Fascinatin’ Factors and Fractions: Sketchpad In Grades 3–6

2009 NCTM Annual Meeting

Session 221             Thursday, April 23  1:00pm

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Summary

Animate your elementary school classroom with activities covering symmetry, animation, factors, fractions, decimals, and more. Build some from scratch; use prepared sketches in others. Attendees will receive teacher notes, student worksheets, and sketches for six activities. Bring a laptop with battery power.

Objectives

- Present new and exciting representations of important elementary school topics
- Show how students and teachers can manipulate these representations to bring them to life
- Address a wide range of 3-6 topics that benefit from such activities
- Describe and model instructional strategies for using them effectively
- Give you the opportunity to work on the activities yourselves
- Send you home with six ready-to-use activities for their classrooms

Activities

Exploration: You’ll experiment with Sketchpad’s tools and use a compass and straightedge to construct several triangles.

Mystery Number: Multiples and Factors. Students use deductive reasoning and their knowledge of multiples and factors to piece together clues and determine the identity of a mystery number.

Zooming Decimals: Precision and Place Value. Students reason about decimals and place value as they name with increasing precision the location of a point on the number line.

Jeff’s Garden: Area Model of Fraction Multiplication. Students use an interactive area model to visualize and make sense of multiplication of fractions. Students come to understand that the product of two fractions, each less than one, is less than either factor.

Participant’s Choice:
- Balloon Flight: Understanding Decimal Numbers
- Comparing Fractions: Number Sense and Benchmarks
- Circle Graphs: Representing Data
- Making a Kaleidoscope: Exploring Rotations

Permission

These activities come from Sketchpad LessonLink. A 30-day preview lets you view all 500 activities and get access to 100 sample activities. Go to http://www.keypress.com/, locate the LessonLink information, and click Register for Preview. The student web page for these activities is http://www.keymath.com/classpass/2009nctm3to6.
Research-Based Instructional Strategies

There are a number of research-based strategies that have been shown to increase student engagement with and understanding of the subject matter, in mathematics and in other subject areas. A few of them are summarized below. (Each strategy lists a source from the Bibliography. Consult these sources for suggestions about implementing the strategy and for information about the research that supports the strategy.)

- **Wait Time**: After you ask a question, give students plenty of time to understand it, consider it, and formulate a response. Allow a minimum of 3 to 5 seconds, or more for complex questions. (Springer & Dick)

- **Revoicing**: Repeat, summarize, or rephrase student contributions to a discussion to focus attention on what the student has said and to encourage further discussion. (Springer & Dick)

- **Collective Reflection**: Ask students to describe the problem-solving process in which they have engaged—what have they learned and how have they learned it? (Springer & Dick)

- **Identifying Similarities and Differences**: Ask students to describe similarities and differences between two different ways of solving the same problem or between two ways of representing the same mathematical concept. (Marzano)

- **Summarizing and Note Taking**: Have students summarize their findings at the end of an activity, preferably through both class discussion and written notes and answers. (Marzano)

- **Reinforcing Effort and Providing Recognition**: Look for opportunities to encourage student effort and point out the connection between effort and achievement. (Marzano)

- **Multiple Representations**: Expose students to a variety of representations of important mathematical concepts. Marzano emphasizes that some of the representations should be nonlinguistic—Sketchpad activities excel at making graphical representations accessible. Often students can recall a Sketchpad image to remind themselves of important concepts and methods. (Marzano)

- **Cooperative Learning**: Have students work in pairs or small groups. Use a variety of groupings, including both short-term and longer-term teams. (Marzano)

- **Generating and Testing Hypotheses**: Explicitly ask students to form and test conjectures, and encourage the process by affirming their efforts to form and express conjectures whether the actual conjectures are right or wrong. (Marzano)

- **Cues and Questions**: Remind students of what they know about a topic at the start of an activity. High-level questions produce deeper learning than recall or recognition questions. (Marzano)

- **Appropriate Feedback**: Provide feedback that’s corrective, timely, and specific to a criterion. The right kind of feedback has a powerful effect on student learning. Feedback that doesn’t depend on the teacher can be particularly effective. (Marzano)

- **Formative Assessment and Self-Assessment**: Use assessment to adapt your teaching to meet student needs. Such formative assessment produces substantial learning gains. Self-assessment helps students understand the purpose of their learning and what they can do to improve. (Black & William)

- **Multiple Solutions**: Take advantage of problems with multiple solutions. Problems with more than one route to a solution capture student interest and inspire mathematical thinking. (Kalman)
Bibliography for Instructional Strategies

The following books and articles can be very useful as you adapt the use of Sketchpad to your classroom and your teaching methods.


  This book lists a number of effective classroom strategies, describes the research that supports them, and is full of practical suggestions for employing them in the classroom.


  This book recommends classifying the mathematical tasks we set for students in terms of their cognitive demand, and provides a classification scheme with commentary and examples. It then provides detailed descriptions of a number of cases (situations in which a teacher sets up and implements a particular task for her middle school mathematics students) and analyzes those cases, looking at various teacher strategies and how they affected the maintenance of cognitive demand and what they implied for student learning.


  This book aims to provide teachers with strategies to help students build algebraic habits of mind. It addresses various broad topics in pre-algebra and algebra, describing obstacles in student thinking to be overcome, strategies for doing so, and lots of annotated examples.


Mystery Number

Name:

Solve each puzzle using as few clues as you can.

EXPLORE

1. Mystery number = _______
   Record the buttons you press and the clues.
   
<table>
<thead>
<tr>
<th>Multiple of</th>
<th>Yes</th>
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   Cross off numbers that can’t be the mystery number.
   Circle numbers that might be.

   1  3  5  7  9  11  13  15  17  19  21  23  25
   2  4  6  8 10  12  14  16  18  20  22  24

2. Mystery number = _______
   
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<th>Multiple of</th>
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   1  3  5  7  9  11  13  15  17  19  21  23  25
   2  4  6  8 10  12  14  16  18  20  22  24
3. Mystery number = ______

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1 3 5 7 9 11 13 15 17 19 21 23 25
2 4 6 8 10 12 14 16 18 20 22 24

4. Mystery number = ______

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<tr>
<th>Multiple of</th>
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1 3 5 7 9 11 13 15 17 19 21 23 25
2 4 6 8 10 12 14 16 18 20 22 24
INTRODUCE

1. Open Mystery Number.gsp. Go to page “Mystery Number.” Distribute the worksheet. Explain that the class’s challenge is to find the computer’s mystery number. The mystery number is a number from 1 through 25. We can ask for clues. Press the Multiple of 2? button. A check mark appears in the Yes column. What did we learn by pressing this button? [The mystery number is a multiple of 2.]

In step 1 of the worksheet, have students enter the button pressed (2) and the clue (Yes) in the chart.

2. Explain that solving the puzzle requires careful reasoning. Our goal is to figure out the mystery number using as few clues as possible. Let’s think about the information we’ve been given and see whether it helps us narrow down our choices. What can you say about the mystery number now that you know it is a multiple of 2? Here are some sample student responses.

   It must be an even number.
   It can’t be an odd number.
   It could be 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, or 24.

   On your worksheets, I’d like you to use the list under the chart. Cross off the numbers that cannot be the mystery number. Students should point out that these are all the odd numbers in the list.

3. Continue working on the puzzle. I’m going to press another button. Press Multiple of 9? A check mark appears under Yes. Ask students to enter this new information on their worksheets. What can you say about the mystery number now? Have students talk with a partner, and then take responses. The mystery number is a multiple of both 2 and 9, so it must be 18. When the class is convinced that the number is 18, demonstrate pressing Show mystery number to check.

4. Draw students’ attention to the Show Sum of the Digits button. Press the button. It displays the sum of the digits in the mystery number—in this case, 1 + 8 = 9. Explain that this button should be pressed for a hint as a last resort. If students have pressed every button in the table and they are not able to determine the mystery number, they should press this button.
5. Solve another puzzle together. Press New Puzzle. Explain that the computer generates a new mystery number from 1 to 25 at random. Ask a volunteer to come to the computer and choose a button to press. Have students record the button and Yes/No clue in the chart in worksheet step 2.

6. Facilitate a discussion of the clue by asking questions such those below. Use the list of the numbers 1–25 on the board and have students use the list on the worksheet to keep track of numbers that have been eliminated and numbers that remain.

   Are there any numbers that cannot be the mystery number? How do you know?

   What numbers might be the mystery number? How do you know?

   Now that you have this clue, do you know the answers to any of the other questions (buttons)?

7. Ask volunteers to press a button for another clue and for any other clues the class may need in order to find the mystery number. As students think about the information they are obtaining, many different mathematical conversations can develop. Here are some scenarios that suggest the reasoning students may use.

   • Student presses Multiple of 8? The answer is No.

     Since the mystery number is not a multiple of 8, it can’t be 16 or 24. But it’s possible that the mystery number is a multiple of 2 or 4 (both are factors of 8).

   • Student presses Multiple of 10? The answer is Yes.

     Every number that is a multiple of 10 is also a multiple of 2 and of 5. The mystery number must be a multiple of both 2 and 5. So we don’t need to check those buttons. Since the number is a multiple of 10, it could be either 10 or 20. To check whether the number is 20, we could press Multiple of 4.
• Student presses *Multiple of 7?* The answer is *Yes.*

_The possibilities are 7, 14, and 21. We can check to see whether the number is a multiple of 2 or a multiple of 3. If it’s a multiple of 3, it has to be 21. If it’s a multiple of 2, it has to be 14. If it’s not a multiple of 2 or of 3, it has to be 7._

• All buttons are pressed and the answers are all *No.*

_The numbers that have buttons (2 through 12) can be crossed out. The numbers 14, 15, 16, 18, 20, 21, 22, 24, and 25 are all multiples of numbers that are less than or equal to 12, so they can be crossed out. That leaves the prime numbers 13, 17, 19, and 23 as well as 1. The only way to tell which of these numbers is the mystery number is to press Show Sum of the Digits._

• The mystery number is a multiple of 5, but no other number.

_The number could either be 5 or 25. There is no way to tell which it is without pressing Show Sum of the Digits._

8. If time allows, solve more puzzles. Each puzzle will expose students to new and interesting properties of multiples and factors.

### SUMMARIZE

9. Facilitate discussion of the strategies students used to find the mystery numbers. Students can choose examples from their worksheet and describe, step-by-step, the reasoning that helped them to deduce the mystery number.

10. _To find the mystery numbers, you had to do a lot of thinking about multiples that numbers have in common. I have two questions for you._ Facilitate as the class discusses each question.

   _Why is a multiple of 10 also a multiple of 2 and 5?_

   _If you know that a number is not a multiple of 10, do you know that it is not a multiple of 2 and 5?_

11. Pose one or more of the problems that follow and have students work in pairs and then share solutions with the class. Alternatively, have students write individually in response to one or more problems. Students will notice that now the mystery number can be a number as large as 30.
• Ann is thinking of a number between 1 and 30. She says the number is a multiple of 8 and a multiple of 3. What is the number? Explain. [24, the only multiple of 8 and 3 that is between 1 and 30]

• Hector is thinking of a number between 1 and 30. He says it is a multiple of 12. You want to know whether the number he is thinking of is 24. What one question about multiples can you ask him to find out? Explain your thinking. [Is the number a multiple of 8? If it is, then the number is 24 and not 12.]

• Sara is thinking of a number between 1 and 30. It has no factors between 2 and 15. What number could she be thinking of? Is there only one possibility? [Sara could be thinking of any prime number between 15 and 30. These are 17, 19, 23, and 29. She could also be thinking of the number 1.]

• Daniel is thinking of a number between 1 and 30. He tells you that it has more factors than any other number in the puzzle. What number is she thinking of? Explain how you know. [The number 24 has more factors than any other number between 1 and 30. Its factors are 1, 2, 3, 4, 6, 8, 12, and 24.]

EXTEND

1. Provide an opportunity for students to solve puzzles in pairs or individually. Students can create puzzles for others to solve using page “Make Y our Own.” If some students would like to play for points, suggest that they give themselves 1 point for each clue they use to find the mystery number. A low score for solving the puzzles is a good score.

2. What questions would you like to pose about mystery-number puzzles? Encourage student inquiry. Some mathematical questions of interest include the ones here.

• What would happen if we showed both the mystery number and all the checkmarks and then dragged the slider? What would we see?

• What numbers in the 1–25 range require the fewest clues to find? Which numbers require the most clues?
What if the computer chose numbers up to 50? Would we need questions about other multiples to be able to identify each number?

Are there other kinds of questions that might allow us to figure out the numbers with fewer clues?

ANSWERS

1–4. Answers will vary because the computer randomly generates mystery numbers and because students’ solutions will vary.
Estimate the location of a point on a number line as you zoom in.

EXPLORE

1. First _________ 2. First _________
   Second _________  Second _________
   Third _________  Third _________
   Fourth _________ Fourth _________
   Fifth _________  Fifth _________

3. First _________ 4. First _________
   Second _________ Second _________
   Third _________  Third _________
   Fourth _________ Fourth _________
   Fifth _________  Fifth _________

5. First _________ 6. First _________
   Second _________ Second _________
   Third _________  Third _________
   Fourth _________ Fourth _________
   Fifth _________  Fifth _________
INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 20 minutes.

1. Open Zooming Decimals.gsp. Go to page “Model 1.” Distribute the worksheet.

2. Ask students to describe what they see. The model displays a number line labeled from 0 to 10. A red point sits on the line. Drag the point to show that it can move anywhere on the number line. Then choose Edit | Undo one or more times to return the point to its original position.

3. **What can you say about the location of the red point?** Give students time to record their answers alongside “First” on the worksheet. Take responses and record them on the board. Sample responses follow.

   *The point is between 6 and 7.*

   *The point is around $6\frac{1}{2}$. *

   *The point is around 6.5.*

   *The point is between 6.5 and 6.75; it’s more than halfway to 7, but it’s not three-quarters of the way.*

4. **The point sits somewhere between 6 and 7. How do you think we can find its location more precisely?** Take responses. Students may suggest dividing the interval between 6 and 7 into more parts.

5. **Let’s take a closer look at what’s happening between 6 and 7.** Press the first Zoom button. A number line will appear directly below the 6–7 interval and slowly expand, as if “zooming in” on this interval.

6. **How does this number line relate to the one above it?** Elicit the idea that the new number line represents a magnified, or “zoomed,” view of the interval where the point lies, between 6 and 7. The dashed lines connecting the two number lines show which portion of the original number line is shown on the number line below it. Explain that the point sitting on the new number line is “the same” as the one above it:
Both points lie at the same location. This may not be immediately clear to students because the points do not sit one directly below the other.

7. **What do the tick marks that sit between 6 and 7 represent? How far is it from one tick mark to the next?** Now, an interval of one has been divided into ten equal parts, so there is an increase of one-tenth, or 0.1, from tick mark to tick mark. Point to each tick mark between 6 and 7, asking the class to count as you go along: *six and one-tenth, six and two-tenths, . . . .*

8. **What can you say about the location of the point now?** Give students time to record their responses alongside “Second” on the worksheet. Take responses and record them on the board. Sample responses follow.

   Now we can estimate the location more accurately.

   We were right that the point is a little closer to 7 than it is to 6.

   The point is between $6\frac{5}{10}$ and $6\frac{6}{10}$.

   The point is between 6.5 (six point five) and 6.6 (six point six).

   The point is around 6.55.

   Give the class time to discuss estimates of the point’s location.

9. **When we zoom in, we gain precision; we can describe the location of the point more accurately. What do you think we’ll see if we zoom in again, this time on the interval between 6.5 and 6.6?** Take responses. Students may or may not predict that the interval will be divided into ten smaller parts, with each part representing a tenth of a tenth—a hundredth.

10. Press the next Zoom button. The new interval 6.5 to 6.6 is shown. Elicit the idea that again an interval has been divided into ten equal parts, but this time a tenth has been divided, not one whole unit. **What is a tenth of a tenth?** Read the location of each tick mark with the class: *six and fifty-one hundredths, six and fifty-two hundredths, and so on.*
11. Ask students to use this magnified view to make a more precise estimate of the point’s location. Students should write their answers on the worksheet alongside “Third.” Take responses and record them. Here are samples of student thinking.

*The point is closer to 6.5 than 6.6.*

*The point is between six and fifty-three hundredths and six and fifty-four hundredths.*

*The point is between 6.53 (six point five three) and 6.54 (six point five four).*

*The point is very close to 6.54. I’d say it’s probably about 6.539.*

12. If you want to continue to thousandths and ten-thousandths, repeat the sequence of steps two more times. Press the next *Zoom* button, watch the interval expand, discuss what the tick marks represent, and ask students to estimate the location of the point. Students’ final estimate of the point’s location will likely be that it lies between 6.5391 and 6.5392. To view the location of the point, reported to eight decimal places, press *Show Location.*

**DEVELOP**

Continue to project the sketch. Expect to spend about 30 minutes.

13. Have students look at all five estimates on their worksheets. *What is different about the estimates you made using each number line?* Students should explain that each time they viewed a new number line a more detailed scale was shown, allowing them to name the location of the point more precisely.
14. Press *Reset* to hide all but the top 0–10 number line. Drag the point to a new location. Repeat the same steps, having students record their estimates.

15. Press *Show Location*. Ask a volunteer to drag the point that sits on the top number line. Students will observe that all five points move simultaneously, because each point represents the same location.

16. To make the movement steadier, press *Animate Point*. **Look at how all five points are moving. What do you notice?** Here are some sample responses.

   The points all move at different speeds.

   The point on the first number line moves the slowest. The point on the fifth number line moves the fastest.

   Every time the point on the fourth number line moves all the way across, the point on the third number line moves one tick mark to the right. That’s because the fourth number line is divided into thousandths and the third number line is divided into hundredths. Every time the point has gone 10 thousandths, it has gone a hundredth.

   Every time the point on the fifth number line moves all the way across, the point on the fourth number line moves one tick mark. That’s because the fifth number line is divided into ten-thousandths and the fourth number line is divided into thousandths. There are 10 ten-thousandths in a thousandth.

**SUMMARIZE**

17. Present problems such as the following. Take responses and have students write each response using decimal notation.

   **A point is closer to 3.6 than it is to 3.7. What are some possible locations of the point?** Here are two sample responses.

   Three and sixty-four hundredths. [3.64]

   Three and six hundred twenty-five thousandths. [3.625]

   **Name a point that is closer to 5.0 than it is to 5.1.**

   **Name a point that is closer to 99.9 than it is to 100.1.**

   **Name a point that is closer to 30.45 than it is to 30.47.**
18. **Suppose I name two decimals for you. Do you think it’s always possible to name another decimal that lies somewhere between them?** Students’ experience with magnifying the number line and viewing ever-finer scales may lead them to believe (correctly) that it is always possible to do so. Provide an opportunity for them to communicate their reasoning and craft explanations in their own words.

**EXTEND**

The colored diamonds serve as a way to indicate where the hidden point sits along the five number lines.

1. Page “Model 2” contains a related model to explore. Students are given the numerical location of an unseen point and must mark the point’s location along the five progressively scaled number lines. To start, students should press **Units** to reveal a number line scaled from 0 to 10. Students should decide on which unit interval the point is located and drag the colored diamond so that it covers that interval. Continuing in this manner with the remaining four number lines, students should identify the intervals on which the hidden point is located and mark them with the colored diamonds. Pressing **Show Answer** reveals the locations of the point. Pressing **New Problem** generates a new number.

2. On page “Model 1,” spend time identifying the location of points on intervals other than 0–10. To change the endpoints of the top number line, double-click **Left Endpoint** = 0, enter a whole number in the dialog box, and click OK.

3. For students who would benefit from more individualized work, provide opportunities to use the decimal model alone or in pairs.

4. Discuss the concept of calibrated scales in tools people use. Begin by discussing measuring tools like tape measures and thermometers that students are familiar with. Continue by discussing reasons people use estimates to the tenths, hundredths, thousands, and ten-thousandths places. Include the idea that the “best” estimate is the one with the level of precision needed in a particular situation: **When you bake a cake, do the directions tell you to bake from 30.5 to 39.5 minutes?**
1. For the next five years, Jeff’s grandmother gives him part of her garden. Jeff uses part of his area to grow pumpkins. What part of the whole garden does Jeff plant in pumpkins each year? Use Jeff’s Garden.gsp to help you complete the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Part of the Whole Garden Jeff Gets</th>
<th>Part of Jeff’s Area He Plants in Pumpkins</th>
<th>Part of the Whole Garden Jeff Plants in Pumpkins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>( \frac{3}{7} )</td>
<td>( \frac{1}{3} )</td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{4}{5} )</td>
<td></td>
</tr>
<tr>
<td>Year 3</td>
<td>( \frac{7}{10} )</td>
<td>( \frac{3}{4} )</td>
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<tr>
<td>Year 4</td>
<td>( \frac{4}{9} )</td>
<td>( \frac{5}{8} )</td>
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<tr>
<td>Year 5</td>
<td>( \frac{2}{3} )</td>
<td>( \frac{5}{6} )</td>
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2. Another year, Jeff’s grandmother gave him \( \frac{4}{5} \) of the garden. He planted \( \frac{1}{2} \) of it in pumpkins. What part of the whole garden did he plant in pumpkins? _________

EXPLORE MORE

3. One year, Jeff was given \( \frac{2}{3} \) of the garden. He planted a part in pumpkins. His grandmother said, “This year \( \frac{1}{3} \) of the whole garden is planted in pumpkins.” Grandmother did not plant pumpkins. What part of Jeff’s garden was planted in pumpkins? _________
INTRODUCE

1. Before using the sketch, pose the problem below. As you do, record the following for students to reference.

\[
\frac{3}{5} \text{ of the garden is Jeff’s}
\]
\[
\frac{2}{3} \text{ of his part is for pumpkins}
\]

Jeff earns extra money by selling produce he grows in his grandmother’s garden. This year, his grandmother will allow him to use \(\frac{3}{5}\) of her whole garden. Jeff has decided that he will use \(\frac{2}{3}\) of his part to grow pumpkins. What amount of his grandmother’s garden will he use to grow pumpkins?

2. Provide paper and explain that students should use a drawing to show this situation. When you come up with an answer, make sure you can explain why it makes sense. Allow students to grapple with this, working in pairs or groups. As you circulate, ask questions to help students persist in reasoning about the problem. Because students often don’t relate the word “of” to multiplying, don’t expect them to think in terms of multiplying the fractions at this point.

What does it mean to have \(\frac{3}{5}\) of a whole?

What could you show next in your drawing that would help you think about the problem?

Do you think Jeff is using more than, less than, or exactly half of the whole garden for pumpkin growing? Does your drawing make sense, given that idea?

3. Lead a discussion of students’ drawings. Invite students’ questions as well as their attempted representations. There is more than one way to draw this situation. Give students time to consider any different drawings that seem correct.

Becoming Familiar with the Model

4. Open Jeff’s Garden.gsp. Go to page “Area Model.” Let’s see how we can use this Sketchpad model to represent the garden problem. Follow these steps.
• The rectangle represents grandmother’s garden, the whole garden.

• To represent Jeff’s part, we need to show $\frac{3}{5}$ of the whole garden.
Change the denominator of the second fraction to 5. Students will see that the rectangle is now divided into fifths by vertical lines.
Change the numerator to 3. Three of the fifths are now colored. Restate that this is Jeff’s part of the garden, $\frac{3}{5}$ of the whole garden.

![Diagram showing $\frac{1}{1} \times \frac{3}{5}$]

• Now we want to show $\frac{2}{3}$ of Jeff’s part. What would his part of the garden look like divided into thirds? Picture this in your mind. Set the denominator of the first fraction to 3, and then drag the point across the colored fifths only. Students will see that two horizontal lines divide the colored area into thirds.

![Diagram showing $\frac{1}{3} \times \frac{3}{5}$]

• How many thirds of his garden will Jeff plant in pumpkins? [Two]
Let’s show $\frac{2}{3}$ of Jeff’s part. Change the numerator to 2 and press Show Product. Two of the horizontal regions are now colored. One of the thirds (the part not in pumpkins) is not.
What part of grandmother’s garden—the whole garden—will Jeff plant in pumpkins? Give students time to think, and then drag the point the remainder of the way across the rectangle.

Ask again, What part of grandmother’s whole garden will be planted in pumpkins? The discussion should yield these ideas: Grandmother’s garden is now divided into 15 equal parts; six of the parts (the colored region) are the amount Jeff will plant in pumpkins; so, Jeff will use \( \frac{6}{15} \) of his grandmother’s garden for pumpkins.

Does this answer make sense? Solicit thinking such as this student sample: You can see that the garden is divided into fifteenths. Jeff’s garden is 9 parts of the whole and 6 of those parts are for pumpkins. I know 6 is \( \frac{2}{3} \) of 9, so \( \frac{6}{15} \) makes sense for the \( \frac{2}{3} \) of Jeff’s garden.

Press Show Numerical Answer for confirmation of the answer.
5. Provide a moment to reflect. *How does this model compare to the way you thought about the problem and the drawing you made at the start of this activity?* Take some responses, or have students discuss in pairs or small groups.

6. Model using the *Reset* button. Pressing the button removes the horizontal lines, the shaded area for the product, and the numerical answer. Students will need to set the fractions that are multiplied back to $\frac{1}{1}$ by changing the numerators and denominators themselves. Doing so will display an undivided rectangle.

**DEVELOP**

**ACTIVITY NOTES**

7. Assign students to computers and tell them where to find *Jeff’s Garden.gsp*. Distribute the worksheet. Explain that students should work on steps 1 and 2. *Using the model, make sense of the problems. Record your answers. Do the Explore More if you have time.*

8. As you circulate, listen for ways students are making sense of fraction multiplication. Here are some things to notice.

- If you need to help some students to reason as they work on Year 1 in the table, pose questions such as these: *Do you need to start with $\frac{3}{7}$? Suppose you started by thinking about $\frac{3}{7}$? What would $\frac{1}{3}$ of $\frac{1}{7}$ look like?*

- Notice any students who are making conjectures about other ways of solving fraction multiplication problems. If some students posit that the numerator in the product can be found by multiplying the numerators of the factors (and the same for the denominator in the product), ask, *Can you explain why that should work? Do you think it will work for every problem of this type?* Don’t confirm for students at this point that this method will always work.
• Some pairs may devise shortcuts for using the model. If you observe shortcuts being used, ask students to explain what they are doing and how they are thinking. If students are making sense of what they are doing, they should continue. For example, students may change both numerators and denominators to match the values in a problem before they drag the point and press Show Product. Or students may manipulate the model as demonstrated, but omit the last step of extending the horizontal lines across the whole garden. Most likely, they are able to mentally extend the lines and determine the pieces the whole rectangle is divided into.

SUMMARIZE

Project the sketch for viewing by the class. Expect to spend about 30 minutes.

9. Students should have their worksheets. Lead a discussion to explore the idea that multiplication by a fraction less than one results in a product smaller than the number being multiplied. (This is true for multiplying a whole number by a fraction and for multiplying a fraction by a fraction.) Begin by asking the following questions.

You started with $\frac{3}{5}$ and multiplied it by $\frac{2}{3}$. Was the product (the amount planted in pumpkins) bigger or smaller than the number you started with, $\frac{3}{5}$?

You multiplied and got a product smaller than the number you started with. Does that make sense?

From their experiences with whole numbers, students assume that multiplication results in a product larger than the factors. Provide time for the class to grapple with this in order to develop a conceptual foundation that makes sense of fraction multiplication. Invite students to model at the computer as the class considers this issue.

The discussion should bring out the idea that multiplying by a fraction involves taking a part of what you started with. Multiplying a fraction by a fraction involves taking a part of a part. For example, $\frac{1}{2} \times \frac{1}{3}$ means taking $\frac{1}{2}$ of $\frac{1}{3}$. You end up with less than you started with.

10. Discuss the problem in worksheet step 2. Note whether any students have related this problem to Jeff’s garden in Year 2 (in worksheet step 1). In Year 2, Jeff plants $\frac{4}{5}$ of half the garden in pumpkins. In step 2, he plants half of $\frac{4}{5}$ of the garden in pumpkins. The answer is the
same in both cases, \( \frac{2}{5} \). This suggests that multiplication of fractions is commutative. If students don’t propose this idea, introduce it. Provide time for the class to explore one or two other problem pairs that are easy to visualize. **What is \( \frac{1}{2} \) of \( \frac{1}{4} \)? What is \( \frac{1}{4} \) of \( \frac{1}{2} \)?**

11. Present the following problem, which gives large values for the denominators. **Another year, Jeff’s grandmother was not well. She gave him \( \frac{7}{8} \) of the garden and kept a small strip for herself. Jeff planted \( \frac{3}{15} \) of his area in green beans. What part of the whole garden did he plant in beans?**

The class is likely to anticipate that the model will display more parts in the solution than students wish to count. The intention here is to prompt them to look for strategies other than counting all the parts, if they haven’t already. Provide a few minutes for students to work on the problem, recording on the back of their worksheets.

Ask students to share how they solved the problem. You may wish to say, **I saw some of you using shortcuts.** Call on students whom you noted using shortcuts, or ask for volunteers to model at the computer.

Some students are likely to have noticed that for all the garden problems, the product can be found by multiplying the numerators and multiplying the denominators of the factors. If this isn’t suggested, ask, **How are the numerators and denominators represented in the model?**

Set the model for \( \frac{2}{3} \times \frac{3}{5} \), the problem you first modeled. Provide ample time for students to compare the symbolic representation with the area model.

![Area Model of Fraction Multiplication](image)

Invite explanations such as these student examples.

**The colored area shows the numerators, \( 2 \times 3 = 6 \), and the whole rectangle shows the denominators, \( 3 \times 5 = 15 \).**
The numerators are represented by two rows of three, and the denominators are represented by the three rows of five.

12. Discuss the Explore More problem, worksheet step 3. This is a working backward problem. Students know the part of the entire garden planted in pumpkins by Jeff and are asked to determine the part of Jeff’s area that is planted in pumpkins.

EXTEND

1. Present problems with missing factors. Have students use the Sketchpad model to determine and/or check their answers.

2. Develop in students the habit of using computational estimation to anticipate and check computations for reasonableness. Present computations like these, and ask students to estimate whether the product in each computation will be greater or less than 1/2. Ask students to explain their reasoning.

   \[
   \frac{5}{6} \times \frac{1}{3}, \quad \frac{4}{5} \times \frac{7}{8}, \quad 4 \times \frac{8}{9}
   \]

3. Have students write another problem like the Explore More. Students should exchange problems and solve. You might use this as an individual assessment.

ANSWERS

1. 

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
<th>Year 4</th>
<th>Year 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{3}{21})</td>
<td>(\frac{4}{10})</td>
<td>(\frac{21}{40})</td>
<td>(\frac{20}{72})</td>
<td>(\frac{10}{18})</td>
</tr>
</tbody>
</table>

2. \(\frac{4}{10}\), or \(\frac{2}{5}\)

3. \(\frac{1}{2}\)
Compare the Fractions

Find handy ways to compare fractions.

EXPLORE

1. Compare the fractions in each pair. Use the symbols $>$, $<$, or $=$ to show how they compare.
   a. $\frac{2}{9}$ $\frac{5}{9}$
   b. $\frac{4}{7}$ $\frac{4}{11}$
   c. $\frac{2}{4}$ $\frac{5}{10}$
   d. $\frac{9}{16}$ $\frac{4}{9}$
   e. $\frac{2}{21}$ $\frac{7}{8}$
   f. $\frac{3}{4}$ $\frac{5}{6}$
   g. $\frac{8}{8}$ $\frac{9}{13}$

2. For each pair of fractions, think about which strategy helps you compare the fractions quickly and easily.
   a. $\frac{6}{11}$ $\frac{24}{50}$
   b. $\frac{8}{9}$ $\frac{5}{6}$
   c. $\frac{3}{12}$ $\frac{8}{12}$
   d. $\frac{30}{60}$ $\frac{7}{14}$
   e. $\frac{40}{50}$ $\frac{4}{4}$
   f. $\frac{7}{17}$ $\frac{7}{100}$
   g. $\frac{11}{12}$ $\frac{2}{15}$
In this activity you will make a circle graph using Sketchpad.

**CONSTRUCT**

These data from the U.S. Census Bureau show the age of the U.S. population in 2000.

<table>
<thead>
<tr>
<th>Category</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18 years old</td>
<td>26</td>
</tr>
<tr>
<td>18–44 years old</td>
<td>40</td>
</tr>
<tr>
<td>45–64 years old</td>
<td>22</td>
</tr>
<tr>
<td>65 years old and older</td>
<td>12</td>
</tr>
</tbody>
</table>

1. Open **Circle Graphs.gsp**. Go to page “Circle Graph.”

2. Construct a large circle.

3. Draw two radii to mark part of the circle. Make sure each radius is connected to the center and to the circumference.

4. The space between the radii will represent “Under 18 years old.”

   Drag the endpoints of a radius so the space looks about the right size.

5. Now you will measure the angle formed by the radii.

   Select the three points of the angle, with the vertex (the center of the circle) second.

   Choose **Measure**| **Angle**.
6. Use the Calculator to compute the percentage of the circle occupied by the space between the radii. Choose Measure | Calculate. Don’t type measurements into the Calculator. Do click on the sketch to enter an angle measurement and to enter 360° into your calculation. Use the calculator keyboard for ÷, *, (multiply), and 100.

\[
\frac{m\angle ABC}{Degrees\ in\ circle} \cdot 100 = 26
\]

7. Drag a point on the circumference until the percentage calculation matches the percentage for “Under 18 years old.”

8. Now you will label this part of the graph. Type the text. Drag the textbox to place it on the graph.

9. Make the other parts of your circle graph. Follow steps 3–8 for each part of the graph.

10. Clean up your graph. Some things are not needed now. Keep the measurements. To hide the labels of points, click on the points with the Text tool. To hide measurements, select them and choose Display | Hide.

11. The calculations verify you have the correct percentages. Add a key with labels to show the category each measurement represents. Add a title.
EXPLORE MORE

12. Add color to your circle graph. Follow these steps for each part of the graph.

   - Press and hold the **Custom** tool icon.
   - Choose the **Color the Part** tool from the menu.
   - Move your pointer over the sketch and click, in order, the center of the circle and then the endpoint of a radius. Move counter-clockwise to the next endpoint and click again.
   - This will color one part of the circle graph yellow. To change the color, choose **Display|Color**.

13. Make a prediction: How will the age of the U.S. population in each category be different ten years from now? Why do you think so? Show your prediction by dragging the radii of the circle.
Making a Kaleidoscope

Make a kaleidoscope by rotating a quadrilateral.

1. Open Making a Kaleidoscope.gsp. Go to page “Kaleidoscope.” You will see three circles, all with center at point C.

2. Construct a point on each circle.

3. Click on points D, E, and F, in that order, to show their labels.

4. Construct a quadrilateral. Select points C, D, E, and F, in order, and choose Construct | Quadrilateral Interior.

5. Follow these steps to rotate the quadrilateral by 90° using point C as the center of rotation:
   - Select point C and choose Transform | Mark Center.
   - Select the quadrilateral interior and choose Transform | Rotate.
   - In the window that pops up, enter 90 for the angle and click Rotate.

6. With the new quadrilateral still selected, rotate it by 90°.

7. Rotate one more time by 90°. You should now have four quadrilaterals.

8. Make the quadrilaterals different colors. Select the interior of each quadrilateral one at a time. Choose Display | Color and pick a new color.

9. Compare the four quadrilaterals. How are they similar? How are they different?

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________
Making a Kaleidoscope

10. Now you will animate your kaleidoscope.
   Select only points D, E, and F. Choose Display | Animate Points.
   Watch what happens.

11. Click the arrow buttons to change the speed of your kaleidoscope. Press the Stop button to stop the motion.

EXPLORE MORE

12. Go to page “Explore More.” Follow steps 2–5 to make a quadrilateral CDEF on the circles.

13. How many shapes will you get if you keep rotating CDEF by 60°? Tell how you know.

_________________________________________________________________
_________________________________________________________________


15. Give two other examples of the number of degrees to rotate a shape and the number of shapes that you will get.

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

16. Go to page “Make Your Own.” Make your own kaleidoscope.
   Choose the number of degrees to rotate the shape. Describe your kaleidoscope.
_________________________________________________________________
_________________________________________________________________
_________________________________________________________________