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Topic
Multiplication—area model

Key Question
How can you determine how many base-ten cubes it would take to cover your rectangle?

Learning Goals
Students will:
• learn to model multiplication by constructing an array,
• learn to use powers of ten to count large arrays, and
• recognize patterns of powers of tens within arrays.

Guiding Documents
Project 2061 Benchmarks
• Add, subtract, multiply, and divide whole numbers mentally, on paper, and with a calculator.
• Express numbers like 100, 1000, and 1,000,000 as powers of 10.

NCTM Standards 2000*
• Understand the place-value structure of the base-ten numeration system and be able to represent and compare whole numbers and decimals
• Understand the effects of multiplying and dividing numbers
• Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use
• Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals
• Use mathematical models to represent and understand quantitative relationships
• Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions
• Select, apply, and translate among mathematical representations to solve problems

Math
Multiplication
Place value
Estimation

Integrated Processes
Observing
Comparing and contrasting
Generalizing

Materials
Base-ten blocks
Butcher paper, cm grid paper, or sample rectangles

Background Information
This investigation has students model the multiplication process in an area array. Prior experience will allow some students to move more quickly than others. Students just being introduced to multiplication will continue with the concrete models for an extended time. Older students will often move directly to the visual record and quickly to a numeric algorithm once the model helps them make sense of prior skills.

In the area model, the two factors give the dimensions of the rectangle, and the number of squares within the rectangle—the area—gives the product. If a rectangle is made from base-ten materials, the squares can be counted rapidly since a flat covers 100 squares, a stick 10 squares, and a cube one square. It is evident that to count the number of squares quickly, the rectangle should be covered with the fewest pieces. This makes it necessary to start with the biggest piece possible. When one factor uses the tens place, sticks will be involved. When both factors have tens in them, flats can be utilized. By being consistent with where rectangles are begun, the patterns become easier to recognize and communicate.

Students may choose a number of ways to record their experience and may switch from one to another as they become more familiar with the process. Young students initially like to make a record on a rectangle that is identical to the sample they receive. They quickly move on to either recording it on a reduced grid; making a box, line, and dot sketch; or developing a rectangle marked into regions.

By building and representationally recording a number of rectangles, students will discover some very
useful patterns. Visually they will see that flats form a rectangle in the lower left corner. The sticks form rectangles in two regions—a rectangle of sticks placed horizontally in the upper left corner, and a rectangle of sticks placed vertically in the lower right corner. The cubes form a rectangle in the upper right corner. As numbers are added to these representations, numeric patterns can be identified. The flats in the lower left corner are tens by tens products. The two regions of sticks are ones by tens products. The cubes in the upper right corner are the products of the ones place.

This visual approach provides a very powerful way for many students to make sense of multiplication processes. Some visual learners begin building the rectangles in their minds and only record the products of each region (the partial products) to sum. Other students become so confident of the patterns that they can make box, line, and dot sketches or a rectangle with regions much faster than finding the solution with traditional algorithms.

Management
1. Initially students will want to trace their solutions onto their sample rectangles. Depending on the level of the students, they may choose to make simple box, stick, and dot sketches for their records.
2. The sample rectangles provided are limited by the print space on a page. Larger rectangles can be made by tracing rectangles onto butcher paper, centimeter grid paper that is available in a roll, or by piecing standard sheets together. These can be laminated for repeated use.

Procedure
1. Give students outlines of rectangles (those provided or teacher generated) and discuss strategies for quickly determining how many unit cubes would be needed to cover the rectangle.
2. Provide the students with base-ten materials and direct them to cover their rectangles. Inform them that in order to make comparisons between groups, they should cover the rectangles with the materials using the following guidelines:
   - Fill from the lower left corner where the x is in the circle.
   - Use the fewest number of pieces by filling with flats first, sticks second, and cubes last.
3. Have each student make an individual record of the group’s solution on the sample rectangle sheets or with a sketch.
4. Working with their group, direct the students to determine a way to count the squares quickly using their record or the model. Have them record the number of cubes that would be needed to cover their rectangle.
5. When the students have completed a number of rectangles, discuss as a class:
   - their counting methods and
   - patterns of arrangement of flats, sticks, and cubes.
6. Direct the students to return to their record and add the following numeric information:
   - Record the dimensions of the rectangle with tens and ones by the respective pieces of the bottom and left side.
   - Within each region, record the dimensions and product of that region.
   - Within a blank margin, record the sum of all the regions’ products.
   - Within a blank margin, record the number of cubes and the rectangle’s dimensions as a multiplication problem.
7. While the students refer to their records, discuss what visual and numeric patterns they observe emerging.

Connecting Learning
1. What strategies did you use to count up the number of cubes rapidly? [flats by 100, sticks by 10, cubes by one]
2. What patterns did you notice in how the types of pieces were placed in each rectangle? [Flats in a rectangle in the lower left corner, a rectangle of sticks placed horizontally in the upper left corner, a rectangle of sticks placed vertically in the lower right corner, cubes in a rectangle in the upper right corner.]
3. What number patterns do you notice in the record of your solution? [tens by tens problem in lower left corner, ones by tens problem in upper left, tens by ones in lower right corner, and ones by ones problem in upper right corner]
4. How can you use your patterns to help you count the number of cubes in a rectangle if you are only told how long each side is?

Extension
Have students outline rectangles or generate the dimensions of a rectangle on their own. Have them apply their patterns and determine the amount of cubes required to cover the rectangle, and then have them check it by building and recording the model.

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### Solutions

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\[ 5 \times 10 = 50 \]
\[ 10 \times 10 = 100 \]
\[ 15 \times 15 = 225 \]

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\[ 2 \times 10 = 20 \]
\[ 10 \times 10 = 100 \]
\[ 18 \times 12 = 216 \]

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\[ 5 \times 10 = 50 \]
\[ 10 \times 10 = 100 \]
\[ 21 \times 15 = 315 \]

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\[ 4 \times 10 = 40 \]
\[ 10 \times 3 = 30 \]
\[ 23 \times 14 = 322 \]

5. 

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\[ 1 \times 10 = 10 \]
\[ 10 \times 3 = 30 \]
\[ 23 \times 11 = 253 \]

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\[ 3 \times 10 = 30 \]
\[ 10 \times 9 = 90 \]
\[ 19 \times 13 = 247 \]
Building RECTANGLES

5.

\[ x \]