Answer to Gollier Problem 58

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(Athey 1999) Because the condition Ross-DARA that we obtained in the previous exercise is very restrictive, it may be interesting to relax it by constraining the set of FSD deterioration of the background risk. Let $f(x,t)$ be the density function of background risk, $\tilde{x}_t$. The marginal indirect utility function can therefore be written as $v_t'(z) = \int u'(z+x)f(x,t)dx$. Using the properties of log supermodular functions, prove that a MLR deterioration in the distribution of the background risk raises the degree of risk aversion of v_t if u is DARA in the sense of Arrow-Pratt.

Proof: Suppose u is DARA. Then,

$$
\begin{aligned}
& -\frac{u''(z,x)}{u'(z,x)} \quad \text{is nonincreasing in } x \\
\Leftrightarrow \quad & \frac{u''(z,x)}{u'(z,x)} \quad \text{is nondecreasing in } x \\
\Leftrightarrow \quad & u'(z,x) \ \ \text{is LSPM.}
\end{aligned}
$$

(by Condition 2 in Lemma 2)[1]

Next, we assume that $\tilde{x}_1$ is dominated by $\tilde{x}_2$ in the sense of the MLR order. Then,

$$
\begin{aligned}
& l(t) = \frac{f(1,\ t)}{f(2,\ t)} \text{ is nonincreasing in } t. \\
\Leftrightarrow \quad & \forall x_1, x_2 \in \mathbb{R}, \forall t_H > t_L : (x_2 - x_1)\frac{f(2,t_H)}{f(1,t_H)} \geq (x_2 - x_1)\frac{f(2,t_L)}{f(1,t_L)} \\
\Leftrightarrow \quad & f(x,t) \text{ is LSPM.}
\end{aligned}
$$

(by Condition 1 in Lemma 2)

[1] **Lemma 2** Suppose that $h : R^2 \to R^+$ is differentiable with respect to its first argument. Then h is LSPM if and only if one of the following two equivalent conditions holds:
1. $\forall x, x_0 \in R, \forall \theta_H > \theta_L : (x - x_0)[h(x,\theta_H)/h(x_0,\theta_H)] \geq (x - x_0)[h(x,\theta_L)/h(x_0,\theta_L)]$.
2. $[\partial h(x,\theta)/\partial x]/h(x,\theta)$ is nondecreasing in θ.

Since the product of two LSPM functions is LSPM, $u'(z+x)f(x,t)$, the integrand of $v_t'(z)$, is also LSPM. By Proposition 20[2], $v_t'(z) = \int u'(z+x)f(x,t)dx = Eu'(z+x)$ is LSPM, i.e., $\frac{v''(z,t)}{v'(z,t)}$ is nondecreasing in t . So, we now know that when $l(t)$ is nonincreasing in t, $\frac{v''}{v'}$ is nondecreasing in t.

Therefore, a MLR deterioration in the distribution of the background risk raises the degree of risk aversion of v_t. ■

[2]**Proposition 20** $H(z) = Eh(z,\tilde{\theta})$ is LSPM if h is LSPM.