Comparing Fractions: Number Sense and Benchmarks

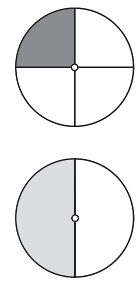
ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Distribute the worksheet and ask students to work on comparing the fractions in step 1 for a few minutes. (For now, they should ignore the write-on line beside each pair of fractions.) Explain that this is a warm-up for the exploration the class will do together. It's not expected that students will be able to compare all pairs.

2. Open **Comparing Fractions Number Sense.gsp** and go to page "Compare." To begin, make sure the model looks like the illustration at right. If students have not used a fraction-circle model before, ask questions to check that students are able to explain the relationships between the numerator of a fraction, its denominator, the number of parts in the circle, and the parts shaded. You might ask, ***What would the top circle look like if I changed 1/4 to 1/8?*** Take responses and then change the denominator of the top fraction by selecting the denominator with the **Arrow** tool and pressing **+** on the keyboard four times. As you do, let students know how you are changing the parameter for the denominator, and pause between each press.



DEVELOP

Continue to project the sketch. Expect to spend about 35 minutes.

3. Facilitate discussion of the fraction pairs in steps 1a–g. Here are suggestions for using the model. Because visual models are being presented, make sure students know when the *numerical* representation is being discussed. Students are asked to write pairs of fractions on the worksheet. This provides an opportunity to check that students are connecting the visual examples with their fraction number sense and with the numerical representation of a fraction.

   A. 2/9 AND 5/9

   Using the model, show 2/9 in both circles and then increase the numerator to 5 in one fraction by selecting the numerator and pressing **+** three times. Pause between each press so that students can see the effect on the circle. ***What can you say about comparing fractions that have the same denominator?***

You may wish to introduce the term *common denominators* now and use it in this activity.

When the denominators are the same, it's easy to compare the fractions. The parts are the same size, so the fraction representing more parts—the fraction with the larger numerator—is greater. With the numerator 5 selected, press + until the numerator is 8 or 9 to further illustrate this.

In worksheet step 1a, have students write another pair of fractions that have the same denominator and different numerators. Then have students use the symbols for greater than and less than to show which fraction is greater.

B. 4/7 AND 4/11

Using the model, show 4/7 in both circles and then, in one fraction, increase the denominator to 11 by selecting the denominator and pressing + four times. Pause between each press so that students can see the effect on the circle. ***What can you say about comparing fractions that have the same numerator?***

When the numerators are the same, it is also easy to compare the fractions. Each fraction has the same number of equal parts, but the size of the parts differs. The more parts, the smaller the parts are. The fraction with the larger denominator is less than the fraction with the smaller denominator.

Exploring extreme cases is worthwhile if the result is an engaging and memorable demonstration of an important concept.

Students may propose using the model to increase the denominator to even larger numbers. Select the larger denominator and, pressing the + key repeatedly, go all the way to 50.

On the worksheet, students should write another pair of fractions that have the same numerator and different denominators, and indicate which fraction is greater.

C. 2/4 AND 5/10

A denominator of 100 makes it impossible to see the divisions (which students enjoy seeing), whereas a denominator of 50 allows students to discern the divisions.

Students should recognize that both of these familiar fractions are equivalent to 1/2. Promote discussion about fractions that are equivalent to 1/2. Test a number of cases using the model, allowing time for students to conclude that for fractions equivalent to 1/2, the denominator is twice the numerator. Students may again suggest trying some very large values for the denominator. The quick way to change a parameter is to double-click the number, enter a new value in the dialog box that appears, and click OK. Use this method when you aren't interested in showing incremental changes.

Students should write another pair of fractions that are equivalent to 1/2, and, therefore, to each other, and use the equal sign to show the relationship.

D. 9/16 AND 4/9

Neither the numerators nor the denominators are the same. Another strategy is needed. A sample student response is this one: *I know that 4/9 is less than 1/2 because 4 is less than half of 9, and I know that 9/16 is larger than 1/2 because 9 is more than half of 16. So, 9/16 is greater.* If no student suggests a successful strategy, start the class off by asking whether 4/9 is greater than or less than 1/2.

When ideas have been considered, model 4/9 in one fraction circle by changing the numerator to 1 and the denominator to 9. Then select the numerator and press **+** until the numerator reaches 4. Stop and ask for students' thinking. ***Are you convinced that 4/9 is less than 1/2? What will the model look like if I press + again?*** Press **+** again to show 5/9. Then press **–** to return to 4/9.

Model 9/16 in the other fraction circle, changing the numerator to 1 and the denominator to 16 to start. Select the numerator and press **+** until the numerator is 9. Again discuss the results.

Students should write another pair of fractions for which relating the fractions to 1/2 is a handy way to compare them. Invite a few students to share the fraction pairs they have written, and model the fractions. (Some students may write pairs of fractions for which thinking about equivalence is an equally handy strategy, for example, 2/5 and 7/10. If students propose this alternate strategy, take time to discuss it.)

E. 2/21 AND 7/8

Thinking about how close the fractions are to benchmarks is again useful. This time students may compare the given fractions to the benchmarks 0 and 1, explaining that 2/21 is close to 0, and 7/8 is close to 1. Ask students to explain how they know that 2/21 is close to 0, and how they know 7/8 is close to 1. ***What can you say about fractions that are close to zero?*** [The numerator is much smaller than the denominator.] ***What can you say about fractions that are close to 1?*** [The numerator is close to the denominator.]

In this model, increasing the numerator until the fraction is greater than 1 will not result in the corresponding picture. Go to page "Improper Fractions" to work with fractions greater than 1.

Show 21/21 in one fraction circle, select the numerator and press **–** repeatedly until the circle shows 2/21. In the other circle, set the fraction to 1/8 and increase the numerator repeatedly until it shows 7/8.

Students should write another pair of fractions for which thinking about the benchmarks 0 and 1 is useful. Invite students to share their fraction pairs.

F. 3/4 AND 5/6

Some students may propose that the fractions are equivalent because each is "*missing one part.*" Show the fractions in the model by double-clicking each parameter and changing its value in the dialog box. Students will see that the fractions are not equivalent. ***Does that make sense? Why aren't the fractions equal to each other? After all, each whole is missing exactly one part.***

Direct students' attention to the model and ask them to observe just one of the fraction circles. Select *both* the numerator and the denominator and press **+** on the keyboard, causing the numerator and denominator to increase by one simultaneously. Repeat several times, pausing after each press. ***What can you say?*** Note whether students are convinced that as the number of parts increases, the size of the missing part decreases.

Elicit the idea that when the missing part is bigger, the given fraction is smaller. Students should reason that 5/6 is greater than 3/4 because 5/6 is 1/6 less than 1, whereas 3/4 is 1/4 less than 1.

Students should write another pair of fractions for which thinking about the "missing part" is useful.

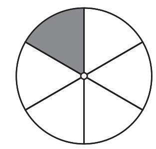
G. 8/8 AND 9/13

Students can compare again using the benchmark 1. ***What can you say about fractions equal to 1?*** [The numerator is equal to the denominator.] Students should recognize that 8/8 is equal to 1 and 9/13 is less than 1.

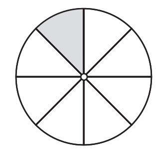
Students may want to model, using the sketch, other pairs in which one fraction is equal to 1 and the other is less than 1.

4. To include discussion of fractions greater than 1, go to page "Improper Fractions." Make sure the models show 1/6 and 1/8 to start. Write the fractions 9/6 and 16/8 on the board, and ask students to think about how they compare. Provide thinking time.

What do you think the model will look like if we show 9/6 and 16/8? Select the numerator in 1/6 and press **+** until 9/6 is shown. Select the numerator in 1/8 and press **+** until 16/8 is shown. ***How can you tell that a fraction is greater than 1?*** [The numerator is greater than the denominator.] ***How can you tell that a fraction is between 1 and 2?*** [The numerator is greater than the denominator, but less than twice the denominator.]



$\frac{1}{6}$



$\frac{1}{8}$

If time allows, have students suggest pairs of fractions to compare in which at least one of the fractions is greater than 1. Discuss students' thinking about the comparisons, and model the fractions.

SUMMARIZE

Working with or without the projected sketch, expect to spend about 15 minutes.

5. Point out that students have devised a number of strategies for comparing fractions. Focus the discussion on the usefulness of looking for a handy, or efficient, strategy when comparing fractions. Ask students to take a few minutes to consider the fractions in worksheet step 2. ***Think about which strategy helps you compare the two fractions in each pair most easily and quickly.***

When most students have had a chance to consider all the pairs, ask about each pair, inviting students to share the strategies they found easiest to use.

(Note that, in general, students should not think that they need to agree on a most efficient strategy when thinking about numbers and computation; what is most efficient depends upon an individual's number sense and experience with computation.)

EXTEND

1. Have students work independently in pairs using pages "Compare" and "Improper Fractions." Give these directions:
   - One student chooses a pair of fractions to compare, trying to stump the other. The student must know which fraction is greater.
   - The second student chooses the fraction he or she thinks is greater.
   - The partners explain their reasoning and model the fractions to check.
   - Students can keep score on paper.

2. Page "Match Up" provides another model for independent practice comparing fractions. The sketch randomly generates pairs of fractions and shows them numerically and in fraction-circle models. Students should determine which fraction is greatest by using the numerical representation as well as by deciding which fraction-circle model shows which fraction. For a hint, students can press *Show Lines.* They can also compare the fraction circles directly by selecting the center of one circle and dragging it onto the center of the other circle.
3. Present three or more fractions and ask students to put them in order. Some students may find it helpful to use the model.