Submission to the *Journal of Answers to Gollier*

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Exercise 82

Consider the following nonseparable intertemporal preferences:

$$U(c_1, c_2) = ln(c_1) + \beta ln(c_2 - kc_1).$$

• *Interpret k depending on whether it is positive or negative.*

• *Show that the optimal consumptions c_1 and c_2 are respectively decreasing and increasing with k.*

Solution 82

Part 1: If k is positive, then consumption is habit-forming, and k represents the degree of habit formation. This means that, in the second period you care about the *growth* in consumption. Intuitively, this would cause a consumer to consume less in the first-period, since they suffer a penalty from that consumption in the second-period.

If k is negative, then consumption is durable, and k represents the degree of durability. This means that you enjoy a boost in the second-period from the first-period consumption. Intuitively, in the two-period model, this will cause an increase in first-period consumption since you enjoy part of that consumption again in the second period.

Part 2: The solution requires explicating a budget constraint. Let $c_1 + c_2 \leq \overline{C}$. It is clear that the constraint will bind. Substituting $c_2 = \overline{C} - c_1$, we can consider the unconstrained problem:

$$\max_{c_1} ln(c_1) + \beta ln(\overline{C} - c_1 - kc_1).$$

This yields a basic first-order condition in terms of c_1. We know that this will define a global maximum by the strict concavity of the objective function:

$$FOC : \frac{1}{c_1} + \frac{\beta(1+k)}{\overline{C} - (1+k)c_1} = 0 \;\Rightarrow$$

$$\overline{C} - (1+k)c_1 = -\beta(1+k)c_1 \;\Rightarrow$$

$$c_1 = \frac{\overline{C}}{(1-\beta)(1+k)}.$$

Denote this optimal first-period consumption as c_1^*. We can determine the marginal change in c_1^* from a change in k by simple differentiation:

$$\frac{\partial c_1^*}{\partial k} = \frac{\overline{C}}{1-\beta}\frac{-1}{(1+k)^2} < 0.$$

The sign of this derivative follows directly, assuming that $\beta < 1$. A rise in k causes a *decrease* in first-period consumption. Since the budget constraint will always bind, a rise in k must therefore correspond to an *increase* in second-period consumption.