or she even begin to attempt to address that student’s needs.

PRINCIPLES AND APPROACHES TO DIFFERENTIATING INSTRUCTION

Differentiating instruction is not a new idea, but the issue has been gaining an ever higher profile for mathematics teachers in recent years. More and more, educational systems and parents are expecting the teacher to be aware of what each individual student needs and to plan instruction to focus on those needs. In the past, this was less the case in mathematics than in other subject areas, but now the expectation is common in mathematics as well.

There is general agreement that to effectively differentiate instruction, the following elements are needed:

- **Big Ideas.** The focus of instruction must be on the big ideas being taught to ensure that they all are addressed, no matter at what level.

- **Choice.** There must be some aspect of choice for the student, whether in content, process, or product.

- **Preassessment.** Prior assessment is essential to determine what needs different students have (Gregory & Chapman, 2006; Murray & Jorgensen, 2007).

Teaching to Big Ideas

The Curriculum Principle of the NCTM Principles and Standards for School Mathematics states that “A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades” (NCTM, 2000, p. 14).

Curriculum coherence requires a focus on interconnections, or big ideas. Big ideas represent fundamental principles; they are the ideas that link the specifics. For example, the notion that benchmark numbers are a way to make sense of other numbers is equally useful for the 1st-grader who relates the number 8 to the more
familiar 10, the 3rd-grader who relates 93 to the more familiar 100, or the 8th-grader who relates π to the number 3. If students in a classroom differ in their readiness, it is usually in terms of the specifics and not the big ideas. Although some students in a classroom where rounding of decimal thousandths to appropriate benchmarks is being taught might not be ready for that precise topic, they could still deal with the concept of estimating, when it is appropriate, and why it is useful.

Big ideas can form a framework for thinking about “important mathematics” and supporting standards-driven instruction. Big ideas find application across all grade bands. There may be differences in the complexity of their application, but the big ideas remain constant. Many teachers believe that curriculum requirements limit them to fairly narrow learning goals and feel that they must focus instruction on meeting those specific student outcomes. Differentiation requires a different approach, one that is facilitated by teaching to the big ideas.

Choice

Few math teachers are comfortable with the notion of student choice except in the rarest of circumstances. They worry that students will not make “appropriate” choices.

However, some teachers who are uncomfortable differentiating instruction in terms of the main lesson goal are willing to provide some choice in follow-up activities students use to practice the ideas they have been taught. Some of the strategies that have been suggested for differentiating practice include use of menus from which students choose from an array of tasks, tiered lessons in which teachers teach to the whole group and vary the follow-up for different students, learning stations where different students attempt different tasks, or other approaches that allow for student choice, usually in pursuit of the same basic overall lesson goal (Tomlinson, 1999; Westphal, 2007).

For example, a teacher might present a lesson on creating equivalent fractions to all students, and then vary the follow-up. Some students might work only with simple fractions and at a very concrete level; these tasks are likely to start with simple fractions where the numerator and denominator have been multiplied (but not divided) to create equivalent fractions. Other students might be asked to work at a pictorial or even symbolic level with a broader range of fractions, where numerators and denominators might be multiplied or divided to create equivalent fractions and more challenging questions are asked (e.g., Is there an equivalent fraction for \( \frac{10}{15} \) where the denominator is 48?).

Preassessment

To provide good choices, a teacher must first know how students in the classroom vary in their knowledge of facts and in their mathematical developmental level. This requires collecting data either formally or informally to determine what abilities and what deficiencies students have. Although many teachers feel they lack the time or the tools to preassess on a regular basis, the data derived from preassessment are essential in driving differentiated instruction.
Despite the importance of preassessment, employing a highly structured approach or a standardized tool for conducting the assessment is not mandatory. Depending on the topic, a teacher might use a combination of written and oral questions and tasks to determine an appropriate starting point for each student.

**TWO CORE STRATEGIES FOR DIFFERENTIATING MATHEMATICS INSTRUCTION: OPEN QUESTIONS AND PARALLEL TASKS**

It is not realistic for a teacher to try to create 30 different instructional paths for 30 students, or even 6 different paths for 6 groups of students. Because this is the perceived alternative to one-size-fits-all teaching, instruction in mathematics is often not differentiated. To differentiate instruction efficiently, teachers need manageable strategies that meet the needs of most of their students at the same time. Through use of just two core strategies, teachers can effectively differentiate instruction to suit all students. These two core strategies are the central feature of this book:

- **Open questions**
- **Parallel tasks**

**Open Questions**

The ultimate goal of differentiation is to meet the needs of the varied students in a classroom during instruction. This becomes manageable if the teacher can create a single question or task that is inclusive not only in allowing for different students to approach it by using different processes or strategies but also in allowing for students at different stages of mathematical development to benefit and grow from attention to the task. In other words, the task is in the appropriate zone of proximal development for the entire class. In this way, each student becomes part of the larger learning conversation, an important and valued member of the learning community. Struggling students are less likely to be the passive learners they so often are (Lovin, Kyger, & Allsopp, 2004).

A question is open when it is framed in such a way that a variety of responses or approaches are possible. Consider, for example, these two questions, each of which might be asked of a whole class, and think about how the results for each question would differ:

<table>
<thead>
<tr>
<th><strong>Question 1:</strong> To which fact family does the fact $3 \times 4 = 12$ belong?</th>
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<tr>
<td><strong>Question 2:</strong> Describe the picture below by using a mathematical equation.</td>
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If the student does not know what a fact family is, there is no chance he or she will answer Question 1 correctly. In the case of Question 2, even if the student is not comfortable with multiplication, the question can be answered by using addition statements (e.g., \(4 + 4 + 4 = 12\) or \(4 + 8 = 12\)). Other students might use multiplication statements (e.g., \(3 \times 4 = 12\) or \(4 \times 3 = 12\)), division statements (e.g., \(12 \div 3 = 4\) or \(12 \div 4 = 3\)), or even statements that combine operations (e.g., \(3 \times 2 + 3 \times 2 = 12\)).

**A Different Kind of Classroom Conversation.** Not only will the mathematical conversation be richer in the case of Question 2 on the previous page—the open question—but almost any student can find something appropriate to contribute.

The important point to notice is that the teacher can put the same question to the entire class, but the question is designed to allow for differentiation of response based on each student’s understanding. All students can participate fully and gain from the discussion in the classroom learning community.

This approach differs, in an important way, from asking a question, observing students who do not understand, and then asking a simpler question to which they can respond. By using the open question, students gain confidence; they can answer the teacher’s question right from the start. Psychologically, this is a much more positive situation.

**Multiple Benefits.** There is another benefit to open questions. Many students and many adults view mathematics as a difficult, unwelcoming subject because they see it as black and white. Unlike, for instance, social studies or English, where people might be encouraged to express different points of view, math is viewed as a subject where either you get it or you don’t. This view of mathematics inhibits many students from even trying. Once they falter, they assume they will continue to falter and may simply shut down.

It is the job of teachers to help students see that mathematics is multifaceted. Any mathematical concept can be considered from a variety of perspectives, and those multiple perspectives actually enrich its study. Open questions provide the opportunity to demonstrate this.

**Strategies for Creating Open Questions.** This book illustrates a variety of styles of open questions. Some common strategies that can be used to convert conventional questions to open questions are described below:

- Turning around a question
- Asking for similarities and differences
- Replacing a number with a blank
- Asking for a number sentence
- Changing the question

*Turning Around a Question.* For the turn-around strategy, instead of giving the question, the teacher gives the answer and asks for the question. For example:
Asking for Similarities and Differences. The teacher chooses two items—two numbers, two shapes, two graphs, two probabilities, two measurements, and so forth—and asks students how they are alike and how they are different. Inevitably, there will be many good answers. For example, the teacher could ask how the number 85 is like the number 100 and how it is different. A student might realize that both numbers are said when skip counting by 5s, both are less than 200, and both are greater than 50, but only one is a three-digit number, only one ends with a 5, and only one is greater than 90.

Replacing a Number with a Blank. Open questions can be created by replacing a number with a blank and allowing the students to choose the number to use. For example, instead of asking how many students there are altogether if there are 25 in one class and 31 in another, students could be asked to choose two numbers for the two class sizes and determine the total number in both classes.

Asking for a Number Sentence. Students can be asked to create a sentence that includes certain words and numbers. For example, a teacher could ask students to create a sentence that includes the numbers 3 and 4 along with the words “and” and “more,” or a sentence that includes the numbers 8 and 7 as well as the words “product” and “equal.” The variety of sentences students come up with often surprise teachers. In the first case, students might produce any of the sentences below and many more:

- 3 and 4 are more than 2.
- 4 is more than 3 and more than 1.
- 3 and 4 together are more than 6.
- 34 and 26 are more than 34 and 20.

Changing the Question. A teacher can sometimes create an open question by beginning with a question already available, such as a question from a text resource. Here are a few examples:

- Rodney has 4 packages of pencils. There are 6 pencils in each package. How many pencils does Rodney have in all?
- Rodney has some packages of pencils. There are 2 more pencils in each package than the number of packages. How many pencils does Rodney have in all?
What number has 3 hundreds, 2 tens, 2 thousands, and 4 ones? You can model a number with 11 base ten blocks. What could the number be?

A cookie has a diameter of 1.75 inches. Express the diameter as a fraction in simplest form. The diameter of a cookie is between 1 and 2 inches. Express the diameter as a fraction in two different ways.

What to Avoid in an Open Question. An open question should be mathematically meaningful. There is nothing wrong with an occasional question such as What does the number 6 make you think of? but questions that are focused more directly on big ideas or on curricular goals are likely to accomplish more in terms of helping students progress satisfactorily in math.

Open questions need just the right amount of ambiguity. They may seem vague, and that may initially bother students, but the vagueness is critical to ensuring that the question is broad enough to meet the needs of all students.

On the other hand, one must be careful about making questions so vague that they deter thinking. Compare, for example, a question like What's a giant step? with a question like How many baby steps are in a giant step? In the first case, a student does not know whether what is desired is a definition for the term, a distance, or something else. The student will most likely be uncomfortable proceeding without further direction. In the second case, there is still ambiguity. Some students may wonder what is meant by “baby step” and “giant step,” but many will be comfortable proceeding; they realize they are being asked about a measurement situation.

The reason for a little ambiguity is to allow for the differentiation that is the goal in the use of open questions. Any question that is too specific may target a narrow level of understanding and not allow students who are not at that level to engage with the question and experience success.

Fostering Effective Follow-Up Discussion. Follow-up discussions play a significant role in cementing learning and building confidence in students. Thus, it is important for teachers to employ strategies that will optimize the effectiveness of follow-up discussions to benefit students at all developmental levels.

To build success for all students, it is important to make sure that those who are more likely to have simple answers are called on first. By doing so, the teacher will increase the chances that these students’ answers have not been “used up” by the time they are called on.

The teacher must convey the message that a variety of answers are appreciated. It is obvious to students when a teacher is “looking for” a particular answer. An open question is designed to ensure that many answers are good answers and will be equally valued.
The teacher should try to build connections between answers that students provide. For example, when asked how 10 and 12 are alike, one student might say that both numbers are even and another might say that they are both between 10 and 20. The teacher could follow up with:

- What other even numbers are there between 10 and 20?
- Which digits tell you the numbers are even?
- Which digits tell you the numbers are between 10 and 20?

Such questions challenge all students and scaffold students who need help.

**Parallel Tasks**

Parallel tasks are sets of tasks, usually two or three, that are designed to meet the needs of students at different developmental levels, but that get at the same big idea and are close enough in context that they can be discussed simultaneously. In other words, if a teacher asks the class a question, it is pertinent to each student, no matter which task that student completed. The use of parallel tasks is an extension of Forman’s (2003) point that task modification can lead to valuable discussions about the underlying mathematics of a situation. Parallel tasks also contribute to the creation of the classroom as a learning community in which all students are able to contribute to discussion of the topic being studied (Murray & Jørgensen, 2007).

For example, suppose a teacher wishes to elicit the big idea within the NCTM Numbers and Operations strand that it is important to recognize when each mathematical operation is appropriate to use. The teacher can set out two parallel tasks:

**Option 1:** Create a word problem that could be solved by multiplying two one-digit numbers.

**Option 2:** Create a word problem that could be solved by multiplying two numbers between 10 and 100.

Both options focus on the concept of recognizing when multiplication is appropriate, but Option 1 is suitable for students only able to work with smaller factors. Further, the tasks fit well together because questions such as the ones listed below suit a discussion of both tasks and thus can be asked of all students in the class:

- What numbers did you choose to multiply?
- How did you know how many digits the product would have?
- What about your problem made it a multiplying problem?
- What was your problem?
- How could you solve it?

**Strategies for Creating Parallel Tasks.** To create parallel tasks to address a particular big idea, it is important to first think about how students might differ
developmentally in approaching that idea. Differences might relate to what operations the students can use or what size numbers they can handle, or they might involve, for example, what meanings of an operation make sense to the students.

Once the developmental differences have been identified, the object is to develop similar enough contexts for the various options that common questions can be asked of the students as they reflect on their work. For example, for the big idea that standard measures simplify communication, the major developmental difference might be the type of measurement with which students are comfortable. One task could focus on linear measurements and another on area measurements. One set of parallel tasks might be:

**Option 1:** An object has a length of 30 cm. What might it be?

**Option 2:** An object has an area of 30 cm². What might it be?

In this case, common follow-up questions could be:

- Is your object really big or not so big? How did you know?
- Could you hold it in your hand?
- How do you know that your object has a measure of about 30?
- How would you measure to see how close to 30 it might be?
- How do you know that there are a lot of possible objects?

Often, to create a set of parallel tasks, a teacher can select a task from a handy resource (e.g., a student text) and then figure out how to alter it to make it suitable for a different developmental level. Then both tasks are offered simultaneously as options for students:

**Original task (e.g., from a text):**
There were 483 students in the school in the morning. 99 students left for a field trip. How many students are left in the school?

**Parallel task:**
There are 71 students in 3rd grade in the school. 29 of them are in the library. How many are left in their classrooms?

Common follow-up questions could be:

- How do you know that most of the students were left?
- How did you decide how many were left?
- Why might someone subtract to answer the question?
- Why might someone add to answer the question?
- How would your answer have changed if one more student had left?
- How would your answer have changed if there had been one extra student to start with?
**Fostering Effective Follow-Up Discussion.** The role of follow-up discussions of parallel tasks and the techniques for encouraging them mirror those for open questions. Once again, it is critical that the teacher demonstrate to students that he or she values the tasks equally by setting them up so that common questions suit each of them. It is important to make sure students realize that the teacher is equally interested in responses from the groups of students pursuing each of the options. The teacher should try not to call first on students who have completed one of the tasks and then on students who have completed the other(s). Each question should be addressed to the whole group. If students choose to identify the task they selected, they may, but it is better if the teacher does not ask which task was performed as the students begin to talk.

**Management Issues in Choice Situations.** Some teachers are concerned that if tasks are provided at two levels, students might select the “wrong” task. It may indeed be appropriate at times to suggest which task a student might complete. This could be done by simply assigning a particular task to each student. However, it is important sometimes—even most of the time—to allow the students to choose. Choice is very empowering.

If students who struggle with a concept happen to select a task beyond their ability, they will soon realize it and try the other option. Knowing that they have the choice of task should alleviate any frustration students might feel if they struggle initially. However, students may also sometimes be able to complete a task more challenging than they first thought they could handle. This would be a very positive experience.

If students repeatedly select an easier task than they are capable of, they should simply be allowed to complete the selected task. Then, when they are done, the teacher can encourage them privately to try the other option as well.

**Putting Theory into Practice**

A form such as the one that appears on the next page can serve as a convenient template for creation of customized materials to support differentiation of instruction in math. In this example, a teacher has developed a plan for differentiated instruction on the topic of measurement. A blank form is provided in the Appendix.

The following fundamental principles should be kept in mind when developing new questions and tasks:

- All open questions must allow for correct responses at a variety of levels.
- Parallel tasks need to be created with variations that allow struggling students to be successful and proficient students to be challenged.
- Questions and tasks should be constructed in such a way that all students can participate together in follow-up discussions.

Teachers may find it challenging at first to incorporate the core strategies of open questions and parallel tasks into their teaching routines. However, after trying
MY OWN QUESTIONS AND TASKS

Lesson Goal: Area measurement

Grade Level: __4__

Standard(s) Addressed:
Choose appropriate customary and metric units to estimate and measure length, perimeter, area, weight, capacity, volume, and temperature, including square feet and square inches . . .

Underlying Big Idea(s):
The same object can be described by using different measurements.

Open Question(s):
Which shape is bigger? How do you know?

Parallel Tasks:

Option 1:
Which shape has a greater perimeter? How much greater is it?

Option 2:
Which shape has a greater area? How much greater is it?

Principles to Keep in Mind:

- All open questions must allow for correct responses at a variety of levels.
- Parallel tasks need to be created with variations that allow struggling students to be successful and proficient students to be challenged.
- Questions and tasks should be constructed in such a way that will allow all students to participate together in follow-up discussions.