Mathematics – Grade Level Assessments and Content Expectations

Grade Six - Session 1
Fluency with Multiplications and Division of Fractions

Participant Packet

Developed by Macomb County Teachers under the leadership of Marie Copeland
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Mathematics - Grade Level Assessments and Content Expectations

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### Grade 6: Fluency with Multiplication and Division of Fractions

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<tr>
<th>N.MR.06.01</th>
<th>N.FL.06.02</th>
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<th>N.FL.06.04</th>
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<tbody>
<tr>
<td>Understand division of fractions as the inverse of multiplication, e.g. if $\frac{4}{5} \div \frac{2}{3} = \square$, then $\frac{2}{3} \times \square = \frac{4}{5}$, so $\square = \frac{4}{5} \cdot \frac{3}{2} = \frac{12}{10}$</td>
<td>Given an applied situation involving dividing fractions, write a mathematical statement to represent the situation</td>
<td>Solve for the unknown in equations such as $\frac{1}{4} \div \square = 1$, $\frac{3}{4} \div \square = \frac{1}{4}$ and $\frac{1}{2} = 1 \bullet \square$</td>
<td>Multiply and divide any two fractions, including mixed numbers, fluently</td>
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</table>

### Instructional Sequence:

1. **Concept of inverse operations**
2. **Meaning of multiplication with fractions**
3. **Meaning of division with fractions**
4. **Write mathematical statements to represent applied situations**

### Important Tips:

- It is more difficult for students to acquire conceptual understanding once they have learned rote procedures (Owen, 1993)
- Development of the meaning of multiplication and division of fractional numbers should emerge from experience with genuine problems (Graeber and Campbell, 1993)
- Estimation of computations with fractions helps all fraction computation make sense.
- For fraction computation, use the same ideas developed for whole number computation.
- Students need an experience with a wide variety of model.
- The numerator of a fraction tells how many parts are being considered.
- The denominator of a fraction indicates the kind or size parts the numerator counts.

### Common Misconceptions:

- Students cannot write a problem that models division or multiplication with fractions.
- Students mistake division of $\frac{1}{2}$ with division by 2.
- Students mistake division of $\frac{1}{2}$ with multiplication by $\frac{1}{2}$.
- Students do not realize they can find a common denominator and divide.
- Students are unable to explain their reasoning.
- Students confuse the use of $\times$ as a multiplication symbol and $x$ as a variable.
- Students don’t understand the use of $\bullet$ as a multiplication symbol.
Models used for division:

**Measurement:**  $8 \text{ ft} \div \frac{1}{2} \text{ ft} = 16$

How many $\frac{1}{2}$ ft lengths are there in something that is 8 ft long?

**Partitive:**  $8 \text{ ft} \div \frac{1}{2} = 16$ ft

If half a length is 8, how long is the whole?

**Product & Factors:**  $8 \text{ sq ft} \div \frac{1}{2} \text{ ft} = 16$ ft

If one side of an 8 square foot rectangle is $\frac{1}{2}$ ft, how long is the other side?

Adapted from: “Knowing and Teaching Elementary Mathematics”, Liping Ma, Lawrence Erlbaum Associates, 1999
Use the fraction strips to model these sums and differences

1. $\frac{3}{4} + \frac{1}{2}$

2. $\frac{7}{8} - \frac{1}{4}$

3. $1\frac{3}{4} - \frac{1}{2}$

4. $2\frac{7}{8} - 1\frac{1}{4}$
5. \[ \frac{2}{2} + \frac{3}{8} \]

7. \[ \frac{1}{4} + \frac{7}{8} \]

6. \[ \frac{5}{8} - \frac{1}{2} \]

8. \[ \frac{1}{4} - \frac{2}{3} \]
Make the one-color rod trains for each rod.

1. \( \frac{1}{2} + \frac{1}{4} \)  
2. \( \frac{7}{8} - \frac{1}{4} \)  
3. \( \frac{1}{2} + \frac{3}{4} \)  
4. \( \frac{1}{2} - \frac{3}{4} \)

1. \( \frac{1}{6} + \frac{1}{3} \)  
2. \( \frac{2}{3} - \frac{1}{2} \)  
3. \( \frac{2}{3} + \frac{5}{6} \)  
4. \( \frac{2}{3} - \frac{5}{6} \)
Write your own problems and model them.

The WHOLE is orange + red

Write your own problems and model them.
Make the one-color trains when orange + yellow make one whole

Write your own problems and model them.
Using the rods to think about fractions

1. If purple is $\frac{2}{3}$ of the whole, what is the whole?

2. $\frac{2}{3}$ of dark green is ______________.

3. If blue is $\frac{3}{4}$ of the whole, what is the whole?

4. $\frac{3}{4}$ of brown is _________________

5. $1 \frac{1}{2}$ of dark green is ________________

6. Blue is $1 \frac{1}{2}$ of _______________
Possible Activities with the Cuisenaire Rods – *The Super Source, ETA Cuisenaire*

**Fraction Game**

Use a dice where

\[ 1 = \frac{1}{2} \]

\[ 2 = \frac{1}{3} \]

\[ 3 = \frac{1}{4} \]

\[ 4 = \frac{1}{5} \]

\[ 5 = \frac{2}{3} \]

\[ 6 = \frac{3}{4} \]

1. This is a game for 2 to 4 players. The object is to collect the most rods.

2. All the rods go into the center

3. Each player rolls the dice to find their fraction and find two rods that represent that fraction.

4. They lose a turn if they cannot find two rods that match the fraction

5. The winner is the player with the most rods.

**Naming Rods**

If you give each rod a value of 1, what are the numerical names for each of the other rods.
Multiplication of Fractions – Area Model

1. \( \frac{1}{3} \times \frac{1}{2} \)
   
   The product of \( \frac{1}{3} \times \frac{1}{2} \) is the same as finding the area of a rectangle whose sides are \( \frac{1}{3} \) and \( \frac{1}{2} \).

2. \( \frac{4}{5} \times \frac{2}{3} \)
   
   The product of \( \frac{4}{5} \times \frac{2}{3} \) is the same as finding the area of a rectangle whose sides are \( \frac{4}{5} \) and \( \frac{2}{3} \).

3. \( 1\frac{1}{4} \times \frac{1}{6} \)
4. \(1 \frac{3}{4} \times 1 \frac{1}{6}\)

5. \(1 \frac{4}{5} \times 2 \frac{2}{3}\)
A Look at Division

Example One

12 ÷ 3

A. If 12 is divided into 3 pieces how big is each piece?

B. How many 3’s are there in 12?

Example Two

3 ÷ 12

A. If 3 is divided into 12 pieces, how big is each piece?

B. How many 12’s are there in 3?
Example Three

\[
\frac{1}{2} \div 4
\]

A. If \( \frac{1}{2} \) is divided into four pieces how big is each piece?

B. How many 4’s are there in \( \frac{1}{2} \)?

Example Four

\[
4 \div \frac{2}{3}
\]

A. If 4 is \( \frac{2}{3} \) of the whole, how big is the whole?

B. How many \( \frac{2}{3} \)’s are there in 4?
Example Five

\[
1 \frac{3}{4} \div 2
\]

A. If \(1 \frac{3}{4}\) is 2 of the whole, how big is the whole?

B. How many 2’s are there in \(1 \frac{3}{4}\)?

Example Six

\[
2 \div 1 \frac{1}{2}
\]

A. If 2 is \(1 \frac{1}{2}\) of the whole, how big is the whole?

B. How many \(1 \frac{1}{2}\)’s are there in 2?
Example Seven

\[
1 \frac{3}{4} \div \frac{1}{2}
\]

A. If \(1 \frac{3}{4}\) is \(\frac{1}{2}\) of the whole, how big is the whole?

B. How many \(\frac{1}{2}\) 's are there in \(1 \frac{3}{4}\)?

Example Eight

\[
1 \frac{1}{8} \div 1 \frac{1}{2}
\]

A. If \(1 \frac{1}{8}\) is \(1 \frac{1}{2}\) of the whole, how big is the whole?

B. How many \(\frac{1}{2}\) 's are there in \(1 \frac{1}{8}\)?
### Grade 6: Fluency with Multiplication and Division of Fractions

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<th>Materials</th>
<th>Key Tips</th>
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</table>
| Add Fractions with Fraction Strips | • Make a strip for each of the following: the unit, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ using a different color for each representation                                                                                   | • 8 ½ x 1 ½ paper strips in different colors                                                  | • Suggested plan:  
  1. Take one whole strip, label it one whole and put it aside.  
  2. Take one whole, fold it in half. How big is each part? What we did is take $1 \div 2$ and found that each part was $\frac{1}{2}$.  
  3. Repeat for $\frac{1}{4}$ and $\frac{1}{8}$ and keep reinforcing the relationship between division and a fraction.  
  • Now model $\frac{3}{4} + \frac{1}{2}$  
    Reinforce that $1 \frac{1}{4}$ is equivalent to $\frac{5}{4}$. Is it possible to use eighth’s to solve the problem?  
  • Discuss common denominators  
  • Start asking questions leading to multiplication and division of fractions. If I take one whole and divide it by $\frac{1}{2}$, how many $\frac{1}{2}$ ‘s do I have? (2) |
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<tr>
<td>Color trains with Cuisenaire Rods</td>
<td>• Use Cuisenaire rods to model relationships with different pieces being the whole.</td>
<td>• Cuisenaire rods</td>
<td>• Discuss different pieces as the whole</td>
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<td></td>
<td>• Model addition and subtraction of fractions</td>
<td>• Worksheets on Cuisenaire rods p 7 – 9</td>
<td>• Discuss how different colored rods can represent $\frac{1}{2}$, etc.</td>
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<td></td>
<td>• Write situations that model the problems</td>
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<td>• Start developing multiplication and division</td>
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<td>Using the rods to think about fractions</td>
<td>• Have students think of the whole and the related parts by changing the length of the whole</td>
<td>• Cuisenaire rods</td>
<td>• Have students explain their reasoning</td>
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<td></td>
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<td>• Worksheet on using the rods</td>
<td></td>
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<tr>
<td>Fraction Game or Naming Rods</td>
<td>• Reinforcing different wholes and their fractional parts</td>
<td>• Worksheet p 11</td>
<td>• Each student must find a rod that represents the whole and the appropriate fraction of that whole they rolled on the dice.</td>
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<tr>
<td>Multiply fractions with the area model</td>
<td>• Discuss the model thoroughly..</td>
<td>• Piece of origami paper</td>
<td>• Have students fold the square into halves.</td>
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<td></td>
<td>• Have participants write a situation that models the problem\</td>
<td>• Multiplication of Unit Fractions handout p 12-13</td>
<td>• Turn the paper over and rotate it 90 degrees and</td>
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<td>• Relate the model to a binomial expansion and the distributive law</td>
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<td></td>
<td>have the participants fold the square into fourths</td>
<td></td>
<td></td>
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<td></td>
<td>• How many pieces have they made? (8)</td>
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<td></td>
<td>• Are they still squares? (no)</td>
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<td></td>
<td>• What is the area of each? (\frac{1}{8})</td>
<td></td>
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<td></td>
<td>• What is the area of the sum of all of them? (1)</td>
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<td></td>
<td>• How can we model (\frac{1}{2} \cdot \frac{3}{4}) with our origami paper?</td>
<td></td>
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<td>• How can we model (\frac{1}{2} \cdot \frac{3}{4}) with our origami paper?</td>
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<td>• Now use the multiplication of fractions worksheet. Use a different color as you</td>
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<td>divide the square</td>
<td>The product of $\frac{1}{3} \times \frac{1}{2}$ is the same as finding the area of a rectangle whose sides are $\frac{1}{3}$ and $\frac{1}{2}$</td>
<td></td>
<td>divide the square</td>
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<td></td>
<td>We start with a square of area one.</td>
<td></td>
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<td></td>
<td>We divide one square one way into thirds and the other way into halves. How many parts do we have? (6)</td>
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<td></td>
<td>Where is the rectangle that is $\frac{1}{3} \times \frac{1}{2}$ in our model?</td>
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<td></td>
<td>What part of the whole does it represent? ((\frac{1}{6}))</td>
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**Division of Fractions**

**The Grand Finale!**

- Show the Lipsing Ma flow chart on division of fractions and point out we have been making connections with all that we have done to help students with the division of fractions
- Have the participants pick a model for each situation, draw it and discuss their models at their table and with the group
- Lipsing Ma flow chart on division of fractions
- A Look at division handout pp 14 – 17
- Emphasize the importance of asking the questions correctly
- Discuss the alternative models
Meaning of multiplication with fractions

Meaning of division with whole numbers

The conception of inverse operations

Meaning of multiplication with whole numbers

Concept of unit

Concept of fraction

Meaning of addition