Developing Concepts

Relative Value – Equivalent Fractions:  ALSO SEE: COMPARISON LESSON
The COMPARISON LESSON formalizes the use of <, >, =. Can be used with Section B of MODELS YOU CAN COUNT ON! or with Section A of FRACTION TIMES.

1.) Informal – Real World and Concrete Modeling): Use of fraction bars, cut of paper, fraction strips, and fraction circles. Do this lesson as part of MODELS YOU CAN COUNT ON (Bar Model Section – B)
   ▪ Look at this fraction bar handout. Tell me what you see.
   ▪ You discovered that some of the pieces were the same size, but had a different name.
   ▪ Here are two fractions, which one is bigger? What is your proof?
   ▪ Think about ½ what does that mean? You discovered 2/4 was the same size piece yet it had a different name. What does 2/4 mean?

2) Pre- formal – Generalizing: Use of a table to organize the investigation of the fraction bars. The table will help students generalize.
   ▪ I want you to use your fraction bars (or circle) to find all the equivalent fractions for each row. (These are fractions that may have different name, but are the same size piece.) You can work with your partner. When you have completed a row, share those results with your table to make sure you agree.
   ▪ Did you find any other equivalent fractions? Put those on your table. We will be sharing our results in a few minutes.
   ▪ Let’s look at the ½ row. What do you notice about all these fractions? Is there a pattern you see? If I had ½ and wanted to make it 2/4 what would I have to do to the pieces?
   ▪ Here are two fractions 1/3 and 1/5. Which one is bigger? How do you know? If 5 is bigger than 3, how come 1/3 is greater than 1/5? Is this always true?
   ▪ What about 2/3 and 4/6? How can you tell which one is greater? Can you use your knowledge of equivalent fractions to help?

3.) Formal – Algorithms (Use multiple strategies when possible)
   ▪ What happened to ½ to make it 2/4? What math can I use to change ½ to 3/6? Can we make a rule? Will it work every time?
   ▪ Let’s write the rule for finding equivalent fractions. You show your work this way. It explains how many times you cut up those pieces to make it look different, but still remain the same.
   ▪ Here is your model: (Have the students tell you the procedure based on the previous procedure. If they are stuck, “What did you do first?
Now try of few more using the model. If you need to check your table or bars, that’s fine.
   ▪ When we compare, read the statement in the same order as you read any other sentence. This symbol < means “less than”, what do you notice about the shape? As you move through the sentence which part of the symbol is first? This symbol > means “greater than” What do you notice about this symbol?
   ▪ Here are some fractions, compare them using the <, >, and =

NOTE: It is important that students understand a ratio is the relationship between 2 numbers. (That’s why they look different, but are the same size.)

Cross curriculum connections:  Measuring with inches, probability, circle graphs. Decimals (part/whole ratios)
Adding/Subtracting Fractions - Like Denominators:

1.) Informal - Real World and Concrete Modeling:
   - If I fill the cup ½ full, then put ¼ of a cup in what do I have? What kind of operation is that? (Add, subtract, multiply, or divide?) Why?
   - How about this one; my cup it ¾ full, I drink ½ of a cup. What do I have left? What operation is that? Why?

Give them some addition/subtraction problems, with like and unlike denominators. Use the fraction bars to develop number sense. Keep saying “putting together” for addition; “taking apart” or “finding the difference” for subtraction. Walk around to make sure they are getting the correct answers, if wrong, “Can you show me how you got this answer?”

2.) Pre-formal - Generalizing: Like Denominators

Try these using your fraction bars……. (Make up some with common denominators so that students can count, and start to look for patterns.)

\[ \frac{1}{4} + \frac{1}{4} = \text{ (Show me this with your bars)} \]

\[ \frac{1}{8} + \frac{3}{8} = \]

Put some more problems up on the board with common denominators (addition and subtraction.)
   - What do you notice about these problems and the answers? Can I find these answers without using the fraction bars?
   - Let’s try one: \[ \frac{2}{5} + \frac{1}{5} = \text{ Write down what you think is the answer. (Walk around to see how they are doing.) Why do you think that? How come the answer is NOT \frac{3}{10}? Why?} \]
   - Let’s get out the fraction bars (circles) and prove this answer. (Put them together to show addition, “Can we count them? How come? (Same size piece.) How do you know? (Same denominator – tells the size of the piece) What does the 5 tell you? (It takes 5 pieces to make one whole)

3.) Formal:
   - Can we make a rule about adding and subtracting fractions with common denominators? (Let’s write the definition for common denominators)
   - Write the rule, and try these examples. If you need to use your fraction bars/circles to help you understand or check your work.

Across the strand connection: Use the clock. (It is a natural for part/whole – 12 pieces for each hour. I worked 2 hours, then 3 hours. What part of the whole 12 hours did I work? What part of the whole day did I work. Write your answer as a fraction.
Adding /Subtracting Fractions – unlike denominators

1.) Informal – Real World and Concrete Modeling:
   - You told me that addition was putting things together. Show me ½ and ¼ using paper. Ok, if this is my problem
     - ½ + ¼ = I add by putting them together. What is my answer? Numerator? Denominator? (If they say 2/6 – go back to the 1/6 pieces, get out 2, put together. Then compare with the pile of ½ and ¼. )
     - We know the denominator is not 1/6, what is the problem with ½ and ¼ when I put it together? (Not the same size piece.) Oh, so before we put them together, RENAME them to get them the same size piece?
     - What can we do with the ½ and ¼ to get them the same size piece? (If they don’t get the connection with equivalent fractions, take them back to the table.) So I want to remove the ½, cut it into 4 pieces, and now I have 2/4.
     - So now my pieces are the same size, can I add them now? Will they go together? What is my answer? Why? What do those numbers mean?

2.) Pre – formal - Generalize
   Let’s look at another problem.

   1/3 + 1/6 = Can we do this? Show me what you have to do first before you put the pieces together.

   This is what it looks like using symbols.

   \[ \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2} \]

   Walk around and do more problems, until you think they “get it”.

3.) Formal - Let’s talk about the steps we did. Let’s write them in our notes.
   (Write the steps, use proper vocabulary (but put in their words, too), use a model right next to it.)

   Then give them some more problems to practice.

   Math across the grade levels. Put up an addition problem. Have students work the problem. Then use a RULER to demonstrate the problem. Put each fraction length end to end. Then measure the TOTAL distance of the line to prove the correct answer.
Addition of Fractions and Mixed Numbers - Renaming:

1.) Informal – Real World and Concrete Modeling (Addition) GROUPS – Pose questions to the group to discuss then randomly call on students for their thinking.

You may want to start with $\frac{1}{2} + \frac{3}{4}$ first to get to the simplifying the answer, using simple fractions.

This lesson starts with mixed #

Using scrap paper (each 8 x 11 sheet represents one whole). Have students work in groups because this model takes up some room.

- Here is another addition problem. Use the paper to represent this problem;
  - Last week at the pizza party, there was 1 $\frac{1}{2}$ pepperoni pizza left over and 2 $\frac{3}{4}$ ham pizzas. What is the total amount of left over pizzas. Write this problem on your paper, using symbols. $1 \frac{1}{2} + 2 \frac{3}{4}$
  - You already told me that addition means putting things together. Can you put these papers together? (You need all the pieces the same size.)
  - What can you do to get the pieces all the same size? (Take the $\frac{1}{2}$ piece, cut it in to $\frac{2}{4}$)
  - Now the pieces are the same size, put all the pieces together, ADD.
  - What do you see when you ADD? (You have 3 whole pieces and five $\frac{1}{4}$ size pieces.)
  - Can you rewrite the answer so it is in a simpler form? (You can put four $\frac{1}{4}$ pieces together to make one more whole; now you have 4 wholes and $\frac{1}{4}$ pieces.

2.) Pre-formal (Adding and renaming)

Ok, now let’s do another problem. $2 \frac{2}{3} + 1 \frac{1}{2} =$ Work in your group and talk about this problem. See if you can find the answer using the model.

- What did you discover this time? What did you do to find the common denominator (get the pieces the same size.)
- Can the strategies you used to find the common denominator (get the same size pieces.) be used with mixed numbers?
- How did you rename your answer to simplify it? (I knew that six $\frac{1}{6}$ size pieces make one whole, so I rewrote my answer to 4 $\frac{1}{6}$

3.) Formal (procedure/algorithm/vocabulary) Students create a table to organize their thinking.

After table, let students work in pairs. Each child finishing the problem by him/herself; then checking with the partner to make sure he/she got the same answer. If there the partners have different answers, then they discuss the problem until a consensus is reached. (Walk around and listen and watch students work to gather information on each child’s progress. If they have something wrong; ask the child to explain their work. Reference the notes you created OR refer back to the informal modeling.

Completed Table with teacher questions, student responses, and notes. This table should be in the students’ notes to be used as a reference, and explanation to parents. Also, the next page can be copied onto a transparency and used on the overhead.
### Activity

1.) Given a real world problem:

Bill found 2 pieces of wood. He measured the wood. One piece was 2 and 2/3 feet and the other piece was 1 ½ feet. He wanted to know the total length of the wood.

2.) Used paper to represent the pizzas. (one 8 x 11 in piece represents one pizza.)

3.) Noticed that the pieces were NOT the same size so I could not put them together (add.)

   Had to cut the ½ piece into 3 pieces (3/6)
   Had to cut the 2/3 piece in ½ (4/6)

4.) Now that all the pieces are the same size, I can put the pieces together.

5.) I want to simplify the answer. I can take 6 of the pieces and make one more whole.

Then add the one more whole to the 3 whole, final answer is:

\[ 4 \frac{1}{6} \]

### Communicating using Numbers

(Vocabulary in italics)

1.) Wrote the problem as an equation.

\[
\begin{align*}
2 \frac{2}{3} & \quad + \quad 1 \frac{1}{2} \\
2 \frac{2}{3} \quad = \quad 2 \frac{4}{6} \\
+ \quad 1 \frac{1}{2} \quad = \quad 1 \frac{3}{6} \\
\hline
3 \frac{7}{6} & \quad = \quad 4 \frac{1}{6}
\end{align*}
\]

- What do you notice? (Pieces are not the same size.)
- What do we have to do? (Get them the same size)
- What is that called? (Common denominator)

- You cut the paper to make it the same size. What do you do mathematically to get the common denominator?
- How did you get your answer?
- Can you simplify this answer?

- I noticed that I do not have a **common denominator**, the pieces are not the same size.

- 2/3 x 2/2 = 4/6
  ½ x 3/3 =3/6

- 1.) Two wholes + 1 whole makes 3 wholes.
  2.) 4/6 + 3/6 = 7/6
  (The 7 in the **numerator** stands for how many pieces. The 6 in the **denominator** is the size of the piece.)

- 7/6 = 1 1/6
  3 +1 1/6 = 4 1/6

- 2 2/3= 2 4/6
  + 1 ½ = 1 3/6
  \[ 3 \frac{7}{6} = 4 \frac{1}{6} \]
Subtracting (Uncommon Denominators with RENAMING)

1.) Informal – (Real World and Concrete Modeling)  GROUPS – Pose the questions to the groups to discuss first.
Back to pizzas. After the class party there was 2 ¼ pizzas left. Mrs. Clark took the remaining pizza to the teacher’s lounge to share. The teachers and the school volunteer parents ate 1 ¾ of the left over pizza. Use the paper to represent the pizza that Mrs. Clark took down to the teacher’s lounge. (One piece of 8 x 11 paper represents 1 whole pizza.)

- The teachers and volunteer parents ate 1 ¾ of the pizza that was on the table. What type of math is that? (Subtraction because it is “take away” or we are finding the difference between the pizza that was on the table, and the pizza left over after 1 ¾ of it was eaten.)
- So if the operation is subtraction, then take away the 1 ¾. What is the problem with that? (I can’t take away three ¼ size pieces. There is only one ¼ size piece.)
- What do you propose we do? (If they don’t know,)“Remember when we worked on equivalent fractions? We cut the pieces to get the all the same size. Can we do that here?” (Yes, we can take the 1 whole pizza, and cut it into four ¼ size pieces. Now we have the four ¼ size pieces from the whole and the ¼ that was already there, so we have 5/4)
- So, you cut up one of the whole pizzas. So what is on the table, now? (There is 1 whole pizza and five ¼ size pieces.)
- The teachers ate 1 and ¾ pizzas. Can you subtract and take away now? (Yes, there is 2/4 pieces left.)

2.) Pre – formal  (Generalizing)

This diagram is NOT to scale:

Kurt’s House 2 ¼ miles

Eric’s House 1 ½ miles

How much further does Kurt live from school? Use the fraction bars to represent the miles. (Work in groups to have enough bars)

- Can you find the difference? What type of math is that? How can you model subtraction? (Take 1 ½ miles away.)
- Can you model this problem the way the bars appear on the table? Do you need to set up the bars differently? How? (Get the same size pieces) What do we call that? (Common Denominator) If they tell you that you have to rename the 2 ¼ miles because you don’t have enough pieces, ask “What is the size of the piece?”
- How do you cut the pieces to get them all the same size. (The 2 ¼ can stay the way it is, just cut up the ½ into 2/4.)
- Now we have 2 ¼ and 1 2/4 can you find the difference? (You have to cut up one of the miles in the 2 ¼ so that you have enough pieces to take away two ¼ size piece.)
- So show me how to do that? (Take one of the whole fraction bars and replace it with 4/4.)
- You renamed the distance that Kurt lives from school. What is it now? (1 and 5/4)
- Can you find the difference now?

3.) Formal: Use symbols and vocabulary

Make another table like the one previously for addition. The table can show the steps and the reasoning behind the steps, by tying the real world experience to the symbol manipulation.

Vocabulary: numerator, denominator, common denominator, renaming, simplifying.